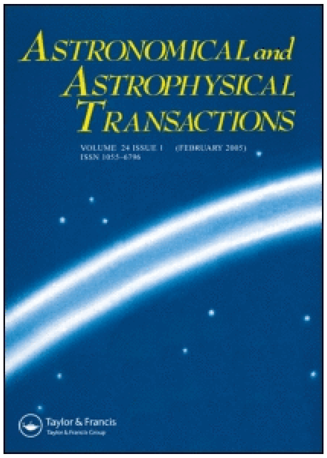

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ON THE CONSTRUCTION OF HIERARCHICAL MODELS OF GALAXIES†

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In the present paper we extend the model of a patchy-structured galaxy described by Nezhinskij and Ollongren (1992) to a hierarchical model and discuss the following:

1. a method of construction of one family of *stable* solutions of the N-body problem;
2. the rate of destruction of patches in the hierarchical model (stellar complexes) by relaxation processes.

KEY WORDS 05.03.1 Celestial mechanics, stellar dynamics – 11.11.1 Galaxies: kinematics and dynamics.

1 INTRODUCTION

Usually it is considered that some solutions of relative equilibrium in the problem of N bodies have a practical value only if $N = 2$, 3, but Nezhinskij and Ollongren (1992) proved that for $N > 3$ such solutions can be realized too in certain dynamical systems modeling **S**- and **SB**-galaxies. It is plausible that we perceive these solutions as patchy rings or ring-like patchy patterns in these star systems.

The solutions of relative equilibrium, which are interesting, are realized in a specific dynamical model of a galaxy and therefore below we shall present its description calling it a *hierarchical* model. The foundations for the choice of the proposed model are well-known facts: the hierarchical structure of quasi-stationary galaxies and the occurrence of patchy of ring-like and nearly ring-like spiral patterns in **S**- and **SB**-galaxies (Nezhinskij and Ollongren, 1992). Giant condensations of matter in the plane of such galaxies, called by Efremov (1984) *stellar complexes*, forming patchy patterns, consist of gigantic clouds of gas and dust, open stellar

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clusters and associations. Therefore globules of matter are chosen instead of point bodies to represent these objects.

2 MODEL OF A GALAXY

Consider the dynamical system, which consists of an oblate ellipsoid ($a = b \geq c$) with a spherical nucleus at the center and $N(= 2M)$ spherical globules. The radii of the nucleus and the globules are small compared to the distances between their centers of mass and the axes of the ellipsoid.

The density ρ of the ellipsoid is invariable in time, symmetric relative to the equatorial plane and the c-axis, and it is a C^1-smooth non-increasing function along any radius directed from the center. Each density function ρ_i ($i = 0, 1 \ldots, 2M$) of the nucleus and the globules is also invariant in time and is a C^1-smooth non-increasing function of the distance from the center of the object with the number i. The mass of the ellipsoid is considerably larger than the mass of the nucleus and the mass of the latter is considerably larger than the mass of each of the globules. The system of the globules consist of M couples of identical masses. The moving bodies can pass through one another. All bodies move under the action of gravitational forces. The globules are in the equatorial plane of the ellipsoid within the ellipsoid and form a central configuration symmetric relative to the center of the ellipsoid, that is: the system of globules rotates around the center of our model with constant angular velocity n. In this system the mass-center of each body must be located at a point of libration.

As a model of a galaxy we choose one of those dynamical models described above in which all points of libration inside globules are stable. Our model of a galaxy is a hierarchical model, i.e. globules consist of a number of bodies of small masses. For the sake of simplicity, we consider that each of these moves under the action of the gravitational forces around the stable point of libration inside a critical surface of Hill, close to the libration point in the center of the globule. Furthermore it does not feel perturbations from other bodies of small masses, belonging to the same globule.

3 ON THE EXISTENCE AND STABILITY OF THE MODEL

For the existence of the hierarchical model described above the following sufficient conditions must hold:

1. the conditions for the point-mass case as formulated by Nezhinskij and Ollongren (1992),

2. the inequalities (for the case $a = b = c$)

$$\rho_i(0) > 9(4\pi r_i^3)^{-1}[m_0 + 4\pi \int_0^{r_i} (\rho(r) - \rho(r_i))r^2\, dr]\,,\ i = 1, 2, \ldots, 2M\,, \tag{1}$$

where r_i is the barycentric radius of body i and m_0 is the mass of the nucleus.

The condition (1) can be derived by considering Hill's surface around the mass i. If (1) is not fulfilled then this surface disappears.

The question on the self-gravitation of stellar complexes embedded in a galactic gravitational field of force is open. Since we consider in this article models of **S**- and **SB**-galaxies, which contain non-selfgravitational stellar complexes, conditions for the existence and stability of points of libration in the centers of globules in our model are equivalent to existence and stability conditions for the galactic models themselves.

We have estimated using a method from Nezhinskij and Ossipkov (1987) that the half-time of destruction of a stellar complex is larger than 10^{10} years. So a hierarchical model can be realized as an actual stellar system.

4 THE SCHEME OF THE COMPUTER CONSTRUCTION OF THE MODEL

Call a gravitational system of ($N > 3$) point bodies a *ring-configuration*, if all the bodies are on some circle or ($N - 1$) bodies are on the circle and one is at the center of the circle and the vector sum of gravitational forces, acting on each body on the circle, is directed to the center of this circle (see Nezhinskij, 1982). If the bodies lie in a plane but meet the geometrical requirements only approximately, we call the system a *quasi ring-configuration*. And finally we call the system a *plane central configuration* if all the bodies are in some plane and the vector sum of gravitational forces, acting on the unit mass at the center of each body, is directed to the center of mass of the configuration and is proportional to the barycentric radius vector of this body.

5 GENERALIZATIONS

1. In these definitions, provided that the system of inequalities (1) is satisfied, we can replace the point bodies by globules of small radii (compared to distances between the bodies).

2. The definitions can be maintained if the system described is accommodated in an ellipsoid of rotation (see Section 2) in such a way that the circle lies in its equatorial plane centered at the center of the ellipsoid or the center of mass, respectively (see Nezhinskij and Ollongren, 1992).

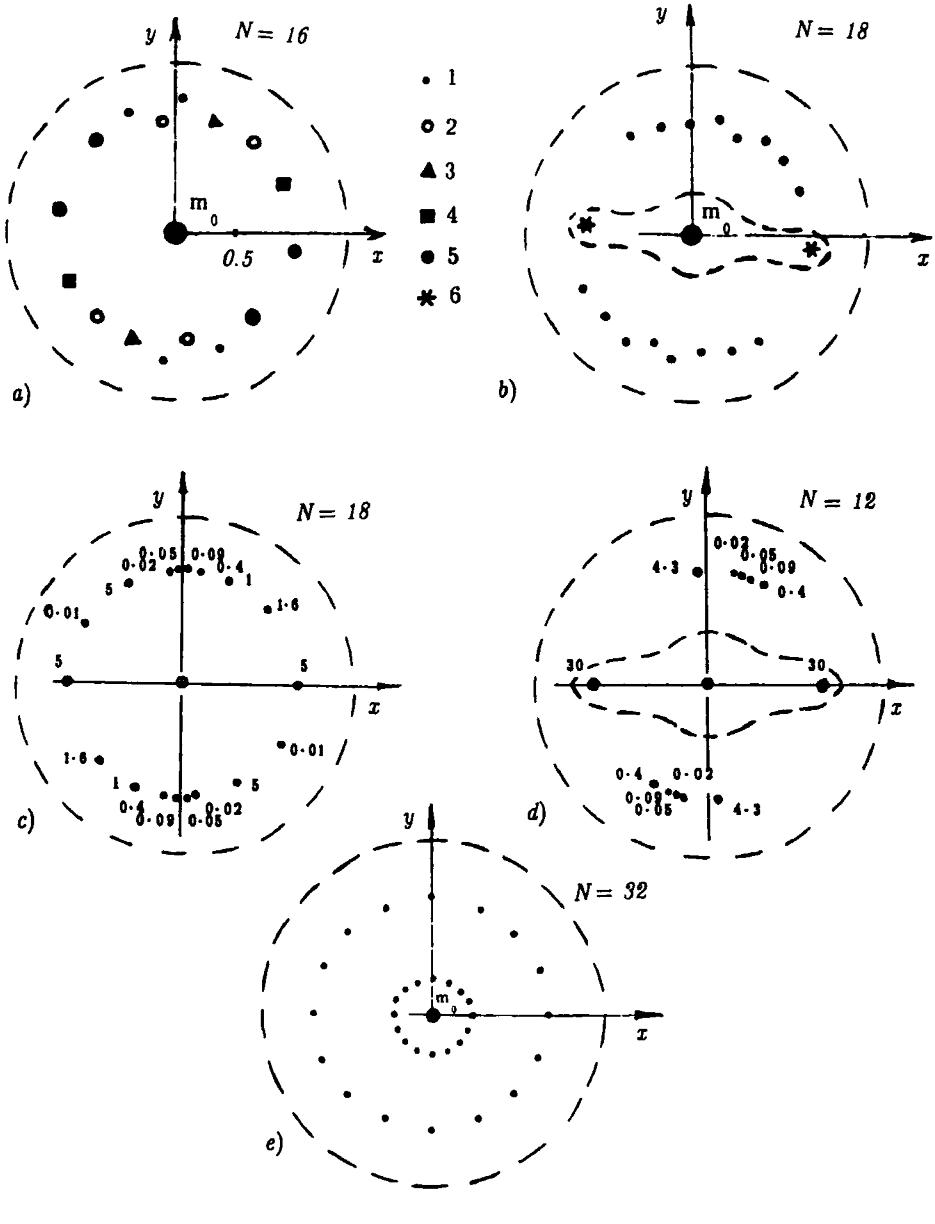

Figure 1 Parameters for the models are chosen in such a way that the total mass sum of the nucleus and the part of the spherical component in the ring-like patchy pattern of globules is equal to $10^{10}M_{\odot}$ and the mean radius of a patchy pattern is 8 kpc. In models *a)* and *b)*, m_0 is equal to 100 in relative units, the density of the spherical component ρ is equal to 1000 and the position of the bodies with masses $m_i = 1, 2, 3, 4, 5, 30$ are marked with the numbers $i = 1, 2, 3, 4, 5, 6$. In models *c)* and *d)*, the potential function of the spherical component is $U(r) = C\ln(1 + r^2/a^2)$, with C and a constants. Close to the position of each globule, its mass is written (the unit of mass is $10^7 M_{\odot}$). Case *e)* is an example of a many-tiered model.

Table.

CONSTRUCTION OF THE RING-CONFIGURATION	The ring configuration exists for any density-function of the ellipsoid, for any N and for any masses of the nucleus and globules. (The proof is in Nezhinskij, 1982)
CALCULATION OF THE COORDINATES OF ALL LIBRATION POINTS OF THE RING-CONFIGURATION	The solution exists for any density function of the ellipsoid, for any N and for any masses of the nucleus and globules. (The proof is in Nezhinskij, 1982)
CONSTRUCTION OF THE CENTRAL CONFIGURATION	The solution exists if the mass of the nucleus is larger than some number $A \geq 0$; A is found experimentally for a chosen density function of the ellipsoid and chosen masses of each of the N globules. (The proof is in Nezhinskij, 1982)
CALCULATION OF THE COORDINATES OF LIBRATION POINTS OF THE CENTRAL CONFIGURATION	The central configuration exists if the mass of the nucleus is larger than some number $B \geq A \geq 0$; B is found experimentally for a chosen density function of the ellipsoid and chosen masses of each of the N globules. (The proof is in Nezhinskij, 1982)
INVESTIGATION OF THE STABILITY OF LIBRATION POINTS OF THE CENTRAL CONFIGURATION	A libration point is stable if and only if (see Nezhinskij, 1983) $2n^2 - U''_{xx} - U''_{yy} > 0$, $n^4 + n^2(U''_{xx} + U''_{yy}) + U''_{xx}U''_{yy} - (U''_{xy})^2 \geq 0$, $(U''_{xx} - U''_{yy})^2 - 8n^2(U''_{xx} + U''_{yy}) + 4(U''_{xy})^2 > 0$ (in these inequalities, U is the gravitational potential)
CONSULT OPERATOR → EXIT	(if all points of libration in the non-zero masses of globules are stable then the operator can decide to exit, otherwise continue)
VARY MASSES OF GLOBULES IN THE NEIGHBOURHOOD OF LIBRATION POINTS OF CENTRAL CONFIGURATION	(The masses of the globules at the stable libration points are varied using a chosen law; the masses at the unstable libration points are decreased)
CONSTRUCTION OF A QUASI RING-CONFIGURATION AND THE CALCULATION OF THE COORDINATES OF THE LIBRATION POINTS	The solution exists if the mass of nucleus is larger than some number $A_1 \geq B \geq A(\geq 0)$; A_1 is found experimentally for a chosen density function of the ellipsoid and chosen masses of all N globules. (The scheme of the proof is the same as in Nezhinskij, 1982)

The existence and stability theorems of families of symmetric central configurations close to ring- (or quasi ring-) configurations (see Nezhinskij and Ollongren, 1992), form the basis of a scheme for the construction of solutions of relative equilibrium in a special case of the N-body problem ($N = 2M$). The essence of the scheme consists of the following: within the bounds of the dynamical model considered (a central configuration) at first we vary the masses of the globules in a neighbourhood of the stable points of libration and then we vary the positions of all the bodies in the dynamical system in order to obtain a new central configuration. This process can be repeated as long as stability of the system is maintained. In this scheme the barycentric system of coordinates (x, y) is used, rotating with constant angular velocity n, in which the system of the globules (the model) is at rest.

In the Table we give a schematic representation of the computer program for the construction of the described models and annotate the basic blocks.

6 DISCUSSION

Several examples of stable models are constructed by the method explained and are shown in Figure 1. An analysis of the models constructed so far shows that a stable patchy pattern of the globules in the equatorial plane of the ellipsoid different from a circle can be built in two cases: A) the gravitational potential of the model is nearly that of a homogeneus ellipsoid in the region where the patchy pattern occurs; B) the patchy pattern is a superposition of several concentric quasi-ring central configurations of globules (Figure 1e).

References

Efremov, Yu. N. (1984) *Vestnik Akad. Nauk SSSR* No. 12, 56.
Nezhinskij, E. M. (1982) *Sov. Astron.* **26**, 485.
Nezhinskij, E. M. (1983) *Sov. Astron.* **27**, 506.
Nezhinskij, E. M. and Ossipkov, L. P. (1987) *Astron. Tzirk.* No. 1474, 1.
Nezhinskij, E. M. and Ollongren, A. (1982) *A. A.* **266**, 140.