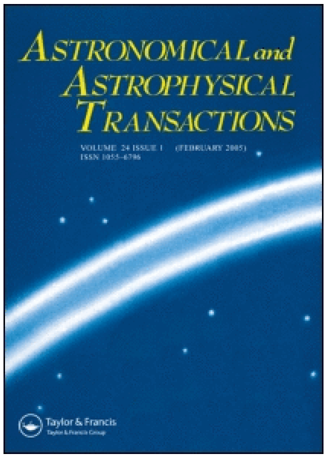

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 May 1995
To cite this Article: Omarova, G. T. (1995) 'On inverse problems in dynamics of gravitating systems', Astronomical & Astrophysical Transactions, 7:4, 267 - 268
To link to this article: DOI: 10.1080/10556799508203272
URL: http://dx.doi.org/10.1080/10556799508203272

Astronomical and Astrophysical Transactions, 1995,
Vol. 7, pp. 267–268
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ON INVERSE PROBLEMS IN DYNAMICS OF GRAVITATING SYSTEMS†

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(Received December 25, 1993)

An analog of Szebehely's equation is found for non-stationary potential on the basis of the angular momentum integral.

KEY WORDS Celestial mechanics, stellar dynamics–inverse problems

Almost two decades ago Szebehely (1974) obtained a linear, first-order partial differential equation for the potential of an autonomous, conservative two-degrees-of-freedom dynamical system, generating a given monoparametric family of planar orbits. This equation is [1]

$$f_x U_x + f_y U_y + 2(U+h)\frac{f_{xx}f_y^2 - 2f_{xy}f_x f_y + f_{yy}f_x^2}{f_x^2 + f_y^2} = 0. \tag{1}$$

Szebehely's equation, as can be seen, is obtained on the basic of the energy integral h.

The inverse problem of celestial mechanics in the non-stationary case is very important now. For example, one may point out the searches for the model problems in celestial mechanics of the variable mass. The non-stationary inverse problem can be defined as follows: For a given family of evolving in time planar orbits determine the corresponding non-stationary potential that produces the given family.

It is known that in the non-stationary problems, generally, we have no energy integral. But the derivation of the pertinent partial differential equations is possible on the basic of other integrals of motion.

Indeed, if we consider the non-stationary problem which contains the angular momentum integral:

$$x\dot{y} - y\dot{x} = k = \text{const}, \tag{2}$$

†Proceedings of the Conference held in Kosalma

then we get the equation [2]

$$
\begin{aligned}
& f_x U_x + f_y U_y + k^2 \frac{f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2}{(x f_x + y f_y)^2} \\
& + 2k \frac{f_x f_{yt} - f_y f_{xt}}{x f_x + y f_y} + 2k f_t \frac{x(f_y f_{xx} - f_x f_{xy}) + y(f_y f_{xy} - f_x f_{yy})}{(x f_x + y f_y)^2} \\
& + f_t^2 \frac{x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy}}{(x f_x + y f_y)^2} - 2 f_t \frac{x f_{xt} + y f_{yt}}{x f_x + y f_y} + f_{tt} = 0.
\end{aligned} \tag{3}
$$

In a more general non-stationary problem with integral

$$
\exp\Big\{ - \int_{t_0}^{t} \alpha(t)\, dt \Big\} (x\dot{y} - y\dot{x}) = c = \text{const}, \tag{4}
$$

where $\alpha = \alpha(t)$ – is a magnitude generally depending on time and characterizing the action of additional forces of the friction nature, we obtain the following equation [3]:

$$
\begin{aligned}
& f_x U_x + f_y U_y + c^2 \frac{f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2}{(x f_x + y f_y)^2} \\
& + 2c \frac{f_x f_{yt} - f_y f_{xt}}{x f_x + y f_y} + 2c f_t \frac{x(f_y f_{xx} - f_x f_{xy}) + y(f_y f_{xy} - f_x f_{yy})}{(x f_x + y f_y)^2} \\
& + f_t^2 \frac{x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy}}{(x f_x + y f_y)^2} - 2 f_t \frac{x f_{xt} + y f_{yt}}{x f_x + y f_y} + f_{tt} - \alpha f_t = 0.
\end{aligned} \tag{5}
$$

Equations (3) and (5) are linear, first-order partial differential equations for the non-stationary potential, generating a given family orbits evolving in time in the non-stationary problems described by the integrals of the form (2) and (4). These equations can be considered as non-stationary analogs of Szebehely's equation (1).

On the basic of the equation (5) we can get a number of model problems often met in celestial mechanics of variable mass [2].

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