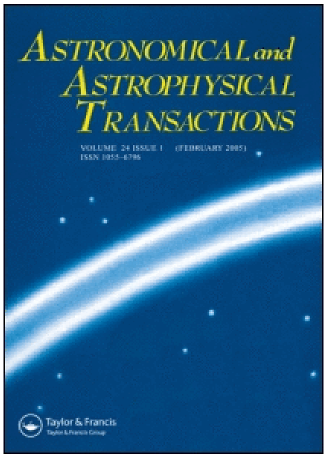

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 April 1995
To cite this Article: Viewga, A. A. (1995) 'The method for studies of galaxy and metagalaxy field structures. Moments of higher orders and the confidence interval', Astronomical & Astrophysical Transactions, 7:2, 155 - 157
To link to this article: DOI: 10.1080/10556799508205409
URL: http://dx.doi.org/10.1080/10556799508205409

Astronomical and Astrophysical Transactions, 1995,
Vol. 7, pp. 155–157
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THE METHOD FOR STUDIES OF GALAXY AND METAGALAXY FIELD STRUCTURES. MOMENTS OF HIGHER ORDERS AND THE CONFIDENCE INTERVAL†

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(Received December 25, 1993)

Agekyan's statistical criterion (Agekyan, 1982) was used for the calculation of higher order moments and finding corresponding confidence intervals.

KEY WORDS Galaxy statistics – methods

A statistical criterion (Agekyan, 1982, Viewga, 1992)

$$A_{2k} = a^{2k}(C_N^2)^{-1}\sum l_{ij}^k, \quad l_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2,$$

$$f(X, Y) = dx_1\, dy_1 \ldots dx_N\, dy_N, \quad 0 \le x_i,\ y_i \le 1 \tag{1}$$

is suitable to calculate the high order moments and to find the probability $P[(A_{2k} - MA_{2k})/\sigma_N <> z]$ with any given accuracy. One has identities:

$$(C_N^m)^n = C_N^m + a_{m+1}C_N^{m+1} + \ldots + a_{mn}C_N^{mn}. \tag{2}$$

In particular,

$$\begin{aligned} (C_N^2)^2 &= C_N^2 + 6C_N^3 + 6C_N^4, \\ (C_N^2)^3 &= C_N^2 + 24C_N^3 + 144C_N^4 + 180C_N^5 + 90C_N^6, \ldots \end{aligned} \tag{3}$$

It is easy to find from (1), that

$$\begin{aligned} MA_2 &= Ml_{12}a^2 = \frac{1}{3}a^2, \\ M(A_2)^2 &= a^4(C_N^2)^{-2}(Ml_{12}^2C_N^2 + 6Ml_{12}l_{13}C_N^3 + 6(Ml_{12})^2C_N^4). \end{aligned} \tag{4}$$

†Proceedings of the Conference held in Kosalma

After integration, (4) can be written as

$$M(A_2)^2 = a^4 (C_N^2)^{-2} \left(\frac{17}{90} C_N^2 + \frac{11}{15} C_N^3 + \frac{2}{3} C_N^4 \right), \tag{5}$$

so that, (Agekyan, 1982)

$$\sigma_N^2 = \frac{2N+3}{45N(N-1)}. \tag{6}$$

In general, one can write

$$M(A_{2k})^n = \frac{a^{2kn}}{(C_N^2)^n} \sum_{s=2}^{2n} \overline{a}_s C_N^s, \quad \overline{a}_s = \sum_{i_r, j_r \le s} M \left(\prod_{r=1}^{n} l_{i_r j_r}^k \right), \tag{7}$$

where $\prod l_{ij}$ are the configurations which are formed by n connections between s objects (all the objects are connected obviously). If $n \geq s$, then some of the connections are repeated; if $n+2 \leq s \leq 2n$, then the configuration consists of some independent subconfigurations. For example, $M(l_{12} l_{34}) = (M l_{12})^2$, $n = 2$, $s = 4$. If all $M \prod l_{ij} \equiv 1$, then (7) transforms to (3) (cf. (4)).

A "spiral" configuration $\xi = \prod_{i=1}^{n} l_{i i+1}$ gives

$$M\xi = \frac{1}{3^n} \sum_{i=1}^{[n/2]} \frac{C_{n-i}^i}{10^i}. \tag{8}$$

The number of configurations of this kind, when $n + 1 = s$, is

$$\frac{n!(n+1)!}{2!} \tag{9}$$

One obtains,

$$M(A_2)^3 = \frac{a^6}{(C_N^2)^3} \left(\frac{29}{210} C_N^2 + \frac{331}{210} C_N^3 + \frac{1733}{315} C_N^4 + \frac{22}{3} C_N^5 + \frac{10}{3} C_N^6 \right), \tag{10}$$

etc. On the next step one considers $\eta = (A_2 - MA_2)/\sigma_N$ and finds semi-invariants from a connection,

$$\kappa_n = n! \sum_{(m_i)} (-1)^{m_1 + \ldots + m_n - 1} (m_1 + \ldots + m_n - 1)! \prod_{j=1}^{n} \left(\frac{M\eta^i}{j!} \right)^{m_j} \frac{1}{m_j!}, \tag{11}$$

$$m_1 + 2m_2 + \ldots + nm_n = n.$$

It was found (Petrov, 1972) that

$$\kappa_3 = \frac{2\sqrt{5}}{7} \frac{2N^2 + 3N + 30}{(2N+3)^{3/2} (N^2 - N)^{1/2}}, \tag{12}$$

$$\kappa_4 = -\frac{2}{7}\frac{2N^3+3N^2+93N-315}{(2N+3)^2(N^2-N)}, \tag{13}$$

$$\kappa_5 = -\frac{120\sqrt{5}}{77}\frac{2N^4+3N^3-87N^2+1125N-1890}{(2N+3)^{5/2}(N^2-N)^{3/2}}, \ldots \tag{14}$$

Then, one has the convergent series, (Petrov, 1972)

$$P(\eta < z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{x^2}{2}}\,dx - \sum_{n=1}^{\infty} Q_n(z), \tag{15}$$

where

$$Q_n(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}\sum_{(s)} H_{n+2s-1}(z)\prod_{j=1}^{n}\frac{1}{m_j!}\left[\frac{\kappa_{j+2}}{(j+2)!}\right]^{m_j}, \tag{16}$$

$$s = m_1 + \ldots + m_n,$$

$$m_1 + 2m_2 + \ldots + nm_n = n$$

and

$$H_m(z) = m!\sum_{j=0}^{[m/2]}\frac{(-1)^j z^{m-2j}}{j!(m-2j)!2^j}. \tag{17}$$

The series applies to any distributions of objects and allows to find the confidence interval accurately.

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