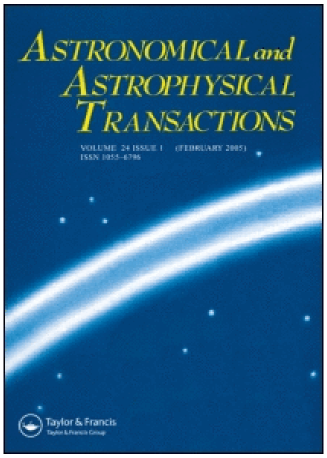

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

The stability of a gravitating ring of free particles

V. A. Antonov [a]; A. A. Vakhidov [a]
[a] Institute for Theoretical Astronomy, St. Petersburg, Russia

Online Publication Date: 01 May 1995
To cite this Article: Antonov, V. A. and Vakhidov, A. A. (1995) 'The stability of a gravitating ring of free particles', Astronomical & Astrophysical Transactions, 7:4, 279 - 281
To link to this article: DOI: 10.1080/10556799508203275
URL: http://dx.doi.org/10.1080/10556799508203275

Astronomical and Astrophysical Transactions, 1995,
Vol. 7, pp. 279–281
Reprints available directly from the publisher.
Photocopying permitted by license only

THE STABILITY OF A GRAVITATING RING OF FREE PARTICLES†

V. A. ANTONOV and A. A. VAKHIDOV

Institute for Theoretical Astronomy, St. Petersburg, Russia

(Received December 25, 1993)

Within the limits of the local hydrodynamical approximation, the stability of a rigidly rotating stellar ring in an external gravitational field is investigated. A criterion of the dynamical stability of the ring, which derives the critical density as a function of the system angular velocity, is obtained.

KEY WORDS Gravitating ring, dynamical stability, local, hydrodynamical approximation

The problem of the stability of a rigidly rotating gravitating ring of free particles is considered. For the solution, the following model is constructed:

1. the matter is homogeneous and cold initially which means that the velocity dispersion (in the case of a stellar system) and the temperature (in the case when the particles are gas molecules) are not taken into consideration;
2. the particle motions are independent of the coordinate measured along the layer rotation axis;
3. the width of the rings is very small compared to the dimensions of the system into which it is embedded;
4. the system rotates at a constant angular velocity Ω;
5. the system potential, density and kinematic characteristics of the system are subject to small perturbations.

The following symbols are introduced:

a – the ring's half-width,

m – mass of a star or other particle,

ν – the initial stellar density,

G – the gravitational constant.

†Proceedings of the Conference held in Kosalma

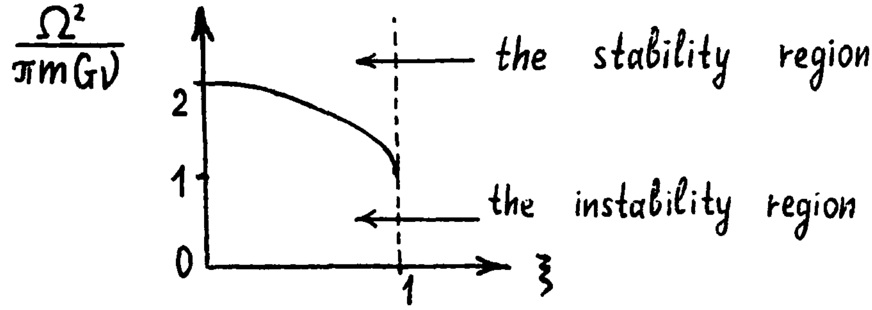

Figure 1

The unperturbed potential of the galaxy is considered to be constant at time of the statement of a question about the stability. This means that the galaxy gravitational field does not have time to react to processes which take place inside the stellar layer.

A Cartesian reference frame is introduced as follows. The origin is located on the middle line of the ring. The x-axis directed along the middle line, the y-axis is directed along the galaxy radius, the z-axis is perpendicular to the plane Oxy, in which the stellar ring is located.

Within the limits of the model under consideration, the system has an infinite extent along the x-axis (along the y-axis, the stars fill the region $-a < y < a$), the perturbations of density, gravitational potential and kinematical characteristics are defined as being proportional to $\exp(i\mu x)$, where $i = \sqrt{-1}$ and μ is the wave number in the transverse direction.

After the linearized hydrodynamical equations for the gravitational system under consideration have been derived and time is separated so far as perturbations are supposed to be proportional to $\exp(\omega t)$, the equation for the perturbation of the potential δU reads

$$\left(\frac{\nu}{\omega^2 + 4\Omega^2} - \frac{1}{4\pi m G}\right)\left(\frac{\partial^2 \delta U}{\partial y^2} - \mu^2 \delta U\right) = 0\,. \tag{1}$$

Using the method of jointing the derivatives of the potential across the layer, the following boundary condition is obtained

$$\frac{1}{\delta U}\frac{\partial \delta U}{\partial y} = \frac{8\Omega i \mu \pi m G \nu + \omega \mu(\omega^2 + 4\Omega^2)}{4\pi m G \nu \omega \mp \omega(\omega^2 + 4\Omega^2)}\,,\ (y = \pm a)\,. \tag{2}$$

The solution to equation (1) with the boundary condition (2) allows to obtain the following criterion (Figure 1) for dynamical stability

$$\pi m G \nu < \frac{\Omega^2}{1 + \sqrt{1 - \xi^2}}\,, \tag{3}$$

where $\xi = \exp(-4\mu a)$ runs within the interval (0; 1) with the variation of the ring width from infinity to zero.

The stability analysis leads to the following conclusion.

The dynamical stability of the model under consideration takes place provided the density ν is constrained by the condition (3) and depends on the product μa.

Quite generally, the critical value of the density lies within the interval

$$0.5\Omega^2 < \pi m G \nu < \Omega^2. \tag{4}$$

The condition (4) shows that the critical density is positive for any value of the ring width, so that regions of stability and instability exist for any system of this form.

The results of this research can be applied to the problem of the stability of the spiral structure of galaxies.