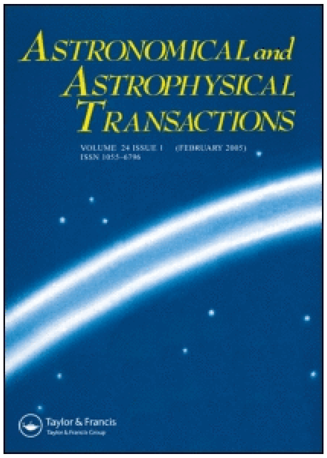

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VELOCITY DISTRIBUTION OF METEOROIDS COLLIDING WITH PLANETS AND SATELLITES. II. NUMERICAL RESULTS

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In the first part of the paper we proposed algorithm for describing velocity distribution of meteoroids colliding with planets and satellites. In the present part we show numerical characteristics of the distribution function. Namely, for each of terrestrial planets and their satellites we consider a swarm of encountering particles of asteroidal origin. They form a field of relative collisional velocities v. We consider momenta μ_k (mathematical expectation of v^k), $k = -1,\ 1,\ 2,\ 3,\ 4$. The data are calculated under two different assumptions: taking into account gravitation of target body or without it. The main results are presented in a series of tables each containing five numbers and several useful functions of them.

KEY WORDS Meteoroids, velocity distribution, collision

1 INTRODUCTION

Collisions in the Solar System play an important role in its history. Consequencies of such events depend essentially on relative velocity of the impactors. In Part I of the present paper (Kholshevnikov and Shor, 1994) we described the method for obtaining the velocity distribution function of meteoroids colliding with planets and satellites. The main idea of the method is as follows. Let us fix a body-target, say the s-th major planet Q_s, and a set of potential projectiles, say minor planets Q. Choose among all the numbered minor planets (Batrakov, 1992) those which have the semi-major axis, a, and eccentricity, e, satisfying the inequalities

$$a(1-e) < a_s(1+e_s), \quad a(1+e) > a_s(1-e_s), \tag{1}$$

where the elements with index s refer to Q_s.

As the lines of nodes and apsides rotate, the orbits of Q and Q_s intersect each other from time to time. The relative velocities at intersection points can be calculated without difficulties. When averaging over all possible intersection points and

Table 1. Velocity distribution characteristics when colliding with Mercury; gravitation is not taken into account; the last row represents λ_k for Maxwell's distribution

k	-1	1	2	3	4
μ_k	0.03588	33.08	1281	56090	2684000
ν_k	27.87	33.08	35.79	38.28	40.48
λ_k	0.8426	1	1.082	1.157	1.224
$\tilde{\lambda}_k$	0.7854	1	1.085	1.162	1.233
		$N = 6$, $\sigma = 13.66$ km/s			

all selected asteroids, we obtain a set of μ_k = mathematical expectation of v^k, v being relative collisional velocity. According to the Carleman theorem (see, e.g., Prokhorov and Rosanov, 1973, § 4.3), the distribution function is uniquely determined by its momenta μ_k. We expect that μ_k for meteoroids of asteroidal nature differ very little from those for asteroids themselves.

More detailed discussion and the description of algorithms one can find in Part I of the present paper. The closed formulae for v and numerical process for averaging over all intersection points are also given there. For μ_2, μ_4 the averaging can be fulfilled also in the analytical form. The results coincide at least to four decimals.

2 MERCURY

For $s = 1$ (Mercury) there are 6 asteroids catalogued in Batrakov (1992) satisfying the inequalities (1).

Table 1 contains five momenta μ_k = mathematical expectation of v^k (first row), v being planetocentric velocity of a collider calculated without taking into account Mercurian gravity. In the second and the third rows, correspondingly, $\nu_k = (\mu_k)^{1/k}$ and $\lambda_k = \nu_k/\nu_1$ are given. The last row represents λ_k for Maxwell's velocity distribution designated $\tilde{\lambda}_k$ (comparing λ_k and $\tilde{\lambda}_k$ one can notice their proximity).

In this paper distances are measured in km, velocities – in km/s. So the first line contains the quantities of different dimensions (km/s)^k; the third and fourth line quantities are dimensionless.

Below Table 1 we give N=number of minor planets – potential colliders and $\sigma = \sqrt{\mu_2 - \mu_1^2}$ = the mean squared deviation of velocity.

We described the collision velocity distribution provided that the planet's gravity is negligible. In reality it is a planetocentric velocity distribution in the meteoroid swarm at the border of the planet's sphere of action. Table 1a contains the same quantities as Table 1 (except Maxwell's momenta) calculated with due regard of planet's attraction. More precisely Table 1a contains characteristics of planetocentric velocity field at a distance R (R being the radius of the planet) from the centre of Mercury. The corresponding correction depends on the parabolic velocity $v_0(R)$ only – see formula (12) in Kholshevnikov and Shor (1994). When calculating, the rotation of the planet was not taken into account. The corresponding correction

Table 1a. Velocity distribution characteristics when colliding with Mercury; gravitation is taken into account

k	-1	1	2	3	4
μ_k	0.03527	33.40	1299	56990	2731000
ν_k	28.35	33.40	36.04	38.48	40.65
λ_k	0.8489	1	1.079	1.152	1.217

$N = 6$, $R = 2439.7$ km, $v_0 = 4.249$ km/s, $\sigma = 13.54$ km/s

Table 1b. Minimal and maximal velocities and length of interval of possible collision for Mercury-crossers; the second and the third columns correspond to the case when gravitation is not taken into account; the fourth and the fifth columns, when gravitation is taken into account

N^*	v_{min}	v_{max}	v^*_{min}	v^*_{max}	$\Omega_2 - \Omega_1$
1566	34.99	54.31	35.25	54.48	180°.
2101	19.03	23.25	19.50	23.63	38.39
2212	17.31	34.19	17.82	34.45	97.49
2340	16.86	17.21	17.38	17.73	11.56
3200	43.65	64.12	43.85	64.26	180.
3838	34.35	35.96	34.62	36.21	32.24

$\Delta v = (v_{\max} - v_{\min})_{\text{mean}} = 10.48$ km/s
$\Delta v^* = (v^*_{\max} - v^*_{\min})_{\text{mean}} = 10.39$ km/s

depends on the velocity direction and place of impact, but it is rather small even for the Earth and Mars.

Now we return to the mean squared deviation σ. Statistical rules demand a factor $b = \sqrt{N/(N-1)}$ when estimating σ from a sample containing N values of a random variable. Since $N = 6$, the factor $b = 1.095$ is important. But we ought to remember that each selected asteroid brings a set of 20–360 points. So in reality we deal with a quantity $N' = 1076$ and the discussed factor may be omitted.

The above numbers 20–360 and 1076 need explanation. If the elements of Q satisfy inequalities more restrictive in comparison with (1),

$$a(1-e) < a_s(1-e_s), \quad a(1+e) > a_s(1+e_s), \tag{2}$$

then a certain position of apsidal line, leading to intersection of the orbits, corresponds to each value of Ω (the longitude of ascending node). We calculated values of v for a set of $\Omega = 0°, 1°, \ldots, 359°$. If the inequalities (1), but not (2) are satisfied, then not all values of Ω lead to intersection. In this case $\Omega \in [\Omega_1, \Omega_2] \cup [-\Omega_2, -\Omega_1]$ and $0° \leq \Omega_1 < \Omega_2 \leq 180°$. If $\Omega_2 - \Omega_1 \geq 10°$, one chooses a 1° step in Ω. If not, one takes exactly 10 steps.

In Table 1b we give the number N^* of the selected minor planet (the first column); minimal and maximal velocities over all possible intersection points when

Table 2. Velocity distribution characteristics when colliding with Venus; gravitation is not taken into account

k	-1	1	2	3	4
μ_k	0.05448	21.02	501.5	13390	393000
ν_k	18.36	21.02	22.39	23.75	25.04
λ_k	0.8734	1	1.065	1.130	1.191
$N = 25$, $\sigma = 7.73$ km/s					

Table 2a. Velocity distribution characteristics when colliding with Venus; gravitation is taken into account

k	-1	1	2	3	4
μ_k	0.04566	23.68	608.8	17000	512200
ν_k	21.90	23.68	24.67	25.71	26.75
λ_k	0.9250	1	1.042	1.086	1.130
$N = 25$, $R = 6051.8$ km, $v_0 = 10.362$ km/s, $\sigma = 6.95$ km/s					

Table 3. Velocity distribution characteristics when colliding with the Earth; gravitation is not taken into account

k	-1	1	2	3	4
μ_k	0.06489	18.15	379.4	8866	225000
ν_k	15.41	18.15	19.48	20.70	21.78
λ_k	0.8490	1	1.073	1.140	1.200
$N = 45$, $\sigma = 7.07$ km/s					

Table 3a. Velocity distribution characteristics when colliding with the Earth; gravitation is taken into account

k	-1	1	2	3	4
μ_k	0.04964	21.65	504.6	12620	335600
ν_k	20.15	21.65	22.46	23.28	24.07
λ_k	0.9306	1	1.038	1.075	1.112
$N = 45$, $R = 6371.0$ km, $v_0 = 11.186$ km/s, $\sigma = 5.99$ km/s					

Table 3b. Minimal and maximal velocities and length of interval of possible collision for Earth-crossers; the second and the third columns correspond to the case when gravitation is not taken into account; the fourth and the fifth columns, when gravitation is taken into account

N^*	v_{min}	v_{max}	v^*_{min}	v^*_{max}	$\Omega_2 - \Omega_1$
1566	29.10	30.36	31.18	32.35	180°
1620	11.19	11.97	15.82	16.39	180
1685	12.59	13.52	16.84	17.55	180
1862	16.43	17.41	19.88	20.69	180
1863	15.57	16.40	19.17	19.85	180
1864	21.64	22.55	24.36	25.17	180
1865	16.06	17.02	19.57	20.36	180
1866	25.14	25.65	27.52	27.99	180
1981	27.56	28.38	29.74	30.51	180
2062	10.69	11.35	15.47	15.94	180
2063	11.19	12.11	15.82	16.49	180
2100	13.34	14.51	17.41	18.32	180
2101	24.43	25.45	26.87	27.80	180
2102	33.16	34.10	34.99	35.89	180
2135	17.60	18.30	20.86	21.45	180
2201	19.88	20.87	22.81	23.68	180
2212	28.45	29.49	30.57	31.54	180
2329	19.87	20.58	22.80	23.43	180
2340	12.12	13.35	16.49	17.41	180
3103	13.64	14.18	17.64	18.06	180
3200	32.82	34.09	34.68	35.88	180
3360	23.42	24.29	25.95	26.74	180
3361	8.60	9.61	14.11	14.75	180
3362	14.61	15.66	18.40	19.25	180
3554	14.02	14.80	17.94	18.55	180
3671	10.69	11.16	15.47	15.80	78.62
3752	29.58	30.24	31.62	32.24	145.95
3753	18.00	19.03	21.19	22.08	180
3757	6.60	6.62	12.99	13.00	12.91
3838	26.33	27.32	28.61	29.52	180
4015	8.28	9.38	13.92	14.60	100.81
4034	14.30	15.28	18.15	18.94	180
4179	11.51	12.86	16.05	17.04	180
4183	16.89	17.89	20.26	21.10	180
4197	24.15	25.12	26.62	27.50	180
4257	24.19	24.71	26.65	27.12	180
4341	20.75	21.70	23.57	24.41	180
4450	17.63	18.62	20.88	21.72	180
4486	16.49	17.54	19.93	20.80	180
4544	10.10	10.85	15.07	15.59	180
4581	10.64	11.65	15.44	16.15	180
4660	6.07	7.51	12.73	13.47	180
4769	15.13	16.15	18.81	19.64	180
4953	23.29	24.19	25.84	26.65	180
5011	12.54	13.55	16.80	17.57	180

$$\Delta v = (v_{\max} - v_{\min})_{\text{mean}} = 0.91 \text{ km/s}$$
$$\Delta v^* = (v^*_{\max} - v^*_{\min})_{\text{mean}} = 0.74 \text{ km/s}$$

Table 4. Velocity distribution characteristics when colliding with Mars; gravitation is not taken into account

k	-1	1	2	3	4
μ_k	0.1002	11.87	164.4	2595	45590
ν_k	9.983	11.87	12.82	13.74	14.61
λ_k	0.8411	1	1.080	1.158	1.231

$N = 165$, $\sigma = 4.85$ km/s

Table 4a. Velocity distribution characteristics when colliding with Mars; gravitation is taken into account

k	-1	1	2	3	4
μ_k	0.08551	13.03	189.6	3067	54540
ν_k	11.69	13.03	13.77	14.53	15.28
λ_k	0.8974	1	1.057	1.115	1.173

$N = 165$, $R = 3390.0$ km, $v_0 = 5.029$ km/s, $\sigma = 4.45$ km/s

gravity is not taken into account (the second and the third columns): these quantities obtained in consideration of planet's gravity (the fourth and the fifth columns); $\Omega_2 - \Omega_1$ in degrees. In particular, we can extract the number $N' = 1076$ from the last column.

3 VENUS - EARTH - MARS

Velocity distribution characteristics for the s-th planet are listed in the Table having the number s (negligible gravity; the letter "a" attached to the number indicates that the gravity is taken into account). To save room, we present the Tables of the 1b type for Mercury and the Earth only.

Calculation of the data of the Tables is described in the previous section.

In the case of Venus, the difference $v_{\max} - v_{\min}$ varies within narrow limits from 0.15 to 0.71, and its averaged value is $\Delta v = 0.46$, that is 2% from $\mu_1 = 21$. This fact reflects the following property. If $e_s = 0$, then all positions of the line of nodes are equivalent and $v_{\max} = v_{\min}$ for each planet-crosser. For Venus $e_2 = 0.007$. In this case we may estimate $N' \approx 4N = 100$ and neglect the factor $b \approx 1.005$.

For the Earth $v_{max} - v_{min}$ varies between 0.02 km/s and 1.44 km/s, $\Delta v =$ 0.91 km/s.

For Mars the limits of $v_{\max} - v_{\min}$ are 0.01 and 6.84, $\Delta v = 2.61$.

For the Earth and especially for Mars, the eccentricities as well as the number of selected asteroids are larger than for Venus. So we put $b = 1$ all the more.

Table 5. Velocity distribution characteristics when colliding with the Moon; gravitation is not taken into account

k	-1	1	2	3	4
μ_k	0.06433	18.25	382.6	8957	227700
ν_k	15.54	18.25	19.56	20.77	21.84
λ_k	0.8518	1	1.072	1.138	1.197
		$N = 45$, $\sigma = 7.04$ km/s			

Table 5a. Velocity distribution characteristics when colliding with the Moon; gravitation is taken into account

k	-1	1	2	3	4
μ_k	0,06315	18.43	388.2	9112	232000
ν_k	15.83	18.43	19.70	20.89	21.95
λ_k	0.8593	1	1.069	1.133	1.191
		$N = 45$, $R = 1737.4$ km, $u_0 = 2.376$ km/s, $\sigma = 6.98$ km/s			

4 THE MOON, PHOBOS AND DEIMOS

Velocity distribution characteristics for a satellite are determined by the same set of asteroids as for the corresponding planet. For the Moon they are represented in Table 5 and 5a, for Phobos and Deimos, in Table 6 and 7. The gravity field of Martian satellites is negligible for our purpose: the parabolic velocity u_0 on their surfaces is less than 0.01 km/s. Remember that we deal with squared velocities and $u_0^2/\mu_2 < 10^{-6}$.

The coincidence of μ_2 calculated by means of numerical and analytical averaging is as perfect as in the case of planets.

5 DISCUSSION

1. It should be noted that the results of our calculations of minimal and maximal collisional velocities of minor planets with Mars (to save room, we do not give them in a special Table) systematically differ from those published by Steel (1985). According to our calculations, the lower boundary of collisional velocities is, as a rule, by several hundred meters per second greater and the upper boundary by several hundred meters per second smaller. Variation of this difference lies within limits from 0 to 1.5 km/s. The minor planet (1727) Mette is a typical example. The perihelion distance of this planet is equal to 1.665 a.u., that is very close to aphelion distance of Mars (1.666 a.u.). Collision of a body having the same orbital elements a, e, i as Mette has with Mars is possible only within small vicinity

Table 6. Velocity distribution characteristics when colliding with Phobos

k	-1	1	2	3	4
μ_k	0.09315	12.43	176.9	2843	50710
ν_k	10.74	12.43	13.30	14.17	15.01
λ_k	0.8637	1	1.070	1.140	1.207
		$N = 165$, $\sigma = 4.73$ km/s			

Table 7. Velocity distribution characteristics when colliding with Deimos

k	-1	1	2	3	4
μ_k	0.09754	12.06	168.6	2683	47550
ν_k	10.25	12.06	12.99	13.90	14.77
λ_k	0.8501	1	1.077	1.152	1.224
		$N = 165$, $\sigma = 4.82$ km/s			

of perihelion. Our calculations give for this planet $\Omega_2 - \Omega_1 = 6^\circ.00$. Moreover, collision velocity with Mars in any point of this vicinity is practically the same and equal to 9.38 km/s. In Steel's paper for this planet v_{min} and v_{max} are given as 8.7 km/s and 10.2 km/s, correspondingly. Such a distinction appears, first of all, as a consequence of different setting up the problem. The orbit of a major planet, say, Mars is treated here as a fixed one with elements referred to the contemporary epoch. One examine impact velocities of the flux of particles having the same fixed orbital elements a, e, i (inclination to the Mars orbit) as the chosen minor planet has and uniformly distributed directions of nodes and perihelions. Minimal and maximal relative velocities over all intersection points with the Mars orbit are found. Final results represent an estimation of impact velocities of meteoroids of asteroidal nature with the planet for the present epoch.

On the contrary, Steel's results correspond to the assumption that inclinations to the invariant plane conserve whereas distribution of nodes and perihelions of both Mars and the minor planet is uniform. This implies a variation of i and extension of the set of velocity values. Velocity distribution turns out to be valid over a period of perihelion revolution of Mars, i.e. about one hundred of thousand years.

2. When examining the Tables above, we can extract the following conclusions.

(1) Compare the third rows of the Tables numbered s, sa ($s = 1, \ldots, 7$) with the fourth one of Table 1. We see that velocity distributions are similar to Maxwell's one, but definitely do not coincide with it. We shall try to find the analytical form of distribution functions in the next paper.

(2) Compare ν_k, σ for fixed k from the set of Tables s or those of sa ($s = 1, \ldots, 4$). In both cases ν_k, σ decrease with s. The reason is evident: the farther from the

Sun, the greater the potential and the smaller the kinetic energy of projectiles are.

(3) Compare the Table s ($s = 1, \ldots, 5$) with the Table sa. Confrontation of the second and the third rows shows that planet's or Moon's gravitation augments ν_k (this is obvious without calculations) and makes λ_k nearer to 1. In other words, gravitation of a target body makes the random variable v "less random". This is reflected also in diminishing mean squared deviation σ.

(4) Compare the quantity ν_k from Table s with that from Table sa, designating latter as α_k. Then we have (Kholshevnikov and Shor, 1994):

$$\alpha_2^2 = \nu_2^2 + v_0^2.$$

For $k \neq 2$ values of α_k differ from $(\nu_k^2 + v_2^2)^{1/2}$ by several units of the third decimal.

(5) Compare Tables 3 and 5. Let us designate β_k the quantity ν_k from Table 5, preserving notation ν_k for Table 3 quantity. Values of β_k are greater than ν_k not more than by 0.06–0.13. According to Kholshevnikov and Shor (1994),

$$\beta_2^2 = \nu_2^2 + 3u^2,$$

u being the circular geocentric velocity of the Moon. Values of β_k differ from those of $(\nu_k^2 + 3u^2)^{1/2}$ by 0.01 ($k = 1, 3, 4$) or 0.03 ($k = -1$). This is less than precision of our data. So in future (after discovering new Earth-crossers) we do not need to fulfill new cumbersome calculations of Table 5 data: it is sufficient to use the formula

$$\beta_k^2 \simeq \nu_k^2 + 3u^2. \tag{3}$$

Further, using the formula

$$\beta_k^2 \simeq \nu_k^2 + 3u^2 + u_0^2$$

for Table 5a quantity ν_k leads to an error in β_k like 0.04 ($k = 1, 3, 4$) and 0.14 ($k = -1$). Remember that u_0 is the parabolic velocity on the Moon's surface.

(6) Compare Tables 4 and 6, 7. For Phobos using the formula (3) leads to an error in β_k like 0.04 ($k = 1, 3, 4$) and 0.21 ($k = -1$); for Deimos, 0.02 and 0.06.

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