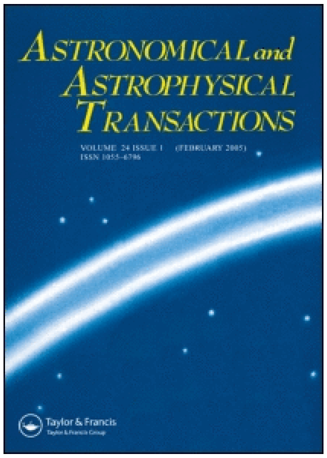

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Initial jeans mass spectra of the three modes of galactic star formation: A theoretical model

V. A. Zakhozhaj [a]
[a] Astronomical Observatory of Kharkov University, Ukraine

Online Publication Date: 01 January 1995
To cite this Article: Zakhozhaj, V. A. (1995) 'Initial jeans mass spectra of the three modes of galactic star formation: A theoretical model', Astronomical & Astrophysical Transactions, 6:3, 221 - 228
To link to this article: DOI: 10.1080/10556799508232069
URL: http://dx.doi.org/10.1080/10556799508232069

Astronomical and Astrophysical Transactions, 1995,
Vol. 6, pp. 221–228
Reprints available directly from the publisher.
Photocopying permitted by license only

INITIAL JEANS MASS SPECTRA OF THE THREE MODES OF GALACTIC STAR FORMATION: A THEORETICAL MODEL

V. A. ZAKHOZHAJ

Astronomical Observatory of Kharkov University,
310022, Kharkov-22, Sumskaya St., 35, Ukraine

(Received August 21, 1993)

We consider the problem of the gravitational instability of gas-dust matter characterized by different spatial distributions consistent with the age of the Galaxy, within the framework of the active phases theory of the Galaxy. Analytical methods are used to obtain the initial mass spectra of the Jeans fragments belonging to the spherical, intermediate and disk components of the Galaxy.

KEY WORDS Gravitation instability, Jeans fragmentation, Galactic evolution

Several developments, namely the results of Larson (1976), the studies of Marochnik and Suchkov (1976) on the distribution of different subsystems as well as a recent paper on the mass spectra of nearby stars (Zakhozhaj, 1990) seem to favour the theory of active phases suggested by Suchkov (1978) as an alternative to hypothesis of continuous star formation. To obtain the rates of star formation at different stages in the evolution of the Galaxy, one needs reliable information on the mass spectrum of fragmenting galactic gas-dust aggregates. In this paper we consider the Jeans fragmentation in the relic gas halo, in the intermediate component and in the Galactic disk.

The density ρ and temperature T of the fragmenting medium determine the Jeans mass ($\mathfrak{m}_j$) given by

$$\mathfrak{m}_j = \frac{d_j T^{3/2}}{\rho^{1/2}} = Y/X, \tag{1}$$

where d_j is a known quantity which can be expressed in terms of fundamental physical constants. Let us suppose that we have obtained expressions for the temperature distribution and density $\psi_i(T)$ and $\chi_i(\rho)$ for the i-th mode of Galactic star formation at the onset of the fragmentation. Then the initial mass spectrum incorporating Equation (1) can be obtained from the following expression:

$$\varphi_i(\mathfrak{m}_j) \propto \mathfrak{m}_j^{-1/3} \int_0^\infty x^{-1/3} \psi_i \left[\left(\frac{\mathfrak{m}_j}{d_j} \right)^{2/3} x^{2/3} \right] \chi_i(x^2)\, dx. \tag{2}$$

1 THE RELIC GAS HALO

In order to describe statistically the process of star generation by means of gravitational instability, let us assume that the relic gas cloud was sufficiently transparent. This assumption imposes a limitation on the density gradient in the direction of the protogalactic cloud center.

On the other hand, fluctuations in matter density can be expected in the gas. The magnitude of such fluctuations is comparable with the mean value. Representing the protogalactic cloud by a polytropic model with the polytrope index n, we obtain

$$\rho = \tilde{S}_1 T^n. \tag{3}$$

Representing the density distribution by the normal law with the mean value $\bar{\rho}$ and dispersion σ_ρ^2, we have

$$f(\rho) = \frac{1}{\sigma_\rho \sqrt{2\pi}} \exp\left[-\frac{(\rho - \bar{\rho})^2}{2\sigma_\rho^2}\right]. \tag{4}$$

Using Equation (3), we obtain the temperature distribution law:

$$f(T) = \frac{nT^{n-1}\tilde{S}_1}{\sigma_\rho \sqrt{2\pi}} \exp\left[-\frac{(\tilde{S}_1 T^n - \bar{\rho}/\tilde{S}_1)}{2\pi\sigma_\rho^2}\tilde{S}_1^2\right]. \tag{5}$$

With the help of Equation (2) we can calculate the initial spectrum for the Jeans fragments as a function of the polytrope index:

$$\varphi_1(\mathfrak{m}_j) \propto \mathfrak{m}_j^{\frac{2n-3}{3}} \int\limits_0^\infty x^{\frac{2n+3}{3}} \exp\left[-\frac{d_j^2}{2\sigma_\rho^2}x^4 + \frac{d_j^2\bar{\rho}}{\sigma_\rho^2}x^2 - \frac{\mathfrak{m}_j^{\frac{2n}{3}}}{2S_1^4\sigma_\rho^2}x^{\frac{4n}{3}} + \frac{\mathfrak{m}_j^{\frac{4n}{3}}\bar{\rho}}{S_1^2\sigma_\rho^2}x^{\frac{2n}{3}}\right] dx, \tag{6}$$

where $S_1 = \tilde{S}_1^{-1} = \text{const}$.

In the completely fluctuating medium of the protogalactic cloud $\rho_c \simeq \bar{\rho}$ (where ρ_c is the density at the cloud center). This corresponds to the condition $n = 0$. In this case integral (6) converges, and the mass spectrum is given by

$$\varphi_1(\mathfrak{m}_j) \propto \mathfrak{m}_j^{-1}. \tag{7}$$

2 THE INTERMEDIATE COMPONENT OF THE GALAXY

The final stage in the evolution of population II stars is connected with the loss of the gas-dust envelope by stars with masses $\mathfrak{m} > \mathfrak{m}_1$ and with the formation of degenerate stars. The density of newly formed gas and dust is smaller than that of the protogalaxy because of a high temperature (a result of supernova explosions at the preceding phase of galactic evolution).

Hence, gravitational instability will not take place. The newly formed gas and dust, having mixed with the initial interstellar gas in a rotating Galaxy, will dilate and cool up to the virial temperature at the expense of emission. After that, the gas-dust component will start collapsing.

The variation in the density of the deposited gas prior to the secondary star formation in the gravitational field of a slowly rotating Galaxy, at angular velocity ω along the axis normal to the Galactic equator (where the gas density is ρ_0), has been well analyzed:

$$\rho(z) = \rho_0 \exp\left(-\frac{3\omega^2}{2\langle v^2\rangle} z^2\right) = \rho_0 \exp\left(-\Omega^2 z^2\right), \tag{8}$$

where v is the velocity of the gas having the Maxwell distribution. The probability for a molecule of gas distributed in a galaxy of radius R to be found in the interval $(z,\ z+dz)$ is given by $f(z)\,dz = \pi(R^2 - z^2)\,dz$ hence, with the help of Equation (8), the distribution function of the gas prior to the secondary star formation is

$$f_2(\rho) = \pi\left(R^2 - \Omega^{-2}\ln\frac{\rho_0}{\rho}\right) \Big/ 2\Omega\rho \ln^{1/2}\frac{\rho_0}{\rho}. \tag{9}$$

Let us assume a Gaussian temperature distribution as in the case of the primary Galactic star formation, then we obtain from Equation (2):

$$\varphi_2(\mathrm{m}_j) \propto \mathrm{m}_j^{-1/3}\left\{S_2 \int_0^1 \ln^{-1/2}\left(\frac{1}{\gamma}\right) \exp\left(-\xi\gamma^2 + \theta\gamma\right) d\gamma \right.$$
$$\left. + S_3 \int_0^1 \ln^{1/2}\left(\frac{1}{\gamma}\right) \exp\left(-\xi\gamma^2 + \theta\gamma\right) d\gamma \right\}, \tag{10}$$

where $S_1 = \text{const}$, $\gamma = x^{2/3}/\xi$, $\xi = \text{const}$, $\theta = \text{const}$.

Expanding the exponent into the Taylor series in terms of γ, with $\gamma \approx 0$, and taking into account the properties of the gamma-function, we come to the conclusion that the sum in the curly brackets is a finite constant, hence:

$$\varphi_2(\mathrm{m}_j) \propto \mathrm{m}_j^{-1/3}. \tag{11}$$

3 THE GALACTIC DISK

After the star formation in the halo had finished, the third stage of the Galaxy evolution – the stage of star formation in the disk – is expected to take place. Unlike the previous ones, the third stage has several phases of star formation. This can be explained using the assumption that every next later phase of star formation takes place in a more and more flattened cloud as a result of gas and dust accumulation at the Galactic equator.

These several phases of stars formation can also be considered as a continuous star formation in the gas-dust component moving around the Galactic center.

In order to describe statistically the process of star formation in the disk, it is necessary to take into account the excess pressure in the Galactic disk resulting from gravitational instability.

We shall restrict our attention to the triggering mechanism for gravitational instability associated with the propagation of spiral density waves along the Galactic disk creating a shock front with excess pressure. In the one-dimensional case, this process can be described using a system of two hydrodynamic differential equations (see Rohlfs, 1977):

$$\frac{d}{dz}(\rho U) = 0, \tag{12}$$

$$\frac{d}{dz}(\rho U^2 + p) = 0, \tag{13}$$

where U is the velocity of the gas relative to the shock front in a direction opposite to the positive z-axis, p and ρ are the pressure and density of the gas, respectively.

Substituting a solution of the system (12)–(13) into Equation (1) and taking into account the equation of state for the ideal gas, we obtain a criterion for gravitational instability in the Galactic disk

$$\mathfrak{m}_j = d_j \frac{(k_1\rho - 1)^{3/2}}{k_2^{3/2}\rho^{7/2}} = k_j \rho^{-2}\left(1 - \frac{1}{k_1\rho}\right)^{3/2}, \tag{14}$$

where $k_1 = \text{const}$,

$$k_j = d_j (k_1/k_2)^{3/2}.$$

Retaining the first term in Equation (14) with the provisions that $(k_1\rho)^{-2} \leq 1$ and that, in Galactic disk, the density of the gaseous component at the z-axis has the distribution $f_3(\rho) = f_2(\rho)$, we obtain

$$\varphi_3(\mathfrak{m}_j) \propto \pm \mathfrak{m}_j^{-1}\left[b \ln^{-1/2}(a\mathfrak{m}_j) - \ln^{1/2}(a\mathfrak{m}_j)\right], \tag{15}$$

where $a = k_j\rho_0^2$ and $b = 2\Omega^2 R^2$.

The sign is fixed by the normalization of the distribution function (15).

4 TESTING THE THEORY

At different stages in the evolution of the Galaxy, the gas-dust matter fragmentation can be described by different mass spectra of the Jeans fragments. The coincidence of the stellar mass spectrum and that of the Jeans fragments can be expected only for the mass range $\simeq (0.1-10)\mathfrak{m}_\odot$ at the main sequence. The fragments with masses

$m > 10m_\odot$ form stars with systematically smaller masses (Masevich and Tutukov, 1988).

Here, it is necessary to take into consideration that:

1. The mass spectrum observed for the stars with masses $m \leq 10m_\odot$ is a sum of the mass spectra of the Jeans fragments having the weights which correspond to the stellar contribution to the density in the Galaxy at different stages of star formation.

2. Within the time T_1 after the moment of the completion of the i-th phase of the Galactic star formation, all the stars with masses $m > m_i$ leave the main sequence. Hence, presently, the following equality is valid for each mass spectrum: $\varphi_i(m)|_{m>m_i} \equiv 0$. Therefore, the observable mass spectrum of stars, within the framework of the active phases theory, is given by a step function.

If the argument value (m_i) at the discontinuities of the first kind is related to the age of stars at the main sequence, it is possible to calculate the time scale for the active phases of the Galaxy: $t_1 = T - T_1$, where T is the age of the Galaxy. Or conversely, we can calculate m_i from a known cosmogonic scale of the Galaxy (Suchkov, 1986).

We have calculated the expected upper mass limits for stars at every stage of star formation which are accessible for observations. The result is that for the first stage of star formation it is possible to observe the stars with the masses $m \leq (0.8 \div 0.9)m_\odot$. Most probably, they are subdwarfs. For the second stage, $m \leq (0.9 \div 1.0)m_\odot$, and for the third one $m \leq (1.1 \div 1.2)m_\odot$.

The fourth stage took place a billion years ago and is still going on. Consequently, there is no restriction on this mass spectrum.

Hence, we obtain the resulting mass spectrum of stars:

$$\varphi(m) = Am^{-1}|_{m<0.9} + Bm^{-1/3}|_{m<1.0} + Cm^{-1}\left[b\ln^{-1/2}(am) - \ln^{1/2}(am)\right]_{m<1.2} + Dm^{-1}\left[b\ln^{-1/2}(am) - \ln^{1/2}(am)\right]_{m>0.1}. \tag{16}$$

As can be easily seen from (16), the most complicated structure of the mass spectrum is expected for the dwarf masses, $m = (0.9 \div 1.2)m_\odot$. Their abundance has been best investigated by means of the nearby star statistics expect for dwarfs (the stars of the first stage of star formation). The number of subdwarfs comes out as ~ 2 percent, and there is no information about their masses (Zakhozhaj, 1990).

In order to test this statistical theory, we have used the most reliable data on the mass spectrum of the main-sequence stars obtained from the statistics of more than 250 stellar masses in the solar neighbourhood, taking into account the data deficit for the total number of stars of low luminosity with the masses $m = (0.1 \div 2.4)m_\odot$ (Zakhozhaj, 1990).

Before using Equation (16) for the mass spectrum approximation, it is necessary to consider the constants a and b in detail.

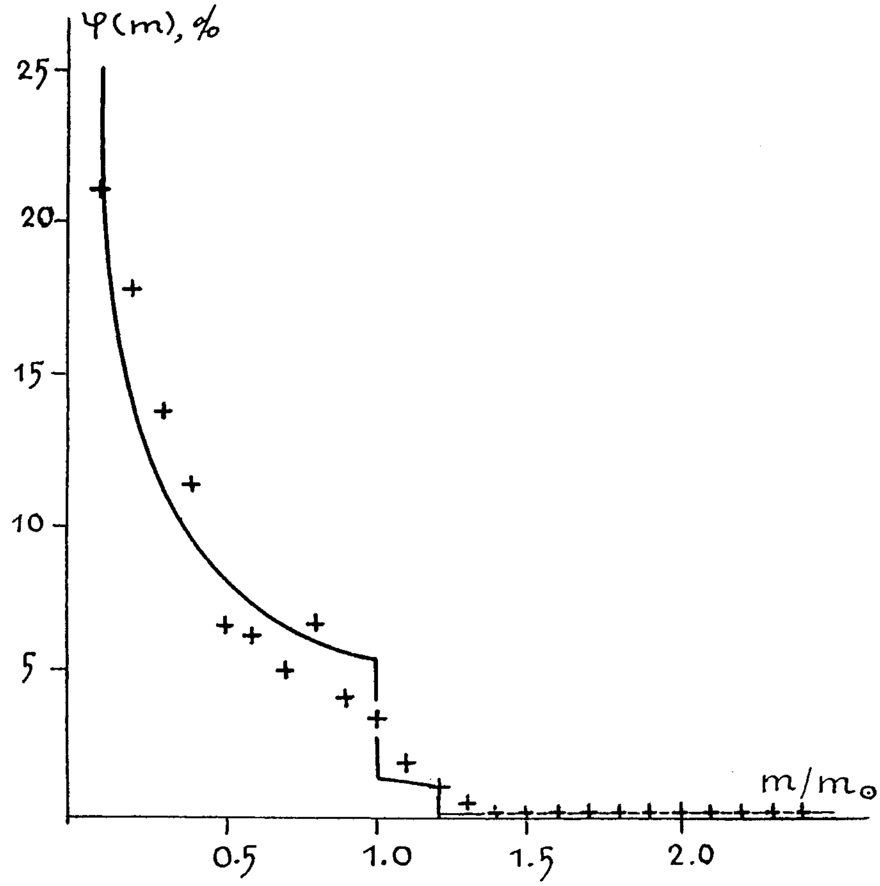

Figure 1

The distribution (9) used to obtain Equation (15) is correct on the condition that $\rho \leq \rho_0 \approx 10^{-24}$ g/cm^3. This inequality is equivalent to the condition: $am_{\rm H} \geq 1$ in Equation (15), where $m_{\rm H}$ is the hydrogen atom mass.

Hence, in order to express the mass in Equation (16) in terms of solar quantities, the constant a should be given by $a = \frac{m_\odot}{m_H} = 1.188 \times 10^{57}$.

To determine the constant b, we have taken into account the radius of the Galaxy ($R \simeq 30$ kps) including the dark matter, rotation frequency ω at the solar distance from the Galactic center, and the root-mean-square velocity of the gas, which obeys a Maxwellian distribution $\langle v^2 \rangle$, obtained from the equality of the kinetic energies of the ionized and neutral gas (for an average temperature). Then $b = 8 \times 10^3$.

Using a and b, we can determine the coefficients B, C, D in Equation (16) by least squares method:

$$B = (3.96 \pm 0.69) \times 10^{-2},$$

$$C = (1.81 \pm 0.23) \times 10^{-5},$$
$$D = (3.45 \pm 0.66) \times 10^{-6}.$$

For the final approximation to the nearby star mass spectrum, we take $A = 0$.

The results of the approximation are presented in Figure 1. The statistical data for the nearby star mass spectrum taken from Zakhozhaj (1990) are marked with "+".

To avoid an obvious correlative interdependency of the functions which describe the third and the fourth stages of star formation, the coefficients C and D have been calculated twice.

First, the value of $D \pm \sigma_D$ for the mass range $[1.3 \div 2.4]\mathrm{m}_\odot$ was obtained, and then the values of $B \pm \sigma_B$, C and D for the masses $[0.1 \div 2.4]\mathrm{m}_\odot$ were calculated. Then the value of D obtained earlier for the masses $[1.3 \div 2.4]\mathrm{m}_\odot$ was tested.

A finite value of $C \pm \sigma_C$ was determined for the mass range $[0.1 \div 1.2]\mathrm{m}_\odot$ assuming that B and C are known. The two values of C are the same.

The coefficients B, C and D enable us to estimate the relative integral contribution of the distribution functions (11) and (15) into the finite mass spectrum. The estimations are given by

$$\begin{aligned} \int \varphi_2(\mathrm{m})\, d\mathrm{m} &= 0.526 \pm 0.089, \\ \int \varphi_3(\mathrm{m})\, d\mathrm{m} &= 0.385 \pm 0.050, \\ \int \varphi_4(\mathrm{m})\, d\mathrm{m} &= 0.089 \pm 0.017. \end{aligned} \tag{17}$$

5 CONCLUSIONS

1. The mass distribution functions obtained for the Jeans fragments describe well enough (with the percentage error less than 20) the mass distribution of the stars generated during the last three periods of star formation.

2. It is impossible to test the mass distribution function for the stars of the first star formation phase because of uncertain information about the subdwarf masses and their distribution function.

3. A study of the resulting standard mass spectrum (16) has shown its significant stability with respect to the choice of the constants a and b. Even the inclusion of higher orders will not affect the results (within 1 percent).

4. From the estimations given in Equation (17), we come to the conclusion that, in the solar neighbourhood, 50 percent of the stellar mass on the main sequence was generated during the second period of star formation, 40 percent of the stellar mass was generated during the third one and 10 percent, during the last period.

References

Larson, R. B. (1976) *M. N. R. A. S.* **176**, 31.
Marsakov, V. A. and Suchkov, A. A. (1976) *Pisma v Astron. Zh.* **2**, 381 (Sov. Astron. Lett., 2).
Masevich, A. G. and Tutukov, A. V. (1988) *Stellar Evolution. Theory and Observation* (Nauka. Moscow) 260 (in Russian).
Rohlfs, K. (1977) *Lectures on Density Wave Theory.* (Berlin: Springer-Verlag; New York: Heidelberg).
Suchkov, A. A. (1986) *The Galaxy.* In: *The Physics of Space.* (Small Encyclopedia. The 2nd edition. Soviet Encyclopedia. Moscow) 62–75 (in Russian).
Zakhozhaj, V. A. (1990) *Astron. Geod. Investig.* (Sverdlovsk) 44 (in Russian).