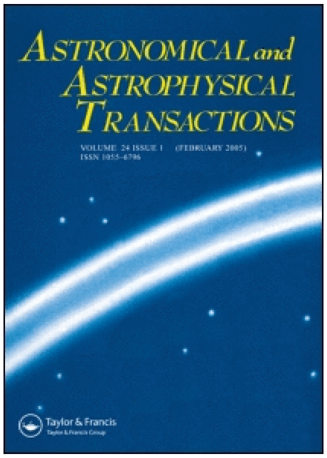

Astronomical & Astrophysical Transactions

The Journal of the Eurasian Astronomical Society

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Two simple self-consistent galaxy models

L. P. Ossipkov [a]
[a] Institute of Computional Mathematics and Control Processes, St.Petersburg, Russia

Online Publication Date: 01 July 1995
To cite this Article: Ossipkov, L. P. (1995) 'Two simple self-consistent galaxy models', Astronomical & Astrophysical Transactions, 8:1, 1 - 5
To link to this article: DOI: 10.1080/10556799508203290
URL: http://dx.doi.org/10.1080/10556799508203290

Astronomical and Astrophysical Transactions, 1995,
Vol. 8, pp. 1–5
Reprints available directly from the publisher.
Photocopying permitted by license only

TWO SIMPLE SELF-CONSISTENT GALAXY MODELS

L. P. OSSIPKOV

Institute of Computional Mathematics and Control Processes, St. Petersburg State University, Bibliotechnaya pl. 2, Peterhof, St.Petersburg 198904, Russia

(Received July 20, 1994)

Two axisymmetric galaxy models with ellipsoidal equipotentials are considered. The first one generalizes the Plummer shpere. The potential of the second model coincides with Parenago's potential at the equatorial plane. Distribution functions depending on two classical integrals of motion are found for both models. They are expressed in terms of Fricke's polynoms.

KEY WORDS Celestial mechanics, stellar dynamics, self-consistent models of galaxies

In recent years, there has been significant progress in constructing self-consistent galaxy models with two-axial distribution (Dejonghe, 1986; Evans, 1993; Hunter and Qian, 1993). New examples of exact models of such kind are of interest. In this note, we consider two models of mass distribution with ellipsoidal equipotentials. The latter means that equation for equipotential surface is

$$\xi^2 = \rho^2 + \varepsilon^{-2}\zeta^2, \quad \varepsilon = \text{const},$$

where ρ and ζ are dimensionless cylindrical coordinates.

First, let us study a model for which the dimensionless potential $\phi(\xi)$ generalizes the well-known Schuster–Plummer spherical potential, i.e.,

$$\varphi(\xi) = (1 + \xi^2)^{-1/2}.$$

This model was mentioned by Ossipkov (1975). The corresponding augmented density $G(\varphi, \rho)$ (i.e., the spatial density considered as a function of potential φ and cylindrical coordinate ρ) is found to have a very simple form:

$$G(\varphi, \rho) = a\varphi^3 + b\varphi^5 + c\rho^2\varphi^5,$$

where a, b, c are constants,

$$a = 2(1 - \varepsilon^{-2}), \quad b = 3\varepsilon^{-2}, \quad c = -3(1 - \varepsilon^{-2}).$$

($4\pi G = 1$ in our units, where G is the gravitational constant). This form of $G(\varphi, \rho)$ implies that the corresponding two-integral distribution function will be of Fricke's (1951) type.

Indeed, let E and L be the energy integral and the area integral, respectively,

$$E = \varphi - (v_\rho^2 + v_\vartheta^2 + v_\zeta^2)/2, \quad L = \rho v_\vartheta$$

where $v_\rho, v_\vartheta, v_\zeta$ are the velocity components in cylindrical coordinates. The even in velocities part of the distribution function $\Psi(E, L^2)$, which is determined by the density law, can be found from the following integral equation:

$$\rho G(\varphi, \rho) = 4\pi \int_0^\varphi dE \int_0^{L_c} (E, L^2)\, dL, \quad L_c = 2^{1/2}\rho(\varphi - E)^{1/2},$$

studied by Lindblad (1940), Kuzmin and Kutuzov (1962) and Lynden-Bell (1962). Following Eq. (2.5.48) of Dejonghe (1986), we find that

$$\Psi(E, L^2) = a_1 E^{3/2} + b_1 E^{7/2} + c_1 L^2 E^{5/2},$$

where the constants a_1 , b_1, c_1 are

$$a_1 = 4(2^{1/2}\pi^2)^{-1} a, \quad b_1 = 64(2^{1/2} 7\pi^2)^{-1} b, \quad c_1 = 32(2^{1/2}\pi^2)^{-1} c$$

The second-order moments of the velocity distribution are readily found using Dejonghe (1986) Eqs (2.3.10), (2.3.11) as

$$\sigma_\rho^2 = \sigma_\zeta^2 = (1 + \xi^2)^{-1/2} \frac{1 + (1 - \varepsilon^{-2})\varepsilon^{-2}\zeta^2}{2[(1 + 2\varepsilon^{-2}) - (1 - \varepsilon^{-2})\rho^2 + 2(1 - \varepsilon^{-2})\varepsilon^{-2}\zeta^2},$$

$$\langle v_\vartheta^2 \rangle = (1 + \xi^2)^{-1/2} \frac{1 - 2(1 - \varepsilon^{-2})\rho^2 + (1 - \varepsilon^{-2}\varepsilon^{-2}\zeta^2}{2[(1 + 2\varepsilon^{-2}) - (1 - \varepsilon^{-2})\rho^2 + 2(1 - \varepsilon^{-2})\varepsilon^{-2}\zeta^2}.$$

As the second example, we consider the potential

$$\varphi(\xi) = (1 + \xi^2)^{-1}.$$

If $\zeta = 0$, it coincides with Parenago's (1950) potential and if $\varepsilon = 1$ it generates Idlis' (1956) sphere. Galactic models with this potential were studied by Genkina (1969). The augmented density is

$$G(\varphi, \rho) = A\varphi^2 + B\varphi^3 + C\rho^2\varphi^3,$$

where A, B, C are constants,

$$A = 2(2 - 3\varepsilon^{-2}), \quad B = 8\varepsilon^{-2}, \quad C = -8(1 - \varepsilon^{-2}).$$

Then the even part of the two-integral distribution function is

$$\Psi(E,L^2) = A_1 E^{1/2} + B_1 E^{3/2} + C_1 L^2 E^{1/2},$$

where A_1, B_1, C_1 are new constants,

$$A_1 = 2(2^{1/2}\pi^2)^{-1}A, \quad B_1 = 4(2^{1/2}\pi^2)^{-1}B, \quad C_1 = 6(2^{1/2}\pi^2)^{-1}C.$$

It is easy to find also that

$$\sigma_\rho^2 = \sigma_\zeta^2 = \frac{2-\rho^2+(2-3\varepsilon^{-2})\varepsilon^{-2}\zeta^2}{3[(2+\varepsilon^{-2})-(2-\varepsilon^{-2})\rho^2-(2-3\varepsilon^{-2})\varepsilon^{-2}\zeta^2]},$$

$$\langle v_\vartheta^2\rangle = \frac{2+(-7+6\varepsilon^{-2})\rho^2+(2-3\varepsilon^{-2})\varepsilon^{-2}\zeta^2}{3[(2+\varepsilon^{-2})-(2-\varepsilon^{-2})\rho^2-(2-3\varepsilon^{-2})\varepsilon^{-2}\zeta^2]}.$$

It is known that this potential has a Stäckel form and admits the third quadratic integral. So it is possible to construct exact three-integral distribution functions generating triaxial velocity distributions (Dejonghe and de Zeeuw, 1988).

The augmented density and the even part of the distribution function can be also found for more general potential considered by Ossipkov (1975),

$$\varphi(\xi) = (1+\xi^2)^{-n}.$$

As for the odd part of the distribution function $\Upsilon(E,L)$, it is determined by the centroid velocity $\langle v_\vartheta\rangle$ considered as a function of φ and ρ. Following Dejonghe and Merritt (1992), this can be called an augmented rotation law. A new method of finding $\Upsilon(E,L)$ was recently proposed by Kutuzov (1993) (see also Kutuzov and Ossipkov, 1993).

Unfortunately, models described above suffer from the following disadvantage. The spatial density of the models is not positive everywhere. It can be proved that this is a general property of finite mass models with ellipsoidal equipotentials (excluding spherical and uniform density cases). For spherical systems, a truncation of models is possible (Idlis, 1956) but it is wrong to truncate non-spherical models with non-ellipsoidal equidensities as it was done by Genkina (1969) and Ossipkov (1975). So models with ellipsoidal equipotentials and non-negative densities must have both unbounded sizes and infinite total mass. A model with

$$\varphi(\xi) = c\ln(1+\xi^2)$$

(Binney, 1982) provides a relevant example. The distribution function for this model was found by Evans (1993). The simplest way of the corresponding generalization of our models will be in adding a uniform background. Then, instead of Genkina's model, we have

$$\varphi(\xi) = (1+\xi^2)^{-1} - \varkappa^2\xi^2, \quad \varkappa = \text{const}.$$

The augmented density

$$G(\varphi,\rho) = G_0(\varphi) + \rho^2 G_1(\varphi),$$

where the functions $G_0(\varphi)$ and $G_1(\varphi)$ are given by

$$
\begin{aligned}
G_0(\varphi) &= 2(2+\varepsilon^{-2})\varkappa^2 + (2+\varepsilon^{-2})[(\varphi_1^2+2\varkappa^2)+\varphi_1(\varphi_1^2+4\varkappa^2)^{1/2}] \\
&+ 2\varepsilon^{-2}\varkappa^{-2}[\varphi_1(\varphi_1^2+3\varkappa^2)+(\varphi_1^2+\varkappa^2)(\varphi_1^2+4\varkappa^2)^{1/2}] \\
&\times [(\varphi_1+2\varkappa^2)-[(\varphi_1+2\varkappa^2)^2+4\varkappa^2(1-\varkappa^2-\varphi_1)]]^{1/2},
\end{aligned}
$$

$$G_1(\varphi) = -4(1-\varepsilon^2)[\varphi_1(\varphi_1^2+3\varkappa^2)(\varphi_1^2+4\varkappa^2)^{1/2}]$$

and

$$\varphi_1 = \varphi - \varkappa^2.$$

This model differs from Genkin's (1966) one in the non-sphericity of the background. Genkin's potential has a Stäckel form and, in principle, exact three-integral distribution functions exist for it, but the equipotentials of the model are quartic. An expression for the augmented density of Genkin's model is much more complicated than even in our case.

Finding distribution functions for unbounded potentials has some specific features discussed by, e.g., Hunter and Qian (1993). Now the corresponding integral equation can be written as

$$\rho G(\varphi,\rho) = 4\pi \int_{-\infty}^{\varphi} dE \int_0^{L_c} \Psi(E,L^2)\, dL$$

(the lower limit of integration is changes now to $-\infty$.). This expression for $G(\varphi,\rho)$ implies that

$$\Psi(E,L^2) = F_0(E) + L^2 F_1(E).$$

It can be shown that $F_0(E)$ and $F_1(E)$ are determined by

$$
\begin{aligned}
F_0(E) &= (2^{3/2}\pi^2)^{-1} d\left[\int_0^E G_0'(E-\varphi)^{-1/2}\, d\varphi\right]/dE, \\
F_1(E) &= -(2^{5/2}\pi^2)^{-1} d\left[\int_0^E G_1'(E-\varphi)^{-3/2}\, d\varphi\right]/dE.
\end{aligned}
$$

Substituting the above expressions for $G_0(\varphi)$ and $G_1(\varphi)$, we see that the distribution function $\Psi(E,\ L^2)$ of our models contains elliptical integrals. We will not write out a rather lengthy formula for it.

References

Binney, J. J. (1982) *MN RAS* **201**, 1.

Dejonghe, H. (1986) *Phys. Rep.* **133**, 217.

Dejonghe, H. and de Zeeuw, T. (1988) *Ap. J.* **328**, 720.

Dejonghe, H. and Merritt, D. (1992) *Ap. J.* **391**, 531.

Evans, N. W. (1993) *MN RAS* **260**, 194.

Fricke, W. (1951) *AN* **280**, 193.

Genkin, I. L. (1966) *Trudy Astrofiz. Inst.* (Alma-Ata) **7**, 16 (in Russian).

Genkina, L. M. (1969) *Trudy Astrofiz. Inst.* (Alma-Ata), **12**, 97 (in Russian).

Hunter, C. and Qian, E. (1993) *MN RAS* **262**, 40.

Idlis, G. M. (1956) *Astron. Zh.* **33**, 20 (in Russian).

Kutuzov, S. A. (1993) In *Mathematical Methods in Studying the Structure and Dynamics of Gravitating Systems*, A. A. Mülläri (ed.), Petrozavodsk Univ. Press, Petrozavodsk, p. 13.

Kutuzov, S. A. and Ossipkov, L. P. (1993) In *Galactic Bulges*, IAU Symp. 153, H. Dejonghe, H. Habing (eds.), Kluwer Publ., Dordrecht-Boston-London, p. 363.

Kuzmin, G. G. and Kutuzov, S. A. (1962) *Bul. Abastumansk. Astrofiz. Observ.* No. 27, 82 (in Russian).

Lindblad, B. (1940) *Stockholms Observ. Ann.* **13**, No. 5.

Lynden–Bell, D. (1940) *MN RAS* **123**, 447.

Ossipkov, L. P. (1975) *Vestn. Leningradsk. Univ.* No. 7, 151 (in Russian).

Parenago, P. P. (1950) *Astron. Zh.* **27**, 329 (in Russian).