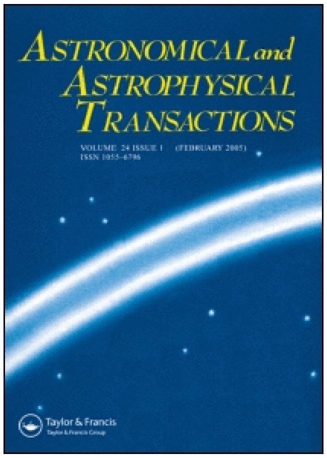

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COLLISIONAL RELAXATION IN A NONINTEGRABLE MEAN-FIELD POTENTIAL†

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We summarize recent investigations showing how and why, for systems characterized by a nonintegrable mean-field potential, collisional effects may be important already on time scales much shorter than the standard relaxation time.

KEY WORDS Stellar dynamics – collisional relaxation

This talk summarizes recent theoretical work investigating how and why, for systems characterized by a strongly nonintegrable mean-field potential U, collisional effects may be important already on time scales much shorter than the standard binary relaxation time, t_R. In what follows, such collisional effects will be modeled by modifying a Hamiltonian flow so as to incorporate noise and dynamical friction, related by a fluctuation-dissipation theorem. The basic objective is to compare deterministic Hamiltonian trajectories with nondeterministic noisy trajectories, and to ascertain the time scale on which deterministic and nondeterministic trajectories with identical initial conditions may be expected to diverge significantly.

Physically, as first stressed by Pfenniger (1986), one anticipates that a chaotic mean field can result in a drastically accelerated collisional relaxation in the configuration and velocity space. The friction and noise both serve to induce perturbations in the unstable deterministic trajectory that will generically be amplified exponential on a time scale t_Λ set by the largest Lyapunov exponent, even if the natural time scale for noise/friction, t_R, is much longer than t_Λ.

As a simple example, one can incorporate noise and friction into a Langevin description by considering a stochastic differential equation of the form $d^2\mathbf{R}/dt^2 + \nabla U(\mathbf{R}) = -\eta d\mathbf{R}/dt + \mathbf{F}_s$, allowing for a constant coefficient of dynamical friction, η, and delta-correlated white noise satisfying $\langle \mathbf{F}_s(t)\rangle = 0$ and $\langle \mathbf{F}_s(t_1)\,\mathbf{F}_s(t_1)\rangle = \Theta\eta\delta_D(t_2 - t_1)$. Here angular brackets denote an ensemble average and Θ a typical "temperature", i.e., a characteristic mean squared velocity. This is to be viewed as

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a perturbation of the deterministic equation $d^2\mathbf{r}/dt^2 + \nabla U(\mathbf{r}) = 0$, with $\delta\mathbf{r} \equiv \mathbf{R} - \mathbf{r}$. Consider in particular a weak noise limit, with $t_R \equiv \eta^{-1} \gg t_\Lambda$. A perturbative calculation then shows that (Kandrup and Mahon, 1993; Kandrup and Willmes, 1993) $\langle|\delta\mathbf{v}|^2\rangle/\langle v^2\rangle \sim \eta t_\Lambda \exp(2t/t_\Lambda$. If, however, one supposes that $t_\Lambda \sim t_{\rm cr}$ with $t_{\rm cr}$ a typical crossing time, and uses the standard scaling $t_R/t_{\rm cr} \sim N/(\log N)$, one then infers that $\langle|\delta\mathbf{v}|^2\rangle/\langle v^2\rangle \sim (\log N/N)\exp(2t/t_\Lambda)$, with an analogous formula for $\langle|\delta\mathbf{r}|^2\rangle$.

This calculation provides a simple explanation of numerical simulations by Pfenniger (1986), who investigated how orbits in a smooth galactic potential can be modified through the insertion of a point mass perturbation of variable mass m. What Pfenniger found was that regular orbits with vanishing Lyapunov exponent experienced perturbations $\delta\mathbf{r}$ that grow linearly, but that stochastic orbits with nonzero Lyapunov exponent experienced instead perturbations $\delta\mathbf{r}$ that grow exponentially. Moreover, for relatively small masses he discovered (a) that the exponential growth rate is independent of the magnitude of the perturbing mass, and (b) that the overall amplitude of the perturbation, i.e., the exponential prefactor, depends on m in a fashion consistent with the preceding perturbative calculation.

It should be stressed that, even though perturbations in position and velocity grow on a very short time scale, perturbations in the particle energy E and other collisionless invariants only grow much more slowly. Suppose, for example, that the mean-field potential U is strictly time-independent. The Langevin equation then implies that $dE/dT = -\eta v^2 + \mathbf{v}\,\mathbf{F}_s$ which, for $|E| \sim v^2$, yields a time scale $\eta^{-1} \equiv t_R$. It follows that, even for a strongly stochastic potential, noise and friction need not after the form of a self-consistent collisionless equilibrium on time scales $\ll t_R$. However, one *can* still envision circumstances under which noise and friction cloud alter important orbital characteristics on relatively short time scales.

Suppose, for example, that one specifies some ensemble of initial conditions and then evolves that ensemble into the future, both with and without noise and dynamical friction. The obvious question is then: on time scales $\ll t_R$, can one infer that the time evolution of the noisy and deterministic evolution will be qualitatively similar, at least in some statistical sense? In a mostly regular phase space region, far from any resonances, one only expects substantial diffusion in the actions on the natural time scale $t_R = \eta^{-1}$. However, if the phase space has an important resonant structure, even weak noise can induce significant effects on relatively short time scales, e.g., by facilitating jumps between nonoverlapping islands (Lieberman and Lichtenberg, 1972). This has obvious implications for the numerical construction of self-consistent models via Schwarzschild's method (Schwarzschild, 1979), or any variant thereof which involves the selection of ensembles of orbits without an explicit consideration of the collisionless invariants. A systematic investigation of these effects is currently underway using the Los Alamos Connection Machine (Habib *et al.*).

It is also possible that, under certain circumstances, friction and noise may serve to trigger an instability for collisionless equilibria. Consider a self-consistent equilibrium, modeled as a time-independent solution of the collisionless Boltzmann equation, i.e., the gravitational Vlasov–Poisson system. By analogy with plasma physics (Morrison, 1980), one knows that this system can be viewed as a constrained Hamiltonian system with respect to an appropriate Lie brackets (Kandrup, 1990), constrained because of the restrictions associated with conservation of phase. One knows, moreover, that all time-independent equilibria are ***energy extremals*** with respect to perturbations that preserve the constraints, so that the first variation of the Hamiltonian H vanishes identically, i.e., $\delta^{(1)}H = 0$. It follows that stability is connected with the sign of the second variation $\delta^{(2)}H$. If, for example, $\delta^{(2)}H > 0$ for all perturbations preserving the constraints, linear stability is assured. However, one anticipates that many/most nontrivial equilibria admit perturbations which preserve all the constraints but still have negative energy, so that $\delta^{(2)}H < 0$. This is, e.g., known to be true for generic axisymmetric equilibria (Kandrup, 1991) and is probably true for many triaxial configurations as well. The existence of such negative energy perturbations does not necessarily signal a linear instability on a time scale $\sim t_{\rm cr}$, since the negative energy modes may all be decoupled from the positive energy modes. However, by analogy with plasma physics and accelerator dynamics, one can envision situations in which the noise and friction couple together the positive and negative energy modes, so as to trigger an instability on a time scale $\ll t_R$.

Both these effects, namely diffusion of orbits in phase space and destabilisation of collisionless equilibria, are known to occur in various other settings, and should almost certainly arise for self-gravitating systems as well. Unfortunately, however, it is extremely difficult to estimate the time scales for these phenomena using theoretical arguments alone, so that one cannot conclude immediately when they may be important on astronomically relevant time scales: for this one would appear to require recourse to numerical simulations. It is, however, clear that, if one chooses to consider nonintegrable mean field potentials, in which a substantial fraction of the mean field orbits are stochastic, one needs to reexamine the conventional assumption that collisional effects are completely irrelevant on time scales $\ll t_R$.

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