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# THE GALACTIC LATITUDE DISTRIBUTION OF THE INTENSITY MODULATION INDEX OF RADIO SOURCES IN THE WIDE FREQUENCY RANGE

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A statistical analysis of the modulation index ( $m$ ) distribution along the galactic latitude for different samples of variable extragalactic radio sources has been performed in a wide frequency range from 80 MHz to 37 GHz. The distributions of the median  $m$  values have demonstrated a striking monotony at all frequencies from meters to millimetres in spite of a considerable scattering in the individual  $m$  values. Some correlation of  $m$  with the galactic latitude was known for low frequency variability and flickering. It was explained by refraction interstellar scintillation (RISS). The same distribution of  $m$  is unexpected for centimetre and millimetre wavelengths.

The galactic latitude distribution of  $m$  obtained are compared with a simple RISS theory. It is shown that the difference between the observed data and the simple theory of RISS is significant at low galactic latitudes  $|b| \leq 15^\circ \div 30^\circ$  almost for all frequencies considered. Some speculations about the galactic latitude distribution discovered are given in the frame of interstellar scintillation. One of them is the flattening of the electron density fluctuation spectrum at low galactic latitudes.

## 1 INTRODUCTION

It is well established now that the radio emission of many extragalactic sources is variable at a time scale from several hours to several years at wavelengths from millimeters to meters. For the majority of variable sources, an amplitude of the variations lies in the range of 10%, and for individual objects may run up to 50%. Usually the bandwidth of the multifrequency variability has a comparable observing frequency. The variations have a statistical character, therefore they are described by the following parameters:

- time scale,
- modulation index,
- decorrelation bandwidth.

By definition, the modulation index is

$$m = \sigma/S_0 . \quad (1)$$

Here  $\sigma$  is the root-mean-square flux and  $S_0$  is the mean flux.

Historically, the variable sources have been separated into the following groups:

	<i>Frequency range</i>	<i>Modulation index</i>	<i>Time scale</i>
Low frequency variability	80–1000 MHz	< 50%	months–years
Flickering	1–2.5 GHz	< 25%	days–month
Intraday variability	≈ 5 GHz	< 25%	hours–weeks
High frequency variability	5–87 GHz	≈ 10%	days–years

All known types of extragalactic sources (including empty fields) can demonstrate all above groups of variability.

There are two alternative explanations of the variability origin. The first explanation suggests that the reason of the variability is a non-steady-state radio emission process in extragalactic sources (e.g., Kellerman and Pauliny-Toth, 1969, 1981). In this situation, the upper limit of the angular size and brightness temperature of observed extragalactic radio source (estimated from the minimum time scale of the variations) differs significantly from the known characteristics of the synchrotron radio emission “canonical” model.

For the majority of variable sources, the estimated angular size is 3–4 orders of magnitude smaller than the angular size derived the “canonical” model, and the brightness temperature is 3–6 orders larger than the upper limit  $10^{12}$  K (due to the Compton scattering).

Such a situation opens the way to a new hypothesis of the microwave radiation origin of extragalactic radio sources, e.g. a non-cosmological redshift, a relativistic bulk motion towards the observer, and a coherent emission (for example, Rees, 1966; Blandford and Konigl, 1979; Quirrenbach *et al.*, 1991; Terasranta *et al.*, 1992).

The second explanation stems from the fact that the variations are due to an interstellar propagation effect. In the context of this explanation, several authors (Shapirovsckaya, 1978; Rickett *et al.*, 1984; Rickett, 1986; Bondi *et al.*, 1992; Shapirovsckaya *et al.*, 1994) showed that for the majority sources observed variations at decimetre and metre wavelengths may be attributed to refractive interstellar scintillation. By now, there are a body of compelling evidence that the parameters of the variations correlate with galactic coordinates in a wide frequency

range (Shapirovskaia *et al.*, 1993; Shapirovskaia *et al.*, 1994) in accordance with the known distribution of the scattering strength in the Galaxy.

The purpose of this paper is to establish how important is the Galaxy influence on the flux variation; and to refine the distribution of the scattering medium. For this, we have performed a detailed statistical analysis of the galactic latitude distribution of the intensity modulation index for the extragalactic source flux variations at frequencies from 80 MHz to 37 GHz, using the observing results of different authors.

## 2 THE SOURCE SAMPLES

The source samples have been observed with different radio telescopes over a period of a few years at low frequencies and over a few months, at high frequencies. The samples satisfy the following requirements:

1. the observation time is equal or greater than several tens of the characteristic time scale
2. the number of variable sources is no less than 25
3. observations are at the wavelength from millimetres to meters.

We have analyzed the following extragalactic radio source samples:

1. 80 and 160 MHz (Slee and Siegman, 1988)

Number of sources - 412.

Number of variable sources - 103 at 80 MHz and 114 at 160 MHz.

Period of observation - 10-15 years.

Out of all variable sources, we chose for the following analysis only those which had been identified optically and with empty fields. The number of these sources is equal to 51 at 80 MHz and 53 at 160 MHz.

2. 408 MHz (Padielli, 1991, private communication)

Number of source - 133.

Number of variable sources - 54.

Period of observation - over 12 years.

Notice that this source sample is the best among all data at our disposal because each source has been observed many times during 12 years. This sample is denoted by 408(54). Forty four sources have been observed over 40 times. These sources from an individual sample that is denoted by 408(44).

It should be mentioned that the full sample at 408 MHz had been studied extensively with a new method which showed a considerable influence of the

Earth velocity on the time scale of the variability (Bondi *et al.*, 1992). This investigation favours the view that in most cases the variability of extragalactic sources is due to the propagation effects.

3. 1410 MHz (Simonetti and Cordes, 1990)

Number of sources – 165.

Number of variable sources with a flat spectrum – 29.

Period of observation – 2 months.

The modulation indices were determined for flat and steep spectrum extragalactic sources. The modulation indices of the steep sources are within the limits of experimental error. By applying the author's method, the modulation index of a flat spectrum source was calculated as follows:

$$m = (m_F^2 - m_S^2)^{0.5},$$

where  $m_F$  is the observed modulation index of the flat spectrum source, and  $m_S$  is the mean modulation index of steep spectrum sources.

4. 5000 MHz (Quirrenbach *et al.*, 1992)

Number of sources – 49.

Number of variable sources – 27.

Period of observation – few months.

The authors calculated in their paper the quantity  $Y = 3 \cdot m$ , where  $m$  is the modulation index. We used these values  $m$  for our analysis.

5. 22, 37 GHz (Valtaoja, 1992, private communication)

Number of variable sources – 50 at 22 GHz and 51 at 37 GHz.

Period of observation – over 6 years.

### 3 THE GALACTIC LATITUDE DISTRIBUTION OF THE INTENSITY MODULATION INDEX

We begin to study the relation between the modulation index and galactic latitude first without a knowledge of the scattering strength distribution in the interstellar medium. Figure 1 shows the distribution of the modulation index  $m$ , along the absolute-value galactic latitude,  $|b|$ , for all studied source samples. Errors in the modulation index are of the order of ten per cent of  $m$ . One can see from Figure 1 that the galactic latitude range is bounded by 80 or 60 degrees, because the source number is too small (from 1 to 3) to consider higher galactic latitudes.

As can be seen from Figure 1, a wide scatter of the experimental data occurs for all frequencies. We believe that this scatter of  $m$  may be due to natural reasons,

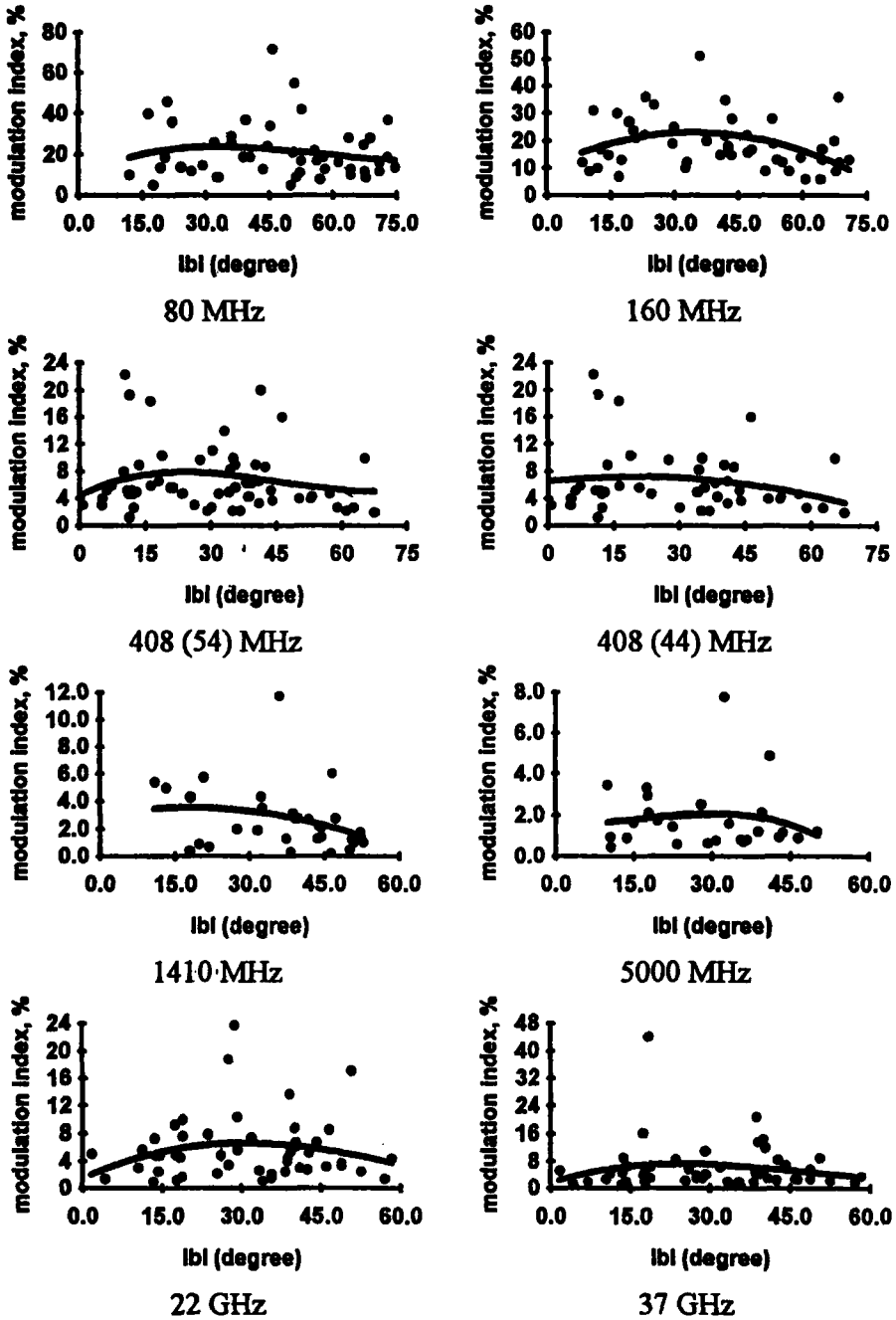


Figure 1 The modulation index,  $m$  (filled circles), for each source against and absolute value of its galactic latitude in the frequency range from 80 MHz to 37 GHz. Errors in the modulation index are of the order of 10% of  $m$ . Solid curve corresponds to a polynomial approximation.

for example, the sources have different intrinsic angular size (see the explanation in Section 4.1).

So, a statistical analysis of the galactic latitude distribution of an average and a median value calculated for some galactic latitude interval is useful. A galactic latitude range from  $0^\circ$  to  $90^\circ$  has been separated by six equal 15-degree bins. In each bin, the number of the modulation index measurements is approximately the same, where it is possible. The average and median values of  $m$  have been calculated in these latitude bins. The use of the median values is preferential for our analysis because the outliers of the modulation indices will be discarded in this case. Therefore the median modulation index  $\bar{m}$ , and the median error,  $\delta\bar{m}$ , have been calculated for each 15-degree latitude bin for all studied extragalactic source samples.

The modulation index distribution in a latitude bin may be considered as Gaussian. Therefore, the median error can be estimated as

$$\delta\bar{m} = 1.25 \cdot \sigma_m / \sqrt{N}, \quad (2)$$

where  $\sigma_m$  is the rms error and  $N$  is the number of sources in the latitude bin.

As can be seen from (2), the median error is larger than the rms error by a factor of 1.25. Note that here the median error is a characteristic of only the modulation index scatter in each latitude bin rather than the measurement error.

The results of the calculations of  $\bar{m}$  and  $\delta\bar{m}$  are presented in Figure 2 for all the samples. These plots have some common main features:

1. Large median errors exist due to a wide scatter of the modulation indices.
2. The median is approximately constant but at least does not increase at low latitudes ( $0^\circ$ – $15^\circ$ ) in comparison which the latitude bin  $15^\circ$ – $45^\circ$ .
3. The median is comparatively smaller at galactic latitudes exceeding  $30^\circ$ – $45^\circ$ .

So, the above-mentioned behaviour of the median modulation index along galactic latitude occurs for all used frequencies except the samples at 80 and 1410 MHz. For these samples, the number of sources in the latitude bin  $0^\circ$ – $15^\circ$  is only two (in another latitude bin of these samples, the number of sources lies in the range from 10 to 15).

It has already been pointed to that the low-frequency, flickering and intraday variability of extragalactic sources may be a result of interstellar propagation effects (a recent paper on this subject: Shapirovskaya *et al.*, 1994). Consequently, some correlation of the modulation indices with galactic latitude was expected at frequencies from 80 to 5000 MHz. In the above-mentioned paper we have revealed some asymmetry in the distribution of the median modulation index towards the Centre- Anticentre-Pole regions of the Galaxy, which points to an important role of the interstellar medium in the variability origin. The median modulation index is the largest towards the galactic Centre and decreases towards the Anticentre and the Pole for the samples analyzed in our paper up to 5000 MHz. A similar analysis has been performed at 22 and 37 GHz. We have obtained similar results – a

decrease of the median modulation index from the Centre to the Anticentre-Pole regions. Up to now, the data for these studies have not been published. From our earlier experience, we expected that the distribution of the modulation index would be nearly similar for all the frequencies studied in this paper. From Figure 2 we can see that this is indeed the case. The results of the following statistical analysis add considerable support to our above assumption.

It was of interest to verify what happens with the behaviour of  $m$  along galactic latitude for independent subsamples selected from our samples. So, all studied samples have been divided according to the following opposite directions:

(C) – in the longitude ( $l$ ) sector towards the Galactic Centre ( $l$  between  $285^\circ$  and  $75^\circ$ );

(AC) – in the longitude sector towards the Galactic Anticentre ( $l$  between  $105^\circ$  and  $255^\circ$ ).

Figure 3 shows the galactic latitude distribution of  $\bar{m}$  for the Centre and Anticentre subsamples at 160 MHz.

For several Centre subsamples, the quantity of the sources is too small to construct any distribution. This is because the majority of the sources for the studied samples have been observed in the Anticentre direction. The analysis performed has shown that the discovered tendency of  $\bar{m}$  exists for the Anticentre subsamples at all frequencies.

In a similar way, we have divided all source samples into two subsamples located in the Northern and Southern sky hemisphere. For the Northern sky hemisphere  $b > 0$ , for the Southern sky hemisphere,  $b < 0$ .

Figure 4 shows the galactic latitude distribution of  $\bar{m}$  for these subsamples at 408 MHz. As can be seen from Figure 2, Figure 3 and Figure 4, the galactic latitude distribution of  $\bar{m}$  for all the subsamples are similar to those for the full samples.

It is vital to verify the statistical significance of the above relation between  $m$  and  $|b|$  by using statistical test.

Let us call the “maximum” bin the latitude bin in which the median modulation index has a maximum. By the  $\chi^2$  test, the galactic latitude distribution of  $m$  on the “maximum” bin ( $15^\circ < |b| < 30^\circ$  for majority frequencies or  $30^\circ < |b| < 45^\circ$  for few frequencies) had been compared with the galactic latitude distribution of  $m$  at higher ( $|b| > 30^\circ$ ) and lower ( $|b| < 15^\circ$ ) galactic latitudes.

The  $\chi^2$  test shows:

1. The largest magnitude of  $\chi^2$  occurs between the “maximum” bin and the range of higher latitudes.
2. The lowest magnitude of  $\chi^2$  occurs between the “maximum” bin and the range of lower latitudes.

Therefore we have obtained that the galactic latitude distributions of  $\bar{m}$  are nearly similar and the results of the  $\chi^2$  test are the same for all the samples and subsamples. So, based on the  $\chi^2$  test, we believe that the modulation indices correlate with galactic latitude. It is important that the  $\chi^2$  test shows the significance of



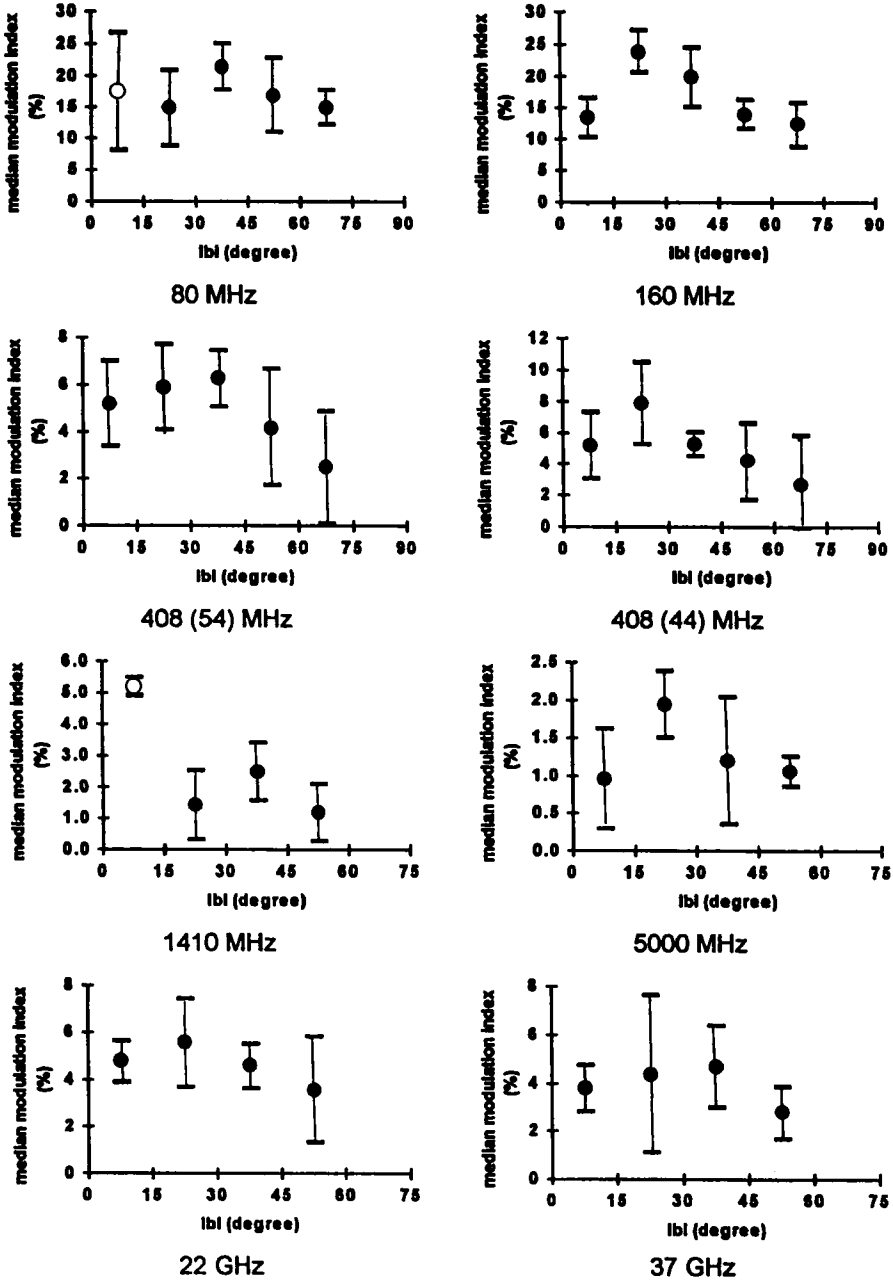
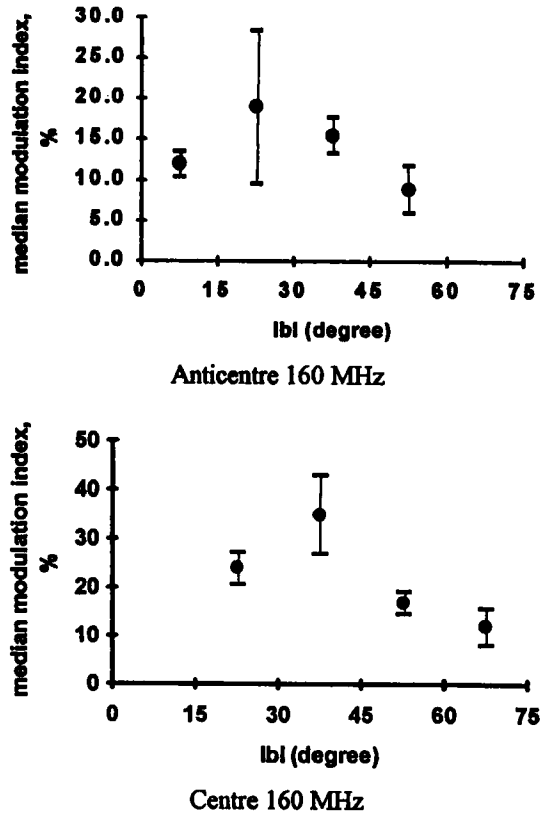


Figure 2 The median value of the modulation index,  $\bar{m}$  (filled symbol), for different  $15^\circ$  latitude bins in the frequency range from 80 MHz to 37 GHz. Median errors have been computed as described in the text. Open symbol indicates an inadequate statistics, i.e., when the number of sources in a latitude bin is equal to only two.



**Figure 3** Filled circles indicate the median values of the modulation index  $\bar{m}$ , for different  $15^\circ$  latitude bins for the galactic Centre and the Anticentre subsamples at 160 MHz. Median errors have been computed as described in the text.

the above shape of the median modulation index distribution ( $\bar{m}$  is approximately constant at low latitude  $0^\circ$ – $15^\circ$  and decreases at latitudes above  $30^\circ$ – $45^\circ$ ).

From the above reasoning, we may compare the obtained relation between  $m$  and  $|b|$  with a simplified theory for the refractive interstellar scintillation (RISS) of extended radio source, presented by Rickett (1986).

#### 4 A COMPARISON BETWEEN EXPERIMENTAL DATA AND RICKETT'S THEORY OF RISS

According to Rickett's model, the refractive modulation index of an extended source is given by

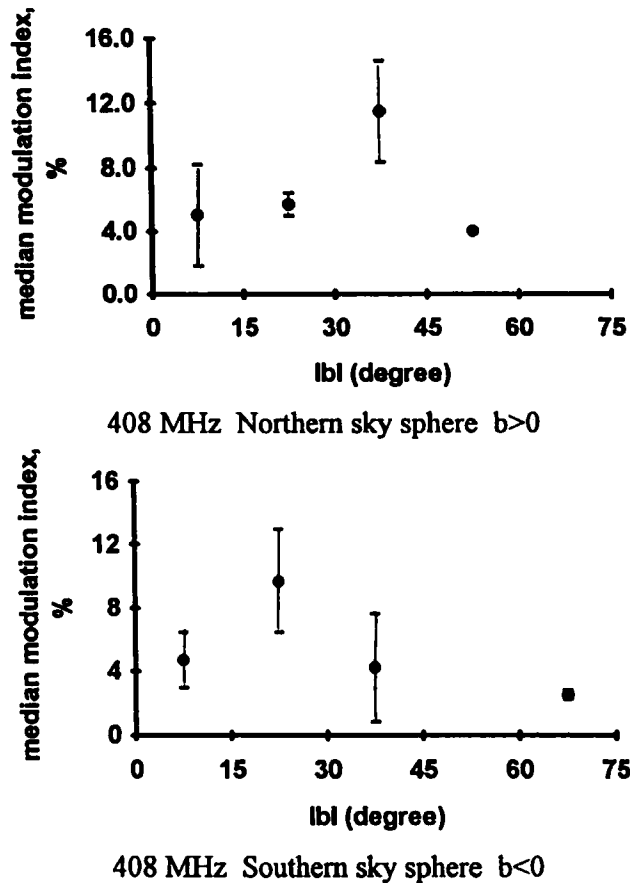


Figure 4 Same as in Figure 3, but only for the Northern and Southern sky hemisphere subsamples at 408 MHz.

$$m_e = \frac{m_r \cdot \Theta_d}{\sqrt{\Theta_d^2 + \Theta_i^2}}, \quad (3)$$

Where  $m_r$  is the modulation index of a point source, which should be of order 0.5,  $\Theta_d$  is the "diffractive" scattering size, or angular broadening size of a point source in the same direction, and  $\Theta_i$  is the intrinsic source size. Rickett's formula assumes that the source brightness distribution is Gaussian in form. This theory uses a two-component model for the distribution of the scattering in the Galaxy. From studies of diffractive interstellar scintillation, Cordes, Weisberg, and Boriakoff (1985) have shown that the distribution can be described by a two-component model. In both components, the spatial spectrum can be approximated as  $C_N^2 \cdot k^{-\alpha}$ , where  $k$  is the wavenumber and  $\alpha \approx 11/3$ . The first component is a  $\pm 500$  pc Galactic disk characterized by  $C_N^2 \approx 10^{-3.5} m^{-6.67}$ . The second component is a layer  $\pm 100$  pc

Table 1

Sample	80	160	408	1410	5000	22000	37000
$a, \%$	16	13	3.7	2.1	1.5	3.3	3.5

with  $C_N^2 \approx 10^{-3}$  to  $1m^{-6.67}$ . The scattering also increases towards the Galactic Centre (Rao and Ananthakrishnan, 1984).

The condition  $\Theta_i \gg \Theta_d$  follows immediately from the scattering data for frequencies exceeding at least 400 MHz. Therefore, we use the following equation obtained for  $\Theta_i \gg \Theta_d$  (Rickett, 1986):

$$m_e \approx 4\lambda_m^2 (\csc b)^{0.5} \Theta_i^{-1}, \quad (4)$$

where  $\lambda_m$  is the wavelength in meters,  $b$  is the galactic latitude,  $\Theta_i$  is the intrinsic source size in *mas*. Thus, the modulation index depends on the galactic as  $m = f((\sin b)^{-0.5})$ . Note that in a more complicated theory,  $m_r$  depends on the wavelength and the scattering angle for point sources.

Then we have carried out a statistical analysis of the comparison between experimental data, namely, the experimental galactic latitude distribution of the modulation index and a theoretical curve following from a simple theory of RISS (see (4)). The theoretical curve  $m = a \cdot (\sin b)^{-0.5}$  (c1) has been fitted to the experimental data by the method of least squares for all the source samples. For each sample, the value of coefficient  $a$  has been calculated and presented in Table 1. Row 1 gives the sample name (named by the observed frequency in MHz). Row 2 gives the values of  $a$  in percentage.

We recall that, from (4) the coefficient  $a$  approximately equals to  $4\lambda_m^2 \Theta_i^{-1}$ . From Table 1 and the above sentence it can be seen that  $\Theta_i$  increases with the wavelength.

From Figure 1 it can be expected that the polynomial approximation is valid for the experimental distribution of  $m$  for almost all frequencies studied. Moreover, the polynomial approximation is usually adopted in the general case, because any function can be expand into the Taylor series. So, if the approximation by  $f = a \cdot (\sin x)^{-0.5}$  is valid for the experimental distribution, then the polynomial is in a close agreement with  $f = a \cdot (\sin x)^{-0.5}$ . From the above, it is of interest to determine the form of the polynomial which has been fitted to the experimental data. So, we have fitted the polynomial of degree 3,  $m = \alpha \cdot b^3 + \beta \cdot b^2 + \gamma \cdot b + \delta$  (c2) ( $\alpha, \beta, \gamma, \delta$  are coefficients) together with theoretical curve (c1) to the experimental distribution of  $m$  for all the samples, except the samples at 160 and 1410 MHz. The approximation by the polynomial of degree 3 without the bends has been impossible for frequencies 160 and 1410 MHz due to a wider scatter of the modulation index. So these samples have been approximated by the polynomial of degree 2. Our actions are readily illustrated by Figure 5. We have obtained that the polynomial shape differs from the fitted theoretical curve shape and agrees with the galactic latitude distribution of  $\tilde{m}$  (see Figure 1). In a similar manner, the

Table 2

Sample	80	160	408	1410	5000	22000	37000
$\sigma_{cv}^t, \%$	65	14	8.9	2.3	2.1	5.5	7.8
$\sigma_{cv}^p, \%$	36	8	4.5	2.2	1.6	2.8	3.3

polynomial has been fitted to the experimental data for the Centre and Anticentre subsamples.

Figure 6 shows the fitted polynomial for the Centre and Anticentre subsamples at 408 MHz. From Figures 1 and 6 we can see that the polynomial shape is nearly similar for the subsamples and all full samples in spite of a wide data scatter. The standard deviation  $\sigma_{cv}^t$  and  $\sigma_{cv}^p$  have been respectively for the theoretical and polynomial curves. The results of these calculations are given in Table 2. Row 1 gives the sample name (named by the observed frequency in MHz). Rows 2 and 3 give, respectively, the standard deviation (in percents) for the theoretical and polynomial curve. Note that the polynomial deviation is less than the deviation for the theoretical curve for all the frequencies.

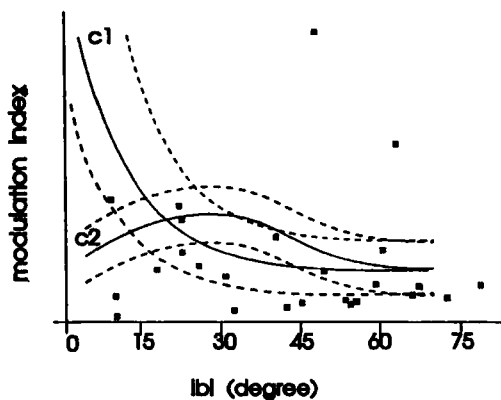
#### 4.1 $\chi^2$ -test

The value of  $\chi^2$  has been calculated by

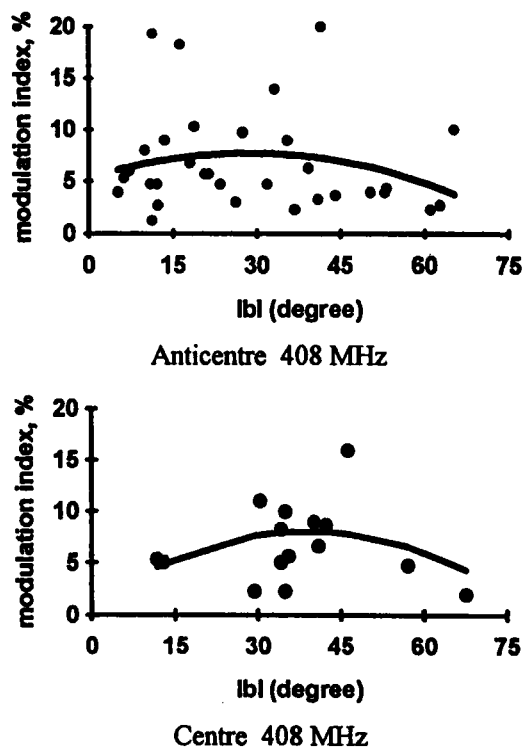
$$\chi^2 = \sum_{i=1}^N [(m_i^e - m_i^{cv}) / \sigma_i^e]^2 \quad (5)$$

where  $m_i^e$  is the  $i$ -th measurement of the modulation index,  $m_i^{cv}$  is  $i$ -th value of the modulation index on the curve,  $\sigma_i^e$  is the experimental error and  $N$  is the number of sources in the samples. The values of  $\chi^2$  have been calculated in two cases. The first case is the comparison between the experimental distribution and the theoretical (c1) approximation. The second is the comparison between the experimental distribution and the polynomial (c2) approximation. This test shows that there is a difference between the experimental distribution and the theoretical approximation, and between the experimental distribution and the polynomial approximation for all the source samples. However, the polynomial approximation is more plausible than the theoretical one because the values of  $\chi_{polyn}^2$  are less than  $\chi_{theor}^2$ . This result was clear from the beginning (from Figure 1).

It is significant that the difference between the polynomial approximation and the experimental distribution can be due to the natural scatter. This scatter follows from the influence of several parameters that appear in the expression for the modulation index (see (3)). The magnitude of the scatter has been estimated on condition that the experimental distribution does not differ from the polynomial distribution. The estimated magnitude of the scattering,  $\sigma_{exp}$ , lies in the range 60–220%. What this means is that the change of any parameter in (3) by a factor 2 may explain all the data scatter. All the results are presented in Table 3.



**Figure 5** Filled symbol indicates the modulation index for each source against the absolute value of its galactic latitude. Solid curves (c1) and (c2) correspond, respectively, to the theoretical approximation  $a \cdot (\sin b)^{-0.5}$  and the polynomial approximation (degree 3). Dotted curves indicate the rms deviations.



**Figure 6** A filled symbol indicates the modulation index for each source against the absolute value of its galactic latitude for the galactic Centre and the Anticentre subsamples at 408 MHz.

Table 3

Sample	80	160	408	1410	5000	22000	37000
$\sigma_{\text{exp, \%}}$	90	60	150	220	140	140	150

#### 4.2 Student's test

Since the  $\chi^2$ -test has established only the tendency of the relation between the modulation index and the galactic latitude, we have performed another statistical test. Using Student's test, we can show that the modulation index does not increase at low galactic latitudes ( $< 15^\circ$ ). Student's test makes possible the estimation of the significance of a difference between two values  $m_1$  and  $m_2$ .

Students coefficient is given by

$$t = \frac{(\bar{m}_1 - \bar{m}_2)}{\sigma} \cdot \sqrt{\frac{n_1 \cdot n_2}{n_1 + n_2}}, \quad (6)$$

were  $\bar{m}_1$  and  $\bar{m}_2$  are the average of the first (experimental points) and the second (fitted theoretical curve) samples respectively,  $n_1$  and  $n_2$  are the numbers of points in the first and second sample and  $\sigma$  is the variance.

In our case, we have calculated the average modulation index,  $\bar{m}_1$  and the standard deviation,  $\sigma_1$ , in the first latitude bin ( $0^\circ$ – $15^\circ$ ) (see Figure 7).  $\bar{m}_2$  is the modulation index in the theoretical curve at  $b = 7.5^\circ$ .  $\sigma_2$  is the standard deviation for the fitted theoretical curve. Then Student's coefficients have been calculated for all the samples. The results of the analysis are presented in Table 4. The calculated coefficient,  $t$ , has been compared with the tabular coefficient  $t(\alpha, n)$ . When  $t(\alpha, n)$  is smaller than  $t$ , then there is a significant difference between  $\bar{m}_1$  and  $\bar{m}_2$ .

From the Table 4 we observe that the difference is statistically significant at high probability for all the source samples, excluding the samples at 1410 MHz.

Row 1 gives the sample name (named by the observed frequency in MHz). Rows 2, 3, and 4 give, respectively, the probability, Student's coefficient, and the tabular coefficient. Since each source sample has been observed with different radio telescopes, these samples can be considered as independent. Therefore, we can calculate the total probability,  $\alpha_\Sigma$ , in a wide frequency band:

Table 4

Sample	80	160	408	1410	5000	22000	37000
$t$	2.79	4.98	1.83	0.38	3.3	2.73	2.41
$t(\alpha, n)$	2.70	3.50	1.70	0.26	2.8	2.70	2.40
$\alpha$	0.99	0.999	0.90	0.20	0.99	0.99	0.98

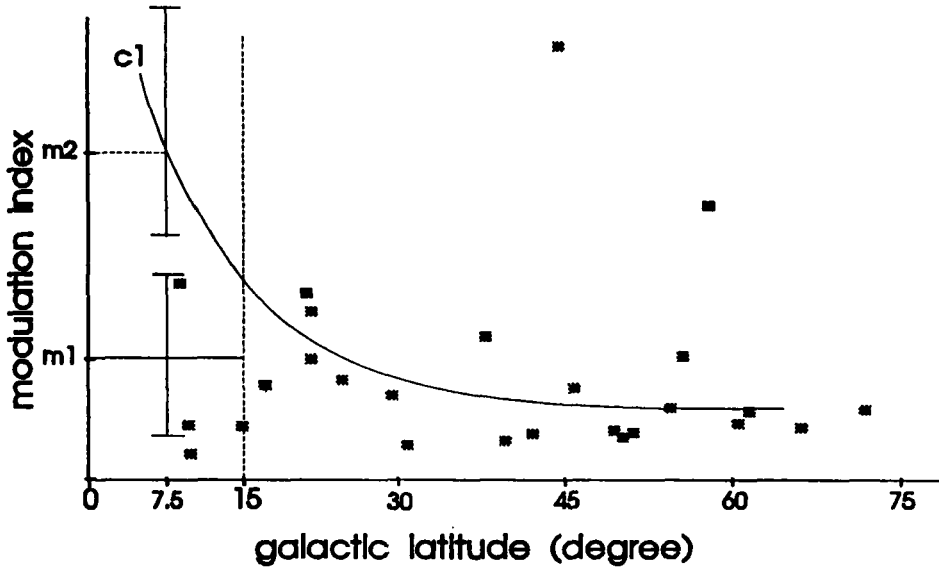


Figure 7 The modulation index for each source against the absolute value of its galactic latitude. Solid curve corresponds to the theoretical approximation  $a \cdot (\sin b)^{-0.5}$ .  $\bar{m}_1$  is the average value of the modulation index in the latitude bin from  $0^\circ$  to  $15^\circ$ .  $\bar{m}_2$  is the modulation index on the theoretical curve at  $b = 7.5^\circ$ .

$$a_\Sigma = 1 - \prod_{i=1}^N p_i, \quad (7)$$

where  $p_i = 1 - a_i$ ,  $a_i$  is the probability at single frequency,  $N$  is the number of the samples. In our case  $a_\Sigma \approx 1 - 1.6 \cdot 10^{-12}$ . This value is near unity, it is a very high probability.

## 5 CONCLUSION

We have used several statistical tests – the  $\chi^2$ -test for a goodness of the fit and Student's test for related samples (Sachs, 1972). The results of these tests confirm each other.

So we have obtained that :

1. Modulation index correlates with galactic latitudes. It gives the possibility to suggest that variability investigated in this paper is produced at all frequencies by the interstellar medium at least for most of the sources. This result confirms the known data at low frequencies and for flickering effects. However this conclusion is unexpected for the millimetre wavelengths so far



as the propagation effects through interstellar plasma are weaker at short wavelengths.

2. Therefore, we can compare the galactic latitude distribution of the modulation index with a simple theory of RISS. The results show that the galactic latitude distribution of modulation index obtained not quite agrees with this simple theory of RISS. As follows from the theory, the modulation index increases with the decrease of galactic latitude. However, we observe that the modulation index is constant at low latitudes and decreases in some cases.

Thus, we suggest several possible explanations of the discrepancy between the experimental data and the theory:

1. If one looks at the formula for the modulation index, (3), the following feature becomes evident:

$$m_e = \frac{m_r \cdot \Theta_d}{\sqrt{\Theta_d^2 + \Theta_i^2}} \Rightarrow \text{If } \Theta_i \ll \Theta_d,$$

$m_e$  depends on galactic latitude, if  $\Theta_i \ll \Theta_d$ ,  $m_e$  does not depend on galactic latitude. One may speculate that  $\Theta_i \ll \Theta_d$  at galactic latitudes below  $40^\circ$ . From a large body of pulsar observations and VLBI measurements of extragalactic sources, it is known that  $\Theta_d$  significantly increases at  $|b| < 10^\circ - 15^\circ$ . However, we believe that this effect is significant only at low frequencies. Nevertheless, the same modulation index distributions are observed up to the mm wavelengths. So, this simple explanation is hardly possible.

2. In the general case,  $m_r$  and  $\Theta_i$  depend on wavelength. In the simple theory of RISS, these values are independent.
3. The Kolmogorov wavenumber spectrum of the interstellar electron density has been used in the simple theory. However, one may believe that the spectrum may be flatter at low galactic latitudes. Then the share of the refraction scintillation (which are produced by strong electron density fluctuations) can decrease at low galactic latitudes, while it is known that  $\Theta_d$  increases at low latitudes (see above) due to the scattering on weak electron density fluctuations.
4. A poor agreement of the results obtained with the simple theory of RISS can be a result of the scattering medium geometry. The large-scale irregularities may be weaker at low latitudes.

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