DINAMICAL PROPERTIES OF GLOBULAR CLUSTERS: PRIMORDIAL OR EVOLUTIONAL?†

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Some observable relations between globular cluster parameters appear as a result of dynamical evolution of the cluster system. These relations are inapplicable to the studies of the globular cluster origin.

KEY WORDS Globular clusters, Dynamical evolution of star clusters

1 INTRODUCTION

There are some well known relations between dynamical parameters of Galactic globular clusters and their galactocentric distances ($R_g$). If this relations are primordial, we can obtain an important knowledge about physical conditions at the globular cluster formation epoch, especially about conditions in the Protogalaxy. But if the relations are evolutional we do not have such a possibility. In any case we should solve the problem about each particular relation, because the solution of this problem is important to resolve a question about the way of the globular cluster formation. Therefore, it is particularly important to find some invariant quantities which keep stable along the evolutional paths of individual clusters.

The galactocentric distances of globular clusters and the diameters containing half of the cluster mass/luminosity in projection ($D_{0.5}$) are mostly invariable characteristics of them. Therefore, they are used for the comparison of the predictions of the cluster formation theory with observational data. In fact, the correlation between these two quantities discovered by van den Bergh et al. (1991) is used.

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However we have shown that, in spite of a relative evolutional stability of the values $R$ and $D_{0.5}$, the relation between them for the globular cluster system could occur as a result of the disruption of some clusters under the action of evaporation provided they are tidally truncated. Consequently, we cannot use the relation for the verification of any theory of the globular cluster formation.

2 THEORETICAL RELATIONS AND OBSERVABLE CORRELATIONS

According to physical theories of globular cluster formation, their characteristics such as mass, diameter, density, virial velocity, chemical composition, and galactic location are connected to each other (Peebles, 1984; Fall and Rees, 1985; Rosenblatt et al., 1988; Murray and Lin, 1992). These predictions are compared with correlations revealed between the observational parameters of the clusters. Actually, there are observable correlations between the radial velocity dispersion and the total luminosity of the clusters (Paturel and Garnier, 1992), between the mass-to-light ratio and luminosity (Pryor and Meylan, 1993), and other more complicated correlations in a multi-parametric space (Djorgovski, 1991, 1993). But what does it mean? Concerning the matter, it is important to know how much the initial properties of particular clusters and their population as a whole have been preserved, and how much they have evolved.

The idea of the modern population of globular clusters as a surviving population which differs very much from the primordial one has been developed for a long time (Spitzer, 1958; Fall, 1977; Surdin, 1978, 1979). The concept of surviving globular clusters was recognized but its consequences are not fully evident and often are not used in the specific investigations. As an example, one can point out the investigation by van den Bergh et al. (1991) where the connections between the globular cluster galactocentric distance ($R_g$) and their diameter containing half luminosity in projection ($D_{0.5}$) was revealed. The correlation is

$$D_{0.5} \propto R_g^{1/2}. \quad (1)$$

Usually the value of $R_g$ is considered as equal to the value of the initial radius of the globular cluster orbit, and the value of $D_{0.5}$, according to results of numerical simulations (Spitzer, 1987), changes very little during the quasi-stationary evolution of the cluster. Therefore van der Bergh et al. (1991) decide to consider the relation (1) as a primordial one which describes the initial population of the clusters. Some authors of theoretical papers (for example, Murray and Lin, 1992) follow them. This point of view leads to quite determined cosmogonical conclusions and specifically to the conclusion about the formation of the clusters inside the Galaxy.

But we will try to show in this paper that the relation (1) naturally appears in a process of dynamical evolution of practically any initial population of globular clusters in a stationary galactic gravitational field. Therefore, it is impossible to apply this relation to the verification of the globular cluster formation theory.
3 THE TIDAL LIMITATION OF THE CLUSTER SIZE

It is well known that globular clusters are fully relaxed systems whose external radius \((r_t)\) is limited by the galactic tidal field (Binney and Tremaine, 1987; Spitzer, 1987):

\[
r_t = \frac{GM}{2g(e)M_G(R_p)}^{1/3},
\]

where \(M\) is the mass of the cluster, \(M_G(R_p)\) is the mass of the Galaxy inside the perigalactic distance of the cluster \((R_p)\), and \(g(e)\) is a weakly varying function of the cluster orbit eccentricity \((e)\). A singular isothermal sphere is an adequate model of the Galaxy in our case:

\[
M_G(R_p) = \frac{R_pV_c^2}{G},
\]

where \(V_c\) is the circular velocity, and \(G\) is the gravitational constant. In this case, \(g(e) \approx 2\) (Seitzer, 1985). If the mass distribution and the luminosity distribution are similar in the cluster, we can express \(D_{0.5}\) in terms of \(r_{hP}\), the radius containing half the cluster mass in projection:

\[
D_{0.5} = 2r_{hP},
\]

which simply depends on \(r_h\), the half-mass radius of the cluster (Spitzer, 1987):

\[
r_{hP} \approx 0.74r_h,
\]

whose value can be expressed in terms of the core radius \((r_c)\) and the tidal cut-off radius for the King model (Fall and Rees, 1977):

\[
r_h \approx 0.70\sqrt{r_c r_t}.
\]

So the formulae (4)-(6) give us the following simple relation:

\[
D_{0.5} \approx r_t 10^{-C/2},
\]

where \(C \equiv \lg(r_t/r_c)\) is the cluster concentration index. Equation (7) is rather accurate in the range of concentrations \(1 \leq C \leq 2\), where 80% of the Galactic globular cluster belong. This is an approximate equation for clusters of very high and very low concentration. From equations (2), (3) and (7) we finally obtain:

\[
D_{0.5} \approx \left(\frac{GM}{2}\right)^{1/3} \left(\frac{R_p}{V_c}\right)^{2/3} 10^{-C/2}.
\]

Consider the simplest case, i.e., circular orbits \((R_g = R_p)\) and equal concentrations \((C = 1.5)\) for all the clusters. In this case the connection between \(D_{0.5}\) and \(R_g\) depends on the value of \(M\), which cannot be arbitrary on the \(D_{0.5} - R_g\) plane.
4 AN UPPER LIMIT ON THE CLUSTER DIAMETER

An upper limit of the cluster mass is determined by the observational limit of $M_{\text{min}} = 2.5 \cdot 10^6 M_\odot$ ($M/L_V = 3$ is accepted), which is caused by the natural exhaustion of the globular cluster luminosity function for large values of $M$. Then a restriction on the cluster diameter follows from the tidal stability condition (8):

$$D_{0.5} \leq D_{\text{max}} = 8.5 \left( \frac{R_g}{1 \text{kpc}} \right) \left( \frac{220 \text{ km/s}}{V_c} \right)^{2/3} \text{pc},$$

which, for $V_c = 220$ km/s, fits well the observable upper boundary of the cluster distribution on the $D_{0.5} - R_g$ plane (see Figure 1).
5 A LOWER LIMIT ON THE CLUSTER DIAMETER CAUSED BY EVAPORATION

The evaporation (dissipation) of a spherical star cluster is theoretically investigated in detail (see Binney and Tremaine, 1987; Spitzer, 1987). According to the result of numerical simulations the evaporation time of an isolated cluster is $t_{ev} \approx 100 t_{rh}$, where $t_{rh}$ is the half-mass relaxation time. The evaporation is more rapid if the size of a cluster is limited by the external tidal field: $t_{ev} \approx (20-30) t_{rh}$. Taking into account that the physical diameter of a cluster reaches the tidal one only at the perigalactic point of the orbit, we can adopt $t_{ev} = 70 t_{rh}$ as a reasonable compromise for our investigation. According to the definition of $t_{rh}$ (Spizer, 1987), we obtain

$$t_{ev} = \frac{1}{m} \left( \frac{M r_h^3}{G} \right)^{1/2},$$

where $m$ is the mean stellar mass, which we assume to be $m = 0.3 M_\odot$. The evaporation of a cluster is very slow at the beginning, but accelerated to the end of the process. Then we can assume as a probable lower boundary for the cluster mass distribution the value of the mass obtained from equation (10) for a cluster evaporated during time $t_{ev}$:

$$M_{\text{min}} = \frac{G M_\odot^2 t_{ev}^2}{10 r_h^3}.$$  

Substituting $M_{\text{min}}$ from equation (11) to be equation (8), we obtain

$$D_{\text{min}} = 0.73 \cdot 10^{-C/4} \left[ \frac{G M_\odot t_{ev} R_p}{V_c} \right]^{1/3}.$$  

We can assume $D_{\text{min}}$ as a conventional lower limit on the cluster diameter caused by the cluster evaporation in the Galactic tidal field. Considering the simplest case of circular orbits and equal values of the cluster concentration parameter, we can obtain a limit on the cluster diameters:

$$D_{0.5} \geq D_{\text{min}} = 2 \left[ \left( \frac{R_g}{1 \text{ kpc}} \right) \left( \frac{t_{ev}}{1.5 \cdot 10^9 \text{ yr}} \right) \left( \frac{220 \text{ km/s}}{V_c} \right) \right]^{1/3} \text{pc},$$

which, at $t_{ev} = 1.5 \cdot 10^9$ yr and $V_c = 220 \text{ km/s}$, really well fits the observable lower boundary of the cluster distribution on the $D_{0.5} - R_g$ plane (see Figure 1).

6 DISCUSSION

Even with very primitive assumptions about circular orbits and equal values of the concentration parameter for all the clusters, the effects of the tidal truncation and evaporation of the clusters make it possible to predict the region of their localization...
on the $D_{0.5} - R_g$ plane. Besides, the upper and lower limits of the cluster diameters are exponential functions of $R_g$ with indices of $2/3$ and $1/3$. Then the prediction for the relation between these two quantities is $D_{0.5} \propto R_g^{1/2}$. It looks absolutely like the observable correlation by van den Bergh \textit{et al.} (1991).

It is possible, in principle, to consider this dynamical effects in more detail, taking into account the ellipticity of the cluster orbits and the distribution of their concentration parameters. We made the investigation like this and showed that the agreement of theoretical and observable boundaries becomes more precise, and the form of the correlation between $D_{0.5}$ and $R_g$ survive.

7 CONCLUSION

We have shown that the observable correlation (1) between the half-luminosity diameter and the galactocentric distance of globular clusters ($D_{0.5} \propto R_g^{1/2}$) has to arise from the dynamical evolution of the cluster system satisfying only one initial requirement of the existence of an upper mass limit.

Although we have no simple answer to the article title question, we have no reasons to use this relation for the verification of globular cluster formation theories, because it is indefinite, if this one is primordial or evolutionary? This relation only gives us an additional confirmation of a monotonic character of the globular cluster mass function on its massive branch.

Apparently, this conclusion is also related to the globular cluster systems of other galaxies, which show similar relation, for example, to the cluster system of the Large Magellanic Cloud (Hodge, 1962; Mateo, 1987), and NGC 5128 (Hesser \textit{et al.}, 1984).

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References

DISCUSSION

Ossipkov: A short note. Dr. Kutuzov has developed a method for estimating the minimum eccentricities compatible with the observed radial velocities. As an example, we considered only a few of RR Lyr stars and we hope to study globular clusters.

Surdin: It would be interesting.

Efremov: Why globular clusters do not form in the present-day disk of our Galaxy, whereas we have young globular clusters in some other galaxies?

Surdin: This depends on the particular conditions in each galaxy: first of all, on the mass spectrum of giant molecular clouds. I guess that in large disk galaxies a maximum mass of the clouds is not so large as in irregular galaxies, because growth conditions are more violent in the disk ones. Consequently, the star cluster masses in the disk galaxies are smaller.