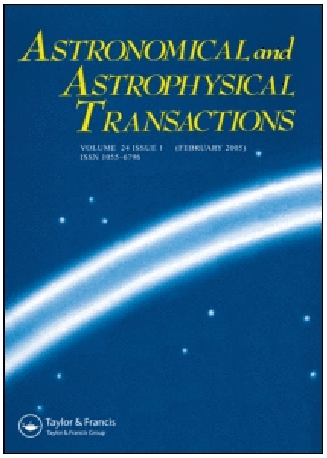

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V. L. Polyachenko [a]
[a] Institute of Astronomy, Russian Academy of Sciences, Moscow, Russia

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BASIC INSTABILITIES OF COLLISIONLESS GRAVITATING SYSTEMS†

V. L. POLYACHENKO

Institute of Astronomy, Russian Academy of Sciences, Pyatnitskaya st. 48, 109017 Moscow Russia

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The paper presents a short summary of basic instabilities in stellar systems, namely: the Jeans, bar-mode and fire-hose (bending) instabilities. Particularly, a variety of Jeans-like instabilities in collisionless systems is emphasized as well as a great number of functions which Jeans instability carries out here. Both these circumstances make the properties of stability in stellar and gaseous systems very different from each other. Then the classification of bar-mode instabilities according to a ratio of the bar pattern angular velocity and the maximal precession speed of nearly-circular stellar orbits is proposed. Some agruments in favor of slow or moderate bars in comparison with fast bars are given. Certain difficulties concerning the current work on the problem of the bending instability are noted.

KEY WORDS Dynamics of stellar systems, Instabilities in gravitating systems.

1 INTRODUCTION

The choice of three instabilities above as principal for stellar systems is quite natural, all the more that the full list of instabilities in gravitating systems is poor enough (e.g., compared with plasma). The bending (fire-hose) instability appears in rather hot and flattened systems; the simplest mechanical analogy of this instability is the behavior of a metallic ruler compressed on its edges. Turning to the remaining instabilities, one must note that the classification itself of a given instability as the Jeans or bar-mode is based on absolutely different principles, so it may lead to some confusion. Indeed, it is natural to classify as the Jeans ones those instabilities which develop due to gravitational clustering (merging) of the matter if the velocity dispersion of attracting objects cannot suppress such a process. As the objects of merging, there can be not only individual stars but also their certain groups. Thus, we include into the Jeans class the instabilities appearing due to a certain physical mechanism (first described by Jeans). On the other hand, the bar-mode

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class includes all instabilities leading to bar formation. In other words, only the geometric structure of the pattern finally arising is important (independent of the physical mechanism of the instability).

But there are at least three different ways for bar formation, one of them being Jeans instability: slow bars (Lynden–Bell, 1979) form as a result of gravitational merging of rather slowly precessing stellar orbits. Different bars form due to fast rotation of galactic discs. Moreover, detailed physical mechanisms for uniform (i) and differential (ii) rotations are quite different in turn; Toomre (1981) described them as the edge instability (i) and swing amplification (ii), respectively.

So, generally speaking, the Jeans class of instabilities intersects the bar-mode class if we define these classes as above. However, there exists a different way for the definition: to consider as the Jeans only those instabilities which lead to the decay of the system into more small-scaled (and more or less rounded) parts. It occurs under the Jeans clustering of a uniform distribution of the matter (as in the original work by Jeans) or, for example, in the case of a cold gravitating disk (Toomre 1964).

Before starting the exposition of the stability theory, one can try to formulate the general purpose of the pure stability theory in application to stellar systems. One of possible formulae (which was pleased to me all last year) is: to determine the way in which the various instabilities depend on the relative populations of the different orbit families: circular, radial, box, tube and so on (de Zeeuw, Franx, 1991).

2 KINDS OF THE JEANS INSTABILITY

The Jeans instability is, of course, the most well-known and important astrophysical instability. For instance, it leads to star formation in a gaseous medium. However, the ways of its manifestation in gas (and, generally, collisional) systems are rather monotonous: as a result of the instability, the initial system breaks into a set of quasi-spherical, collapsing objects. Such a monotony is apparently connected with the isotropy of pressure quickly restored due to often collisions of particles. In anisotropic systems, one should expect a wider variety of the Jeans instability's manifestations. Indeed, here the Jeans instability may play a role even opposite in comparison with its usual "rounding" function – as in Figure 1,*e* which demonstrates the action of the radial orbit instability converting an initially spherical system into an ellipsoidal one. This instability, like most others which tend to equalize temperatures in a gravitating medium (i.e., which tend to make the medium more nearly isotopic), is basically of a Jeans nature (Polyachenko and Fridman, 1988).

We defined above the Jeans instability as some kind of "merging" of gravitating matter. Figure 1 shows that the phenomenon of merging may proceed not only among individual particles (as in Figures 1,*a*, *b*), but also among orbits of particles (Figure 1,*e*), planes of orbits (Figures 1,*c*, *d*) and so on. It depends on concrete geometry of the system under consideration and on a kind of prevailing orbits. Note that the description of the large-scale Jeans instability as it occurs in the set of

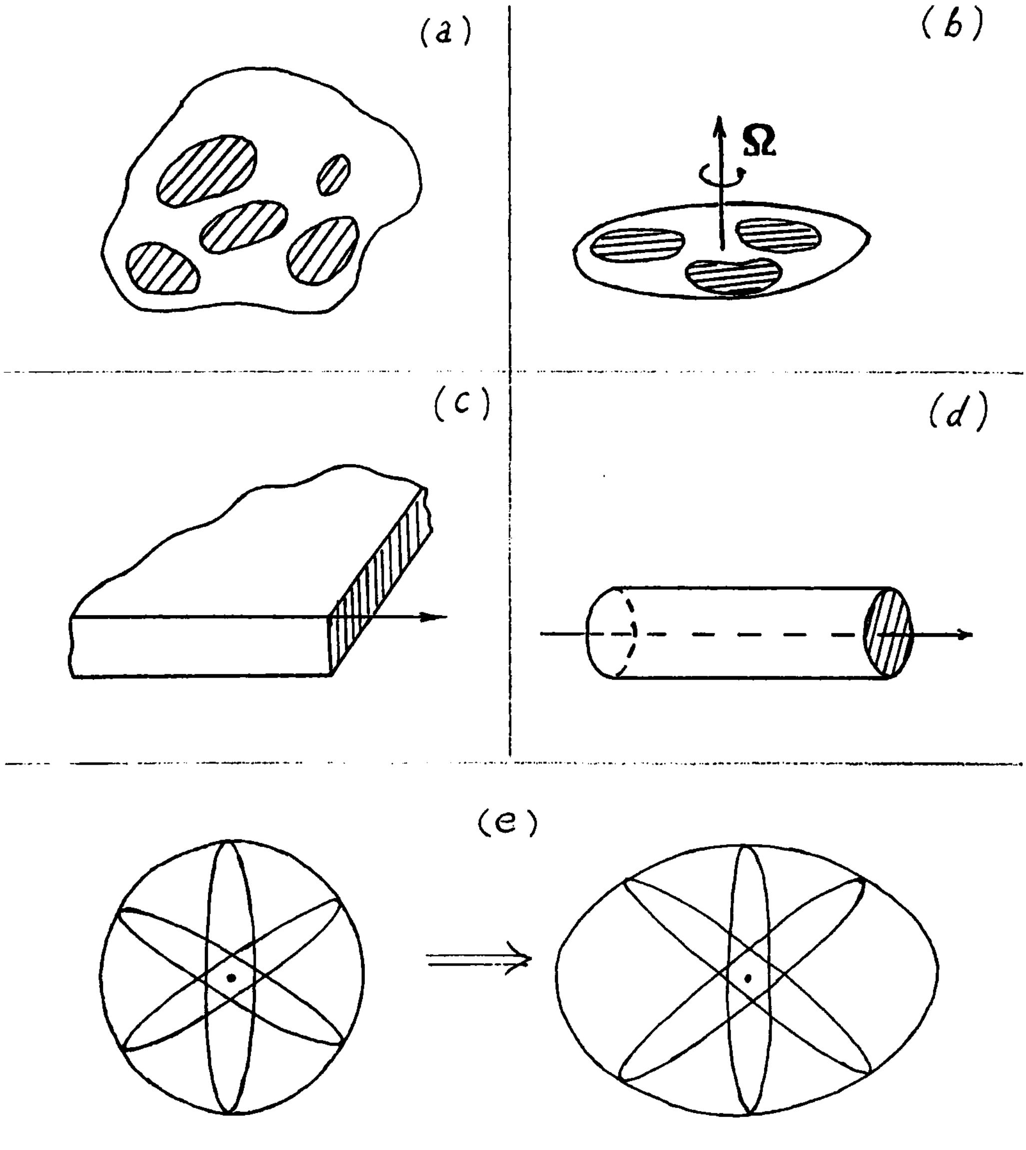

Figure 1 Different kinds of the Jeans instability
(a) uniform medium
(b) rotating disk
(c) flat layer
(d) cylinder
(e) the radial orbit instability for a spherical system.

planes of orbits was first used by Antonov and Nezhinskii (1973). They investigated the stability problem of a circular cylinder with infinite generatrix (along the Z-axis; see Figure 1,*d*) within the framework of the following model. The particles with identical values of the coordinate Z and velocity v_Z, should conventionally be grouped into discs. Let us assume these discs to be "rigid", i.e., assume that also in the perturbed state they move as a whole. The thermal velocities of particles in the (x, y) plane in this model are taken into account indirectly by the fact that the distribution of the surface density in the initial stationary model is ascribed to the disc. Assume the discs to remain always oriented parallel to the (x, y) plane and to pass freely through each other. Then the problem of perturbed motions becomes mainly one-dimensional; it can easily be solved and leads to a simple dispersion equation. Exact stability study for the particular cylindrical model (with circular orbits on the (x, y) plane; Mikhailovskii and Fridman, 1971) performed a little earlier showed that the model described gives qualitatively correct results up to the margin of stability.

Perhaps, the most interesting kind of Jeans-like instabilities is the radial orbit instability (Figure 1,*e*). At present, this instability is studied in detail but mainly by numerical methods (see Fridman and Polyachenko, 1984). An analytic theory was recently developed by the author (Polyachenko, 1991, 1992a) within the framework of a more general theory for low-frequency modes of gravitating systems when a characteristic frequency $\omega \sim \Omega_{pr} \ll \Omega_1$, where Ω_{pr} is the orbital precession angular speed and Ω_1 is the frequency of radial oscillations of stars. For such modes, the integral equations can by easily derived; for an initially spherical system, the equation is:

$$\chi(r) = \frac{4\pi G}{2l+1} \int r'^2 \, dr' F_l(r, r') B(r'), \tag{1}$$

where $\chi(r)$ is the radial part of the perturbed potential $\Phi_1 = \chi(r) e^{-i\omega t} Y_{lm}(\Theta, \varphi)$, $Y_{lm}(\Theta, \varphi)$ is the spherical harmonics; G is the gravitational constant, $F_l(r, r') = r_<^l / r_>^{l+1}$, $r_< = \min(r, r')$, $r_> = \max(r, r')$;

$$B(r) = \sum_{s=-l}^{l} \alpha_s^l \int v_\perp \, dv_\perp \, dv_r s \left(\frac{\partial F_0}{\partial L} + \Omega_{pr} \frac{\partial F_0}{\partial E} \right) e^{-is\delta(\mathbf{I}, \omega)} \frac{\overline{\Phi^{(s)}}}{\omega - s\Omega_{pr}}, \tag{2}$$

$$\overline{\Phi^{(s)}} = \int_0^{2\pi} \chi(r(\mathbf{I}, w_1)) \exp\left[is\delta(\mathbf{I}, w_1) \right] dw_1; \tag{3}$$

$\mathbf{I} = (I_1, I_2, I_3)$ and $\mathbf{w} = (w_1, w_2, w_3)$ are the action-angle variables; F_0 is the equilibrium distribution function; v is the particle's velocity, E, L is the energy and the angular momentum, respectively; $\delta(\mathbf{I}, w_1) = \varphi - w_2 + w_1/2$ is the known function, $\alpha_s^l = (l+s)!(l-s)! / \left[\left(\frac{l+s}{2}\right)! \left(\frac{l-s}{2}\right)! 2^l \right]^2$.. Similar equations occur for a disc and cylinder systems; they can be simplified if we turn to the systems with nearly-radial orbits. For the simplest case of the cylinder with radial orbits, the following integral equation can be obtained (Polyachenko, 1991):

$$-\omega^2\psi(E) = \int dE_1 K(E, E_1)\psi(E_1), \tag{4}$$

$$K(E, E_1) = 4\pi G \left(\frac{\partial\Omega_{pr}(E_1, L)}{\partial L}\right)_{L=0} \frac{\Omega_1(E)}{\pi}\varphi_0(E_1) \times \int_0^{r_{\max}(E)} \frac{r\,dr}{\sqrt{E_1 - \Phi_0(r)}\sqrt{E - \Phi_0(r)}}, \tag{4'}$$

where $F_0 = \delta(L)\varphi_0(E)$ and $\Phi_0(r)$ is the equilibrium potential. This equation has the eigenvalue $(-\omega^2) \sim 4\pi G\overline{\rho_0}R^2(\partial\Omega_{pr}/\partial L)_{L=0}$ where R is the size of the system, for a mode with no nodes in E. An instability $(\omega^2 < 0)$ occurs if the orbits precess in the forward sense, and the relation $(\partial\Omega_{pr}/\partial L)_{L=0} > 0$ holds (the opposite relation may be fulfilled only in quite exotic cases).

The stability problem for rotating spherical systems was also studied recently. Polyachenko (1992a) derived an analytic dispersion relation for the simplest model. This dispersion relation was studied in detail (Polyachenko, 1992b; Fridman and Polyachenko, 1992). It turns out that the marginal value of the precession speed dispersion increases due to rotation, i.e. the latter plays the role of a destabilizing factor. (The opposite conclusion by Polyachenko, 1992a was made from an incorrect limited form of the dispersion relation.)

3 BAR-MODES AND BARS

Figure 2 shows the location for the angular speeds of all three different kinds of bars or bar-modes. Analyzing the associated structures (rings, spirals) appearing as the response of the disc surface density to a bar forcing, we demonstrate (Pasha and Polyachenko 1993a,b) that Lynden-Bell's slow bars and moderate bars provide a consistent understanding of some typical features in barred galaxies.

Bar-enveloping inner rings get linked to the inner Linblad resonance (ILR) with the $m = 2$, dominant bar mode, and the outer rings, to the $m = 4$ ILR, next in both importance and location. Remarkably, the resonance radius ratio is $r_4/r_2 \approx 2.2$ which falls in the observed peak of the outer to inner ring axis ratio R/r. Fast bar theories give worse values: the currently recognized one, 2.6, probably means that the fast bars are a minority in the barred galaxies.

There are some additional supports in favor of slow or moderate bars. (i) Using the bar ending near ILR enables one to lessen or to remove the well-known difficulty with the disk response to the potential of a bar ending near corotation which appears to be too weak (e.g., see Sellwood and Wilkinson, 1992). (In its turn, avoiding this difficulty requires some artificial speculations implying two different values of the pattern speed). This follows from an evident estimate for the ratio of density

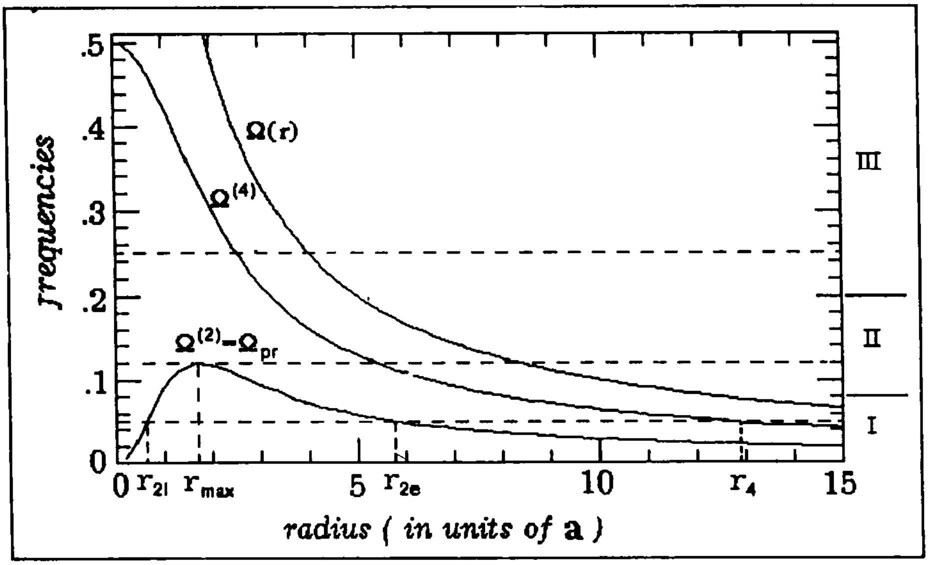

Figure 2 $\Omega^{(m)}(r) = \Omega - \kappa/m$ ($m = 1,2,\ldots$), where $\Omega(r)$ is the angular velocity, $\kappa = (4\Omega^2 + r d\Omega^2/dr)^{1/2}$; $\Omega^{(2)}(r) = \Omega_{pr}(r)$. $V(r) = V_0 \tanh(r/a)$ fits a typical "flat" rotation curve of the galaxy disc. All frequencies are in units of V_0/a. The curve $\Omega_{pr}(r)$ peaks at the turnover radius r_{max} and is crossed by a straight line of the slow bar angular speed Ω_b twice, at r_{2i} and r_{2e}, giving rise to a couple of inner Lindblad resonances with the $m = 2$ bar mode. No intersection of Ω_b and Ω_{pr} occurs in the case of fast bars. The moderate bar's dashed line corresponds to the "near-resonance" case. $\Omega_b = \Omega^{(4)}(r)$ determines the location of $m = 4$ *ILR* at $r = r_4$. For slow bars (*region I*) Ω_b is considerably less than $(\Omega_{pr})_{max}$. Moderate bars (*region II*) have $\Omega_b \sim (\Omega_{pr})_{max}$. For fast bars (*region III*) Ω_b is considerably larger.

responses (corotation to ILR at the same radius, with the same potential), which is less or about $\Omega_{pr}/\Omega(r) \ll 1$ ($\Omega(r)$ is the angular velocity of rotation). (ii) The simplest analytical support to the correct orbital alignment with the slow bar follows from the same relations (Equation (10) in Sellwood and Wilkinson, 1992) with which the alignment of orbits relative to fast bars was earlier demonstrated. It is easily to show that, for the power-law form of the perturbed potential, $\Phi_1 \sim r^{-n}$, the orbits inside the bar (between *ILR*s) align along the bar for

$$n > 2\Omega/(\Omega - \Omega_p). \tag{5}$$

Near corotation, this inequality cannot be satisfied for realistic values of n, so that the orbits are orthogonal to a fast bar ending at about corotation radius (R). A slow bar ends near the outer *ILR*, well inside R, and (5) shows now the orbits aligned along it already for moderate values of n (slightly above 2). These are suitable for centrally peaked bar mass distributions in late type *SBs*. (In early *SBs*, with a nearly constant density along the bar, this slow bar is rather made of nearly radial orbits.) We therefore observe that the slow bar concept can be compatible with the evidence concerning the point of the bar-orbit alignment.

4 THE FIRE-HOSE INSTABILITY

This important instability has been studied less extensively compared with the two others. Perhaps, this instability can help to solve the problem of maximal flattening of ellipticals (Polyachenko and Shukhman, 1979). Some progress in this problem (Merritt *et al.*, 1991) was connected with the study of a number of new distribution functions for collisionless ellipsoid recently derived. However, there is a principal difficulty for all this work: sufficiently "good" (realistic) distribution functions for E-galaxies are so for unavailable. An unexpected possibility to explain the peanut form of many galaxies with the help of the fire-hose instability has attracted attention since recently (Sellwood and Wilkinson, 1992).

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