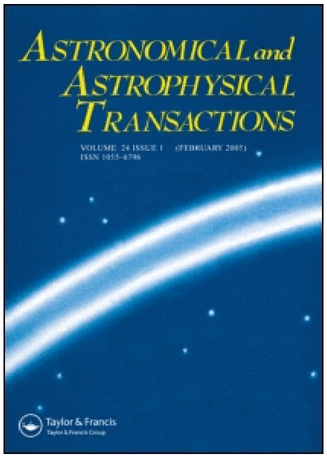

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Stochastic properties of the gravitational **N**-body problem

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STOCHASTIC PROPERTIES OF THE GRAVITATIONAL N-BODY PROBLEM[†]

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The paper summarized the results of a numerical investigation of the exponential orbit instability and "violent relaxation" in gravitating systems.

KEY WORDS Stellar dynamics – N-body problem – instability

This talk summarized detailed numerical simulations which quantify two basic senses in which a self-gravitating system of point masses evolves so as to "forget" certain aspects of the initial conditions.

1. Dating back to the pioneering work of Miller (1964), it has been known that, viewed *microscopically* in the many-particle phase space, N-body simulations are exponentially unstable towards small changes in initial conditions. Given unperturbed and perturbed initial data, $\{\mathbf{r}_i^u(0)\ \mathbf{v}_i^u(0)\}$ and $\{\mathbf{r}_i^p(0)\ \mathbf{v}_i^p(0)\}$, the total configuration $\sum_{i=1}^N |\mathbf{r}_i^p(t) - \mathbf{r}_i^u(t)|^2$ and the corresponding velocity, energy, and angular momentum perturbations grow exponentially on a relatively short time scale. This is, moreover, a genuine instability, and not simply a manifestation of numerical roundoff error, etc.

Recent simulations (Kandrup, Smith, 1991; Kandrup, Smith, 1992; Kandrup, *et al.*, 1993) have demonstrated that this instability is a robust phenomenon. The basic qualitative features are largely independent of: the form of the initial conditions, e.g., whether or not the system is in virial, or collisionless, equilibrium; the amplitude or form of the perturbations; or a possible distribution of masses. The characteristic time scale associated with the instability, as determined by the growth of the total N-particle perturbation, or as probed by various statistics of the individual particles, is, for $N \gg 2$, comparable to, but somewhat shorter than, a characteristic crossing time t_{cr}. When expressed in units of t_{cr}, the time scale for the instability is a *decreasing* function of particle number N but, for $N > 100-200$,

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exhibits at most a very weak dependence on N. The total perturbations in position, velocity, energy, and angular momentum all grow exponentially, on essentially the same time scale, unit the perturbations become *macroscopic* in size and begin to saturate. In this regime, the perturbed and unperturbed systems continue to decohere, although the difference in position and velocity grow more quickly than differences in energy and angular momentum.

To assess the relative importance to this instability of close encounters and the bulk mean field, one can modify the two-body potential by introducing a softening parameter ϵ, and then investigate the effect of increasing ϵ. The net conclusion of such an investigation (Kandrup, *et al.*, 1992) is that the instability is slowed by increasing ϵ, but that it is *not* turned off completely, even for ϵ larger than the typical interparticle spacing.

These conclusions make sense theoretically. For generic perturbations of generic initial data, one anticipates that close encounters (Heggie, *et al.*, 1989) and a bulk mean field (Kandrup, 1990) can each individually trigger an instability on a time scale $\sim t_{\rm cr}$. Moreover, for generic perturbations of generic initial data, the probability that two nearby trajectories do *not* diverge goes to zero exponentially with increasing N (Kandrup, 1990).

2. Lynden-Bell's (1967) original theory of violent relaxation incorporated the idea that, at a coarse-grained level, a self-gravitating systems will typically evolve on a time scale $\sim t_{\rm cr}$ towards the "most likely" state consistent with (a) the holonomic constraints associated with conservation of number, energy, angular momentum, and (perhaps) some third integral, and (b) a coarse-grained conservation of phase. One implication thereof is that particles will typically "forget" all aspects of their initial conditions on a comparable time scale. This idea is not supported by numerical simulations. As first indicated by van Albada (1982), the rapid evolution towards a statistical quasi-equilibrium is not accompanied by a complete shuffling of the individual particles.

Recent simulations of spherically symmetric collapse and of colliding polytropes (Kandrup, *et al.*, 1993) have been analyzed to quantify this phenomenon. Specifically, particles were binned in terms of various aspects of their initial conditions, and the evolution of the different bins then analysed individually. The principal conclusion is that, in a coarse-grained sense, particles "remember" their initial binding energies: If the particles are binned in terms of their initial binding energies, and the mean energy per bin tracked during the subsequent evolution, one does not find that the means evolve towards a common value on a time scale $\sim t_{\rm cr}$. There is, however, a significant growth in the energy dispersions for the individual bins, which indicates that these nonholonomic *mesoscopic* constraints are less pronounced on a *microscopic* level. This fact is corroborated by the evolution of $\mathcal{R}(t)$, the rank correlation of binding energies, which compares the ordering at $t = 0$ with the ordering at later times t. The systematic decrease in $\mathcal{R}$ with increasing t is partially collisionless and partially collisional in origin.

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DISCUSSION

Bisnovatyi-Kogan: What is the difference between "virial" and "collisionless" equilibrium?
Kandrup: Virial equilibrium only implies a balance between the total kinetic and potential energies, T and W, namely $2T/|W| = 1$. Collisionless equilibrium implies a time-dependent solution to the collisionless Boltzmann (i.e. Vlasov) equation.
Bisovatyi-Kogan: What do you think about the existence of some intermediate (between violent and collisional) type of relaxation, claimed by Gurzadian et al.?
Kandrup: If the proposed timescale of Gurzadyan and Savvidy really exists, there should be conditions under which a small perturbation grows on a timescale $\tau \sim N^{1/3} t_{\rm cr}$, with $t_{\rm cr}$ a typical crossing time. I and my collaborators have performed many simulations with total particle number N varying from 16 to 4000, and found absolutely no evidence that $\tau/t_{\rm cr}$ increases with increasing N. Rather, we find that $\tau/t_{\rm cr}$ is a decreasing function of N, although the N-dependence is weak for $N > 100 - 200$. I should also say that the Gurzadyan–Savvidy prediction disagrees with theoretical predictions of other researchers. Using a different line of reasoning, Goodman, Heggie and Hut have argued that close encounters will always induce an instability on a timescale $\tau \sim t_{\rm cr}$ (or, more precisely, $\tau \sim t_{\rm cr}/\log N$).

And, using the same approach as Gurzadyan and Savvidy, I was led to the prediction $\tau \sim t_{\rm cr}$, not $\tau \sim N^{1/3} t_{\rm cr}$. My disagreement with Gurzadyan and Savvidy results because (in their language) I compute an "average" curvature that is different from theirs.
Ivanov: There are observations of the dependence of the velocity dispersion of stars on age. Wielen (1977) showed that the velocity dispersion depends on time as $t^{1/2}$. Have you considered the agreement of your simulation with the observations?
Kandrup: No, I have not.
Ivanov: I would like to make some comments on the issue of Professor Kandrup's report. Professor Chernin and myself studied the dynamics of model triplets of

galaxies. We employed autocorrelation functions, Fourier spectra and the Lyapunov's maximum exponents to detect the dynamical stochastization and to investigate its properties. We found that increase of the softening parameter leads to a significant change in the properties of chaos and more predictable dynamics.
Kandrup: We observe rather similar effect in the simulations which we have performed. For simulations with N larger than about 300, increasing the softening parameter from essentially zero to a value somewhat larger than a typical interparticle spacing slows down the overall rate for the instability by a factor of 5.