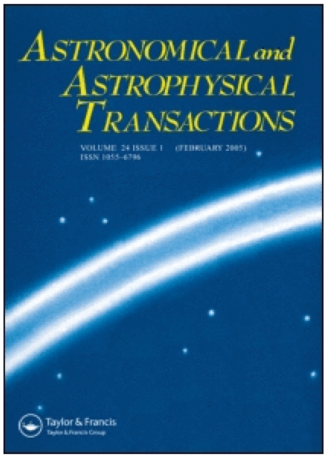

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THE RESTRICTED PHOTOGRAVITATIONAL THREE-BODY PROBLEM: A MODERN STATE

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We discuss a modern state of the three-body problem when an infinitesimal mass is effected not only by gravitation but also by light pressure from one (or both) of the primaries. This problem, called the photogravitational one, attracted much attention during the last ten years. Many aspects of the libration point locations and their stability for all values of radiation pressure and mass ratios are shown and discussed. A retrospective chronological review of the results is given.

KEY WORDS Restricted three-body problem, radiation pressure, libration points, stability

Investigations on photogravitational problems (in particular, the restricted photogravitational three-body problem which may be considered as a generalization of the classical one) take an important place in modern celestial mechanics. Because of this it is of interest to make clear the modern state of the problem and its generalizations.

The photogravitational three-body problem is known in the following main versions: 1) a particle of infinitesimal mass is affected by radiation pressure from only one of the main gravitating mass-points providing its mass to be reduced by this repulsion pressure, 2) a particle is affected by radiation pressure from each of the primaries, both having the reduced masses, 3) only one or both of the gravitating and radiating primaries have a spherical or ellipsoidal form.

It is quite natural to try to develop the formulated problem in the same direction as for the classical restricted three-body problem, and it was done (analyses of the surfaces of zero velocity, the libration points and their stability and so on). Then the results obtained for the classical problem will follow as particular cases from the photogravitational problem when radiation is assumed to be absent. On the other hand, the radiation effects give rise to some new aspects of the problem.

1 THE BASIC FORMULATION OF THE PROBLEM WITH ONE AND TWO RADIATING BODIES

1.1 Basic Equations

Consider the problem in the first formulation when the mass-reduction factor Q of the body with mass M is determined by the value of a resultant force F_Σ of two collinear forces: the gravitational force F_{gr} and the radiation pressure force F_{pr} acting on a particle P of infinitesimal mass m:

$$F_\Sigma = F_{\text{gr}}(1 + F_{\text{pr}}/F_{\text{gr}})r^0 = QF_{\text{gr}}r^0.$$

Here the value of each force (their projections on the unit radius-vector r^0) is

$$F_{\text{gr}} = -G\frac{Mm}{r^2} < 0, \quad F_{\text{pr}} = E\frac{A}{r^2}, \tag{1}$$

where G is the universal gravity constant, A is the transverse section area of the particle P, E is the intensity of radiation.

Then Q may be written in the form

$$Q = 1 + \frac{F_{\text{pr}}}{F_{\text{gr}}} = 1 - \frac{E}{GM} \cdot \frac{A}{m},$$

showing its dependence on the sailness A/m of a particle, which in general may be variable.

For describing the motion of the particle P introduce, as in the classical problem, the rotating Cartesian system Oxyz with the origin O at the barycenter of the primaries M and m (Szebehely, 1967). Let R_1, R_2 and R_0 be the distances of the particle P from the primaries and the origin O, respectively, so that

$$R_1^2 = (x+\mu)^2 + y^2 + z^2, \quad R_2^2 = (x+\mu-1)^2 + y^2 + z^2,$$
$$R_0^2 = x^2 + y^2 + z^2,$$
$$\mu = \frac{m}{M+m}.$$

Then the system of the equations of motion is

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x},$$
$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y},$$
$$\ddot{z} = \frac{\partial U}{\partial z}, \tag{2}$$

where the potential U is

$$U = \frac{Q(1-\mu)}{R_1} + \frac{\mu}{R_2} + \frac{1}{2}(x^2 + y^2). \tag{3}$$

These equations are not integrable, of course, but they have nine families of particular solutions – the relative equilibrium states – five of which are analogous to the libration points of the classical restricted three-body problem.

1.2 The Libration Points Analogous to the Classical Ones

The first study on the libration points of the photogravitational problem in the above formulation was carried out by Radzievsky (1950) who found out that their location strongly depends on the mass reduction factor Q. First, he considered a planar case (1950) and later, a nonplanar one (1953a).

One of the peculiarities of the libration points of the photogravitational problem is that their appearance is related to the distinct evolution patterns of the cavities of the zero-velocity surface. For example, the points L_1 and L_2 can exist simultaneously or L_2 can appear before L_1. Besides, if the points L_4 and L_5 form, in classical case, an equilateral triangle with the primaries then, in the photogravitational problem, with one radiating body they form an isosceles triangle.

Radzievsky has also generalized the photogravitational problem to the case of two radiating bodies (primaries) with the mass reduction factors Q_1 and Q_2. Then the force function, instead of (3), will be

$$U = \frac{2Q_1(1-\mu)}{R_1} + \frac{2Q_2\mu}{R_2} + \frac{1}{2}(x^2+y^2), \tag{4}$$

and for the triangular libration points we shall have

$$R_{1,2} = Q_{1,2}^{1/3}, \tag{5}$$

from which it is clear that the triangular libration points can exist only for positive Q_1 and Q_2 when gravitation prevails. Since all physically possible values of Q_1 and Q_2 are subject to the inequality $Q_{1,2} \leq 1$, then the whole set of these points entirely fills a domain bounded by two circles of unit radius centered at each primary with masses μ and $1-\mu$. In this case, triangular points already do not form an isosceles triangle (besides the case $Q_1 = Q_2$), so that

$$R_{1,2} = Q_{1,2}^{1/3}.$$

Hence, the conditions of existence of these points may be represented by the inequalities

$$Q_1 \geq O, \quad Q_2 \geq O, \quad Q_1^{1/3} + Q_2^{1/3} \geq 1. \tag{6}$$

1.3 The Coplanar Libration Points (Not Having Analogies in the Classical Case)

It was shown (Radzievsky, 1953a) that these points (L_6 and L_7) are located in the XZ plane symmetrically with respect to the X-axis along the curve which starts at one of the primaries and asymptotically approaches the Z-axis.

1.4 Zero-Velocity Surfaces

Radzievsky (1950, 1953a) has carried out a comparative analysis of zero-velocity surfaces with the classical case yet schematically, whereas Colombo (1966) gave a first real geometrical picture of the surface section for the point L_2 in the "Sun-Earth-Dust Particle" system thereby proving the appearance of L_2 before L_1.

A three-dimensional picture of the topological features of the zero-velocity surface for a photogravitational problem with one radiating center was given by Ragos and Zagouras (1988) by means of computer calculations.

A detailed analysis of the zero-velocity surfaces for two radiating centers and for all real values of the parameters μ, Q_1 and Q_2 was carried out by Lukyanov (1988 b).

1.5 Astronomical Applications of the Photogravitational Problem

A possible astronomical application of the photogravitational problem was first pointed out by Radzievsky (1953a, b; 1966): a capture of a particle by a planet (basing on the Jacobi integral), the origin of comets in the framework of the restricted three-body problem, and the theory of the origin, structure and evolution of zodiacal clouds. The zodiacal luminescence phenomena are interesting in connection with the "Sun–Jupiter–Particle" libration point. A possible correlation of possible fly-by's with aperiodic increase in the luminescence of cloud particles was also noted.

Only space-born measurements have revealed weak fluctuations of the brightness of the zodiacal light thereby indicating the existence of dust arcs at the circle axis of the librations (L_4, L_5): if a dust-arc passing through the Sun exists, then each half a year it will result in an asymmetry of the brightness of an opposition observed from the Earth. Later on, these arcs were also predicted by Schuerman (1980) and a program of their observations was proposed by Giovane *et al.* (1985).

2 A PROBLEM WITH A SINGLE RADIATING CENTER (NONCONSERVATIVE PERTURBATIONS, LINEAR STABILITY)

Further progress in the investigation of the problem is associated with including, into equations of motion, some perturbation factors, i.e., adopting more complicated models with allowance for radiation pressure and considering the stability of the libration points with or without these perturbations.

2.1 A Model of a Light Pressure Taking into Account the Dynamical Effect of Radiation

The equations of motion of the restricted three-body problem in a rotating barycenter system with the potential (3) were derived first by Colombo *et al.* (1966) in order to investigate some features of a hypothetic near-earth dust cloud (the "Sun–Planet–Particle" system). Similar equations were derived by Chernikov (1970) for a rotating heliocentric system. Later on, these equations, using the potential (4), were derived and investigated by Schuerman (1980a, b).

The first estimations of the errors in determining the positions of the libration points because of the neglect of the dynamical effect were made by Colombo *et al.* (1966). It was shown that all the points, though in different degree, have some displacement because of this. For example, the triangles $M\ L_4 m$ and $M\ L_5 m$ are not only isosceles but fail to be similar ones. The locus of the positions of the libration points $L_1, \ldots, L_5$ was considered by Chernikov (1970) taking into account the dynamical effect.

2.2 Linear Stability

The first study of the linear stability of the five libration points in the case considered was carried out by Colombo *et al.* (1966). In this work, the libration points in the planar case were found independently of Radzievsky (1950, 1953). Stability was investigated both with taking into account the dynamical effect and without it (as a particular case) from the point of view of destruction of the stability under the action of the dynamical effect which is treated as an external nonconservative perturbation.

The first investigation of the linear stability of the libration points $L_1, \ldots, L_7$, was carried out by Chernikov (1970). It was as well made both including the dynamical effect and without it. Instability of the collinear libration points was established from general physical relations taking into account an analogy with the classical case. A more detailed analytic investigation of the collinear libration points L_1, L_2, L_3 for the "Sun–Earth–Particle" system when $Q \in [1/2; 1]$ was carried out by Filyanskaya (1972). In all these works, the stability analysis was fulfilled on the basis of the linearized equations by considering roots of the characteristic equation. Another and more simple method for proving instability of the collinear libration points was used by Perezhogin (1976) who showed the impossibility of their gyroscopic stabilization from which instability follows according to Kelvin–Chetaev's theorem.

The first investigation of the stability of the coplanar libration points L_6, L_7 in the linear approximation was carried out by Perezhogin (1976): with the help of computer calculation of the roots of the characteristic equation it was shown that for all possible values of the Sun's system parameters these libration points are unstable according to Lyapunov (later Lukyanov (1987) showed that, for some other parameter values, the libration points L_6, L_7 may be stable).

Astronomical applications of the stability problem of the libration points were indicated by Matas (1975), Mignard (1982, 1984) and Perezhogin (1985). Some methods of keeping a spacecraft at the libration points of the photogravitational problem were considered by Polyakhova (1986) and Jumanaliev and Kiselev (1986).

The results on the stability mentioned above were generalized by Bhatnagar (1979a, b) and Sharma (1982, 1987) to the case when one or both primaries are oblate spheroids and one of them is a source of radiation. Stability was investigated in two cases: 1) the bigger body radiates being an oblate spheroid and having the equatorial plane coinsiding with a plane of motion, and 2) the nonradiating smaller body is a spheroid under the same assumptions.

Further studies of the stability of the libration points developed in the following directions: 1) investigations of the linear stability in the case of two radiating centers and further generalization of the problem, including a theory of periodic orbits in the neighborhood of the libration points; 2) extending the range of the mass-reduction factor Q beyond the bounds of the photogravitational problem; 3) a nonlinear analysis of the stability in the case of one or two radiating centers including a resonant situation. Shaboury (1990a, b; 1991, 1992) had investigated the cases of triaxial satellites.

3 THE PHOTOGRAVITATIONAL PROBLEM WITH TWO RADIATING CENTERS

3.1 The Case of Two Positive Reduction Factors

After Radzievsky's basic works (1950, 1953) the case of two radiating centers was investigated again by Schuerman (1980a, b) who considered the triangle libration points and derived the conditions of their stability to the first approximation in small parameters Q_1 and Q_2 belonging to the interval [0; 1].The positions of these points (L_4, L_5) were found as intersections of the two circles (5) under the condition (6). Besides this, the orbits near L_4 and L_5 were considered and it was shown, basing on the linearized system, that they can be unstable untwisting around the libration point. It must be noted that the restriction (6) used by Schuerman does not admit coplanar libration points. This drawback was overcome in further investigations.

3.2 The Photogravitational Problem for All Possible Values of Q_1 and Q_2

A) Collinear libration points. Investigation of all collinear libration points with two radiating centers for all possible values of Q_1 and Q_2 was carried out practically simultaneously by Lukyanov (1984) (the paper was accepted for publication on 30.09.1982) and by Kunitsyn and Tureshbaev (1983) (the paper was accepted for publication on 21.12.1982). In both works, the number and positions of these points depending on the values of the parameters μ, Q_1 and Q_2 were determined using different methods. In the latter paper, it was also shown that in the case of equal masses ($\mu = 0.5$) the interval libration points can be stable to the first approximation. This result disproved a mistaken assertion of instability of collinear libration points (Schuerman, 1980; Giovane *et al.*, 1985). In their later papers, Kunitsyn and Tureshbaev (1985a, 1985b) considered the linear stability of the collinear libration points for all physically possible values of the parameters μ, Q_1 and Q_2. It was shown that the external points are always unstable as well as the interior ones when $Q_1 < O$ and $Q_2 < 0$. But for the interior points, there is some region of stability though it is inaccessible if only one of the primaries radiates.

A similar investigation of the collinear libration points was carried out by Simmons *et al.* (1985) almost at the same time, who considered the whole admissible interval for the mass-reduction factor. The main attention was paid to the transformation of one type of the libration points to another. With the help of the linearized equations of the perturbed motion, the stability of all libration points was investigated and the conclusion about their possible stability mentioned above was confirmed.

B) Triangular libration points. The stability of the triangular libration points, using the linearized equations of motion, was determined practically simultaneously by Simmons *et al.* (1985) and Kunitsyn and Tureshbaev (1985c). The region of stability in the Q_1, Q_2 plane obtained in the first paper was determined providing its bounds are represented by two conjugate ellipses. Another simple geometrical interpretation of this region was given in the second work by means of the introduction of radius-vectors of the libration points and the angles between them and the Ox-axis. Such coordinates give a very simple picture of the stability region since its bounds are presented by arcs of two circles having the segment Mm as their mutual chord. The radii of these circles are functions of μ. By means of such interpretation one may see, that for some values of μ, the stability region breaks up into two subregions separated from each other by an instability gap. All the above results on the stability of the triangular libration points can be treated using this interpretation.

C) Coplanar libration points. Radzievsky (1950, 1953) was the first to point out on the existence of two coplanar libration points L_6 and L_7 providing Q_1 and Q_2 have opposite signs. A more detailed investigation of these points including their stability in the linear approximation was performed by Kunitsyn and Tureshbaev (1985d, 1986) and by Ragos and Zagouras (1988). If one of the sets of inequalities, either $Q_1 > O, Q_2 < O, \mu < 1/2$ or $Q_1 < O, Q_2 > O, \mu > 1/2$, is satisfied, i.e., the mass of the body with positive Q exceeds that of the body with negative Q, then two more points L_8 and L_9 arise (Lukyanov, 1984b).

Investigation of the stability of all coplanar points with the help of the roots of the six-order characteristic equation showed (Simmons *et al.*, 1985) that the points L_8 and L_9 are unstable and the points L_6 and L_7 can be stable for all values of μ.

A discussion of the results obtained by Simmons *et al.* (1985) on the stability of the coplanar points for $\mu \neq 0.5$ was initiated by Tureshbaev (1986) and Ragos and Zagouras (1988) as aimed at the following two questions: 1) Is it true that the points L_6 and L_7 are stable only if both the primaries radiate or one of them may be unluminous? and 2) Is it true that the stability region exists only if a smaller body radiates more intensively? This discussion started because of a discrepancy between the text and the figures in the paper of Simmons *et al.* (1985). Ragos and Zagouras (1988, 1990, 1991a, b) made things clear showing by means of computer calculations that: 1) the points L_6 and L_7 can be stable also when only one of the primaries radiates and 2) if $\mu \neq 0.5$ these points can be stable and it does not matter which of the primaries (the smaller or larger one) radiates more intensely. It is of interest to compare the above works with the results of Perezhogin (1976) who proved the stability of the points for the system "Sun–Jupiter–Particle" ($\mu \approx 10^{-3}$, $Q_1 < O$, $Q_2 = 1$), i.e., for the case when the more massive body radiates. Apparently, this result, which seemed to confirm the conclusions of Simmons *et al.* (1985), allowed these authors to use Perezhogin's (1976) results as an example of application of their theory. But in fact this agrees with the conclusions of Tureshbaev (1986) and Ragos and Zagouras (1988) because of the nature of evolution of the stability region of the points L_6 and L_7 when μ decreases. This evolution was investigated by Tureshbaev (1986).

D) A periodic orbit of the photogravitational problem. Radzievsky (1953) was the first to point out the existence of a periodic orbit about the collinear point L_2 in the XOZ plane when only one of the primaries radiates. Later, Filyanskaya (1972) showed that the periodic orbits described using exponents with purely imaginary arguments exist around all collinear libration points. Then Sharma (1982, 1987) considered the periodic orbits in the problem with one luminous body taking into account the effect of nonsphericity of the luminous body. These orbits may be used as holo-orbits for parking large orbital stations (Perezhogin, 1985). The construction of the holo-orbits around the points L_4 and L_5 was fulfilled by Freitas and Valds (1980). For the case when both primaries radiate, Tureshbaev (1986) proved the existence of periodic orbits around the coplanar points L_6 and L_7 taking into account a small eccentrisity of the primaries providing the period of these orbits is the same as that of the primaries. A more detailed investigation of these orbits around the stable points L_6 and L_7 was carried out by Ragos and Zagouras (1988). These periodic orbits are symmetric with respect to the XOZ plane and have three angular frequencies.

Resonance effects in the motion of interplanetary dust were studied by Kogan (1987) basing on the restricted photogravitational three-body problem and corresponding periodic solutions with allowance for the Pointing–Roberson effect. It occurs that dissipation due to a dynamical effect of radiation forms a resonance structure in a planar model of the system "Sun–Jupiter–Dust Particle".

E) A wider range for the mass-reduction factor and other generalizations. Further investigations of the libration points of the photogravitational problem with both bodies luminous were carried out by Lukyanov (1984a, b; 1986b) who allowed for a wider range of variation of the mass-reduction factor considering all possible real values of Q_1 and Q_2. Of course, for $Q_{1,2} > 1$ this brings one beyond the framework of the photogravitational problem but it may be possibly useful if other perturbation factors are considered (for example, Coulomb forces).

It was shown, in particular, that some more collinear libration points may arise in this case. Investigations of stability on the base of the linearized equations were carried out as well.

Further, Lukyanov (1987, 1988a) in the same generalized formulation carried out investigations of the existence and stability of the coplanar (1987) and triangular (1988a) libration points which, on one hand, have verified the above results and, on the other hand, have provided a new interpretation of the stability region in the Q_1, Q_2 variables.

Then (Lukyanov, 1986a, c) an attempt was made to consider variable values of the mass-reduction factor. It was shown, for example, that for existence of the collinear points it is necessary that a linear dependence between Q_1 and Q_2 takes place. From this case it is easy to go on to the photogravitational problem with variable masses of the primaries (Lukyanov, 1989a, b).

4 NONLINEAR ANALYSIS OF THE STABILITY OF THE LIBRATION POINTS

Since the equations of motion of the photogravitational problem can be written in the Hamiltonian form, the stability conditions of the libration points derived from the linearized equations can be considered as necessary ones, because, as it is known, stability of a Hamiltonian system is possible only in the critical case when all the roots of the characteristic equation are purely imaginary or zero-valued. It was established by Lyapunov that then a nonlinear analysis of the perturbated motion equations is necessary. On the other hand, instability determined by means of considering the roots of the characteristic equation is preserved, in accordance with Lyapunov's results, in spite of the presence of any nonlinear terms small enough. It is this situation which takes place in all the above cases of instability of the libration points of the photogravitational three-body problem.

It should be noted that in the case of the linear stability of the libration points we have not such a simple situation (as well as in the classical problem) when the quadratic part of the perturbed Hamiltonian is a positive definite function and the stability of the nonlinear system can be established basing on the Lagrange-Dirichlet's theorem. In accordance with Birkhoff's results, the stability can be proved for a model system in which all terms of any order exceeding or equal some n are omitted. Such kind of stability is called Birkhoff's full stability and it occurs if internal resonances (i.e., integer-valued linear relations between the characteristic exponents) are absent. Along with this, results of the KAM theory (Kolmogorov, 1954; Arnold, 1963a, b; Moser, 1968) may be used from which one can assess the stability of the whole system but for almost all possible initial perturbations in the sense of Lebesgue's measure. In order to determine such a stability, it is necessary to transform the initial Hamiltonian to a normal form and then to check the conditions of nonsingularity according to the KAM theory theorem. A strong instability, if it occurs, may take place only in the presence of internal resonances, provided the resonances of the third and the forth order are the most important ones.

A nonlinear analysis of the stability of the triangular libration points with a single luminous body was first carried out by Kunitsyn and Perezhogin (1978, 1980). For the points L_4 and L_5, the Hamiltonian for a planar perturbed motion was normalized provided the forth order terms are included and then the conditions of the KAM theory theorems were checked up. This normalization was carried out as well in the cases of the resonances of the third and the forth order. The results of these investigations can be formulated as follows: the points L_4 and L_5 are stable in the planar case according to Lyapunov except for some values of Q and μ which correspond to internal resonances of the third and the forth order and maybe to a degenerate case when

$$c_{20}\omega_1^2 + c_{11}\omega_1\omega_2 + c_{02}\omega_2^2 = 0$$

(here ω_1 and ω_2 are the frequencies of the main oscillations). It was pointed out that stability of these points takes place for all planets of the Solar system. A similar analysis for the points $L_4, \ldots, L_7$ was carried out in three dimensions also by Perezhogin (1982). The stability in the sense of the KAM theory and the formal stability have been proved in the whole region of the linear stability except the values of Q and μ mentioned above. An estimation of the Arnold diffusion rate was made which allows to establish the stability of the points L_4 and L_5 for a large time interval.

Later, Perezhogin and Tureshbaev (1987) considered nonlinear stability of the triangular points L_4 and L_5 for the case of two luminous bodies. Again, the conditions of stability in the sense of KAM theory and a formal stability were corroborated in the whole region of parameters Q_1, Q_2 and μ where the linear stability takes place and on the part of the resonance curves of the third and the forth order.

Similar results were obtained for the points L_6 and L_7 (Perezhogin and Tureshbaev, 1989). Nonlinear stability of L_4, L_5 was investigated also by Kumar and Choudry (1986, 1987, 1987–1988, 1990) and by Gozdziewski and others (1991).

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