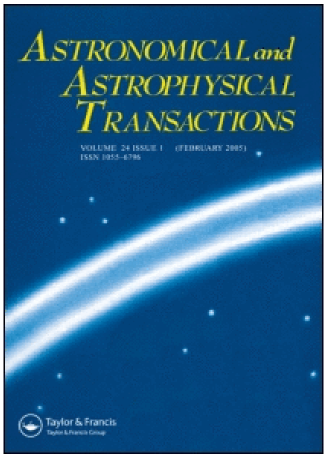

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 January 1995
To cite this Article: Aquilano, R., Barreto, W. and Núñez, L. A. (1995) 'The oscillations of relativistic radiating spheres', Astronomical & Astrophysical Transactions, 6:3, 187 - 196
To link to this article: DOI: 10.1080/10556799508232066
URL: http://dx.doi.org/10.1080/10556799508232066

Astronomical and Astrophysical Transactions, 1995,
Vol. 6, pp. 187–196
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THE OSCILLATIONS OF RELATIVISTIC RADIATING SPHERES

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(Received April 28, 1993)

A method proposed by L. Herrera and collaborators in 1980 to study General Relativistic Spheres in the free streaming out is extended to handle a general radiating spheres. We have applied this method to study a specific scenario, where the luminosity profile is obtained for radially oscillating contracting spheres, and the evolution of the boundary surface is analyzed when pulsating profiles are provided. It is found that, in both cases the oscillatory frequency of the surface coincide with the frequency of the pulsating profile.

1 INTRODUCTION

Recently, it has been reported that the rapid variations of the luminosity of a young supernova remnant, generally interpreted as due to its rotation, can be alternatively explained as radial oscillations of the compact object (Abramowicz, 1989; Lindblom and Mendell, 1994). Ramaty *et al.* (1980) and, recently, Hameury and Lasota (1986) describe γ-ray bursters as objects that transform a fraction of their gravitational energy into gamma radiation via oscillations of the neutron star surface. Energy fluctuations in stellar binary systems endowed with thermomechanical oscillations can be used to describe X-ray bursters (Aquilano *et al.*, 1987, 1988, 1990).

We present results on a close relation between the oscillation of the surface and the pulsation of the radiation. Within the modeling, free streaming out approximation for the radiation is considered.

2 THE FIELD EQUATIONS AND THE METHOD

Let us consider a nonstatic, spherically symmetric distribution of matter formed by a fluid and unpolarized radiation (null fluid). In radiation coordinates (Bondi, 1964), the metric takes the form

$$ds^2 = e^{2\beta}\left((V/r)du^2 + 2dudr\right) - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right), \tag{2.1}$$

where β and V are functions of u and r. Here $u = x^0$ is a timelike coordinate, $r = x^1$ is the null coordinate and $\vartheta = x^2$ and $\varphi = x^3$ are the usial angle coordinates. The u-coordinate is the retarded time in flat spacetime, and therefore u-constant surfaces are null cones open to the future. This latter fact can be readily noticed from the relationship between these coordinates and the usual *Schwarzschild coordinates* (T, R, Θ, Φ)

$$\begin{aligned} u &= T - \int \frac{r}{V} dr, \quad \vartheta = \Theta, \\ r &= R, \quad \varphi = \Phi. \end{aligned} \tag{2.2}$$

It is assumed that for a local observer moving with a radial velocity ω, the space-time contains

i) an isotropic fluid of density $\hat{\rho}$ and pressure $\hat{P}$,

ii) isotropic radiation energy density $3\mathcal{P}$,

iii) unpolarized radiation energy density $\hat{\varepsilon}$ travelling in the radial direction.

For its moving observer, the covariant energy-momentum tensor can be written as

$$\hat{\mathbb{T}}_{\mu\nu} = \begin{pmatrix} (\hat{\rho} + 3\mathcal{P} + \hat{\varepsilon}) & -\hat{\varepsilon} & 0 & 0 \\ -\hat{\varepsilon} & (\hat{P} + \mathcal{P} + \hat{\varepsilon}) & 0 & 0 \\ 0 & 0 & (\hat{P} + \mathcal{P}) & 0 \\ 0 & 0 & 0 & (\hat{P} + \mathcal{P}) \end{pmatrix}. \tag{2.3}$$

The energy-momentum tensor describing this fluid can be written, for a non comoving observer, in the radiation coordinates as (for details see Herrera and Núñez, 1990)

$$\mathbb{T}_{\mu\nu} = (\rho + P)\mathbb{U}_\mu \mathbb{U}_\nu - P g_{\mu\nu} + \hat{\varepsilon}\mathbb{K}_\mu \mathbb{K}_\nu \tag{2.4}$$

where $P = \hat{P} + \mathcal{P}$, $\rho = \hat{\rho} + 3\mathcal{P}$.

The four-velocity is given by

$$\mathbb{U}_\mu = \delta^0_\mu e^\beta \left(1 - \omega^2\right)^{-1/2} \left(\frac{V}{r}\right)^{1/2} + \delta^1_\mu e^\beta \left(\frac{V}{r}\right)^{-1/2} \left(\frac{1-\omega}{1+\omega}\right)^{1/2}, \tag{2.5}$$

the null fourvector is

$$\mathbb{K}_\mu = \delta^0_\mu e^\beta \left(\frac{V}{r}\right)^{1/2} \tag{2.6}$$

Outside the matter, equation (2.1) should represents Vaydia's metric, therefore:

$$\beta = 0, \quad V = r - 2m(u), \quad \text{and} \quad E = \frac{m_0(u)}{4\pi r(r - 2m(u))}. \tag{2.7}$$

In these expression m has been considered as an integration function depending on u. This function is the "mass aspect" defined by Bondi and collaborators (Bondi *et al.*, 1962) and in the static limit it coicides with the Schwarzschild mass. Inside the matter, the configuration $m(u)$ is generalized to $\mathfrak{m}$ by considering everywhere

$$V = e^{2\beta}(r - 2\mathfrak{m}(u, r)). \tag{2.8}$$

Now Einstein's field equations can be written as

$$\frac{\rho + P\omega^2}{1 - \omega^2} + \varepsilon = e^{-2\beta}\frac{r}{V}\mathbb{T}_{00} = \frac{1}{4\pi r(r - 2\mathfrak{m})}\left(-\mathfrak{m}_0 e^{-2\beta} + \left(1 - \frac{2\mathfrak{m}}{r}\right)\mathfrak{m}_1\right), \tag{2.9a}$$

$$\frac{\rho - P\omega}{1 + \omega} = e^{-2\beta}\mathbb{T}_{01} = \frac{\mathfrak{m}_1}{4\pi r^2}, \tag{2.9b}$$

$$\frac{1 - \omega}{1 + \omega}(\rho + P) = \mathbb{T}_{11}e^{-2\beta}\frac{V}{r} = \frac{r - 2\mathfrak{m}}{2\pi r^2}\beta_1, \tag{2.9c}$$

and

$$P = -\mathbb{T}_2^2 = \frac{-\beta_{01}e^{-2\beta}}{4\pi} + \frac{1}{8\pi}\left(1 - \frac{2\mathfrak{m}}{r}\right)\left(2\beta_{11} + 4\beta_1^2 - \frac{\beta_1}{r}\right) + \frac{3\beta_1(1 - 2\mathfrak{m}_1) - \mathfrak{m}_{11}}{8\pi r}. \tag{2.9d}$$

Observe that it is possible to obtain algebraically the physical variables (ω, ρ, P, and ε) from the field equations (2.9a–d), once the functions $\beta(u, r)$ and $\mathfrak{m}(u, r)$ are given. This is only valid for the physical system considered in the present paper, i.e. an isotropic fluid configuration plus a free streaming radiation field. For richer physical environments, such as anisotropic fluids, thermal conducting fluids, viscous fluids and charged fluids and when the surface phenomena are involved, additional information has to be provided (Herrera and Núñez, 1990 and references therein)

Now, defining two auxiliary functions:

$$\tilde{\rho} = \frac{\rho - P\omega}{1 + \omega} \tag{2.10a}$$

and

$$\tilde{P} = \frac{P - \rho\omega}{1 + \omega}, \tag{2.10b}$$

which will be hereafter referred to as the *effective density* and the *effective pressure*, respectively, and integrating equations (2.9b) and (2.9c), it can be obtained that

$$\tilde{\mathfrak{m}} = \int_0^r 4\pi s^2 \tilde{\rho}\, ds, \tag{2.11a}$$

and

$$\beta = \int_{a(u)}^{r} \frac{2\pi s^2}{s - 2\tilde{\mathrm{m}}} (\tilde{\rho} + \tilde{P})\, ds; \tag{2.11b}$$

where $r = a(u)$ defines the boundary of the fluid sphere.

Consiquently, $\mathrm{m}(u, r)$ and $\beta(u, r)$ are written as functions of $\tilde{\rho}$ and $\tilde{P}$ in the same way as they are expressed in terms of ρ and P for the static limit. To complete the HJR method outlined below to be consistent, it is nessesary to match the interior solution to the Vaidya metric at the boundary surface. This matching can be also carried out either by using the Darmois–Lichnerowicz Conditions or by requiring the continuity of the functions β and m across the boundary surface and requiring that

$$-\beta_{0a} + \left(1 - \frac{2\mathrm{m}_a}{a}\right)\beta_{1a} - \frac{\mathrm{m}_{1a}}{2a} = 0 \tag{2.12}$$

(Herrera and Jiménez, 1983).

Using that β is continuous and $\beta = 0$ for the Vaydia metric, we may expand it near the boundary surface $r = a(u)$:

$$\beta_{0a} + \dot{a}\beta_{1a} = 0; \tag{2.13}$$

where $\dot{a} = \frac{da}{du}$. Substituting expression (2.13) into (2.12) and using equations (2.9b) and (2.9c), it is obtained

$$\dot{a} = \frac{(\omega_a \rho_a - P_a)\left(1 - \frac{2\mathrm{m}_a}{a}\right)}{(\rho_a + P_a)(1 - \omega_a)}. \tag{2.14}$$

On the other hand, the matter velosity can be written in the radiative coordinates as

$$\frac{dr}{du} = \frac{V}{r} \frac{\omega}{1 - \omega}. \tag{2.15}$$

Therefore, it follows that

$$\dot{a} = \left(1 - \frac{2\mathrm{m}_a}{a}\right) \frac{\omega_a}{1 - \omega_a}. \tag{2.16}$$

The crucial point of the HJR method is the system of ordinary differential equations for quantities evaluated at the surface. The first of these equations is (2.16). Scaling the radius a, the total mass m and the timelike coordinate u by the total initial mass $m(u = 0) = m(0)$, i.e.,

$$A = a/m(0), \quad M = m/m(0), \quad u/m(0) \Rightarrow u$$

and defining

$$F = 1 - 2M/A, \quad \Omega = \frac{1}{1 - \omega_a},$$

equation (2.16) can be written as

$$\dot{A} = F(\Omega - 1). \tag{2.17}$$

The second *Surface Equation* emerges from the evalution of equation (2.9a) for $r = a + o$, and it takes the form

$$\dot{M} = -F \cdot E. \tag{2.18}$$

As stated above, the total luminosity E is defined by

$$E = \left(4\pi r^2 \varepsilon\right)_{r=a(u)}. \tag{2.19}$$

Now, using the definition above and Equation (2.17), we can rewrite equation (2.18) as

$$\frac{\dot{F}}{F} = \frac{2E + (1-F)(\Omega - 1)}{A}. \tag{2.20}$$

The third *surface equation* can be obtained from the field equations (2.9b), (2.9c) and (2.9d) evaluated at $r = a$, together with condition $F_a = 0$ or, equivalently, by recalling the conservation equation (2.16). Finally, using the *effecive variables* (2.11) and (2.12), and after some straightforward manipulations, we arrive at

$$e^{2\beta}\left(\frac{(\tilde{\rho}+\tilde{P})}{(1-2\mathrm{m}/r)}\right)_o - \frac{\partial \tilde{P}}{\partial r} - \frac{(\tilde{\rho}+\tilde{P})}{(1-2\mathrm{m}/r)}\left(4\pi \tilde{P} + \frac{\mathrm{m}}{r^2}\right) = -\frac{2}{r}\left(P - \tilde{P}\right), \tag{2.21}$$

which is a generalization of the Tolman-Oppenheimer-Volkov (TOV) equation for hydrostatic support in dynamical radiative situations. We stress the conspicuous role played in this expression by the *effective variables* $\tilde{\rho}$ and $\tilde{P}$.

Equations (2.17), (2.20) and (2.21) conform to *the System of Surface Equations (SSE)*. This system may be integrated (numerically in most of the cases), for any given radial dependence of the effective variables. For completeness we outline here a brief resume of the HJR method for radiating fluid spheres in the free streaming out approximation (see Herrera and Núñez 1990, for details):

1. Take a static interior solution of the Einstein equations for a fluid with spherical symmetry.

2. With the r dependence of $\tilde{\rho}$ and $\tilde{P}$ and using (2.11a) and (2.11b) we get m and β up to three functions of u.

3. For these unknown functions of u, we have a system of ordinary differential equations (2.17, 2.20, 2.21) for the quantities evaluated at the surface: *the surface equations*. The first two equations (2.17) and (2.20) are model independent, and the third one depends of the particular choice of the equation of state.

4. One has three *surface equations* (2.17, 2.20, and 2.21) corresponding to: the boundary radius A, the velocity of the boundary surface (related to Ω), the function $\mathfrak{m}$ evaluated at $r = a(u)$ (related to F) and the "total luminosity" FE. Providing one of these functions, the *system of surface equations* can be integrated for any particular set of initial data.

5. By substituting the result of the integration in the expressions for $\mathfrak{m}$ and β, these metric functions become competely determined.

6. The complete set of matter variables for energy density ρ, pressure P, radial matter velocity ω, and radiation energy flux $\hat{\epsilon}$ can be algebraically found for any part of the sphere by using the field equations (2.9a–d).

3 THE MODEL

We shall work out three models previously studied in the free streaming out radiation flux approximation (Herrera *et al.*, 1980). They are: Schwarzschild-like, Tolman-VI-like and Tolman-V-like solutions. In the static limit, the Schwarzschild-like homogeneous solution represents an incompressible fluid of constant density. The equation of state of the static Tolman VI solution approaches the one of a highly relativistic Fermi gas and, therefore, with the corresponding adiabatic exponent of 4/3. Finally, for the Tolman V solution, the relation $P/\rho \sim 1/3$ is maintained during the contraction at the center of the distribution.

In order to close the *system of surface equations*, we have first provided a pulsating pattern for the luminosity profile, and, second, a contracting and oscillatory evolution for the boundary surface is given. This is

$$A(u) = \frac{A_0 - A_f}{e^{\left(\frac{u-\chi}{\tau}\right)} + 1} + A_f\,(1 - \delta \sin \phi u)\,, \tag{3.1}$$

where A_0 and A_f, respectively, represent the initial and the final radius of the configurations, δ is the variation over the the final radius in the oscillatory motion; χ and τ control the contracting evolution as in any Fermi-like distribution function.

The first model considered is the Schwarzschild-like homogeneous model. The effective density is assumed to be

$$\tilde{\rho} = f(u) = \frac{3}{8\pi}\,\frac{1-F}{A^2}, \tag{3.2a}$$

and the effective pressure can be easily computed as

$$\tilde{P} = \tilde{\rho}\frac{(3-2\Omega)(W^2 - r^2)^{1/2} - (W^2 - a^2)^{1/2}}{3(W^2 - a^2)^{1/2} - (3-2\Omega)(W^2 - r^2)^{1/2}}, \tag{3.2b}$$

where

$$W^2 = \frac{3}{8\pi f(u)}.$$

The second model is the Tolman VI-like model expressed in terms of the effective variables:

$$\tilde{\rho} = \frac{3h(u)}{r^2} \tag{3.3a}$$

and

$$\tilde{P} = \frac{\tilde{\rho}}{3}\left(\frac{1-9k(u)r}{1-k(u)r}\right). \tag{3.3b}$$

Obtaned of equations (2.10a) and (2.10b), and therefore, $k(u)$ can be expressed as

$$k(u) = \frac{1}{8\pi}(1-F). \tag{3.4a}$$

$$k(u) = \frac{4\Omega - 3}{3a(4\Omega - 1)}. \tag{3.4b}$$

The last model studied is the Tolman-V-like model, and its effective variables are

$$\tilde{\rho} = \frac{1}{8\pi}\left(\frac{w(u)}{r^2} + z(u)r^{1/3}\right) \tag{3.5a}$$

and

$$\tilde{P} = \frac{1}{8\pi}\left(\frac{w(u)}{3r^2} + \frac{3}{5}z(u)r^{1/3}\right) \tag{3.5b}$$

Functions $w(u)$ and $z(u)$ can be expressed in terms of the surface variables as

$$w(u) = \frac{1}{28}(1-F)(5-2\Omega) \tag{3.6a}$$

and

$$z(u) = \frac{5}{84\pi}\frac{(1-F)(4\Omega-3)}{A^{7/3}}. \tag{3.6b}$$

In the case where the radiation profile is given, Schwarzchild-like homogeneous and Tolman-VI-like solutions have been integrated using

$$A(0) = 10.00, \;\; F(0) = 0.80, \;\; \text{and} \;\; \Omega(0) = 0.80$$

as a set of initial conditions. Corresponding radiation profiles have been given as

$$L \propto 1 - \sin\beta u \tag{3.6}$$

We have run several models with these initial conditions and luminositied.

On the other hand, if $A(u)$ is given, the integration has been carried out using:

$$F(0) = 0.80, \;\; A_0 = 10.0, \;\; A_f = 9.0, \;\; \chi = 5.0, \;\; \tau = 8.0 \;\; \text{and} \;\; \phi = 10^4$$

Variation from $\delta = 10^{-9}$ to $\delta = 10^{-11}$ has been used to run Schwarzschild-like.

Figure 1 displays the evolution of the contracting boundary for the models considered. In Figure 2, a typical given oscillatory radiation profile is sketched. Figure 3 contains the oscillatory radiations profile for the Schwarzschild-like equation of state in the case the evolution of the boundary surface is provided. Finally, Figure 4 displays the same for the Tolman-V and Tolman-VI models.

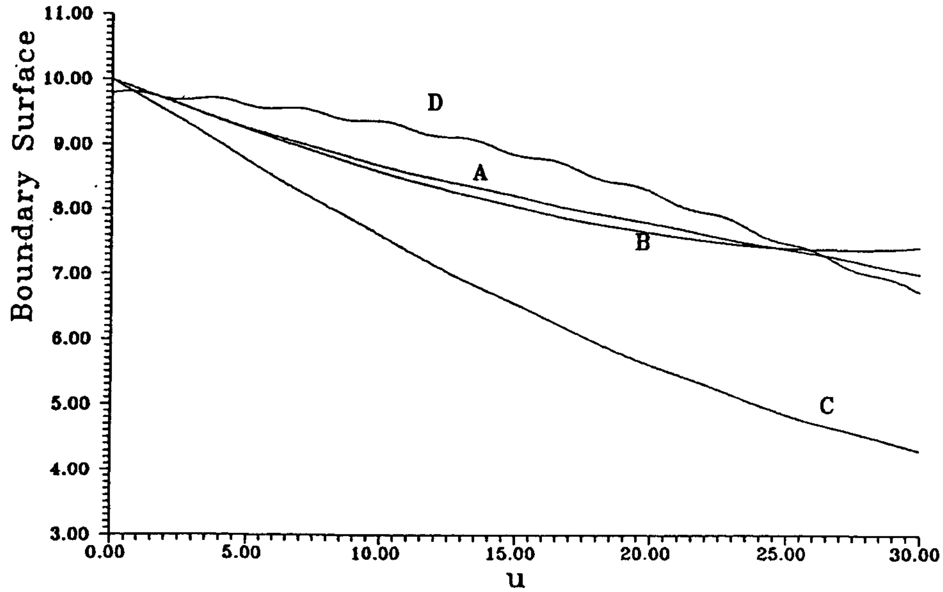

Figure 1 The evolution of the boundary surface for different models. Curves A, B, and C describe the collapse when the pulsating luminosity profile is given, for the Schwarzschild-like, Tolman-VI-like and Tolman-V-like models, respectively. Curve D represents the contracting pattern adopted for the boundary.

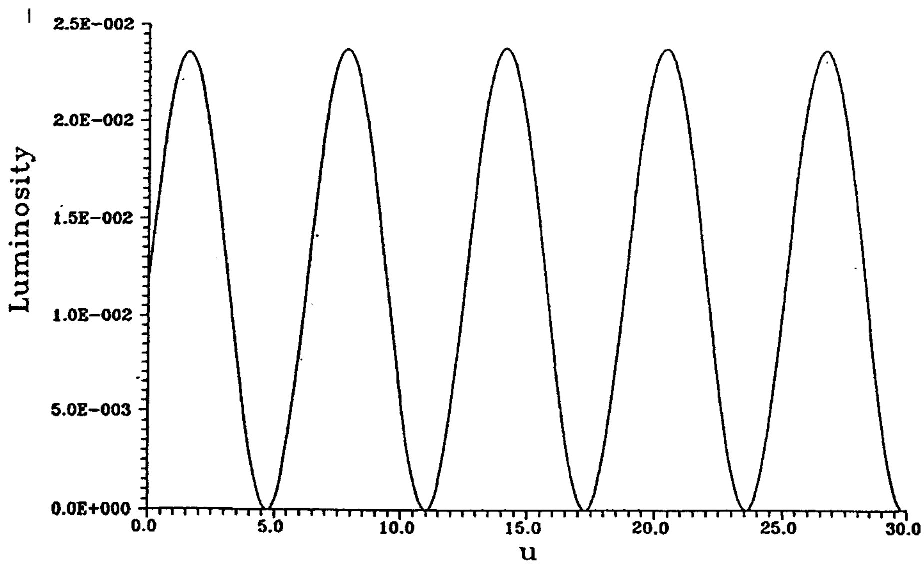

Figure 2 A pulsating radiation profile provided as input to close the system of surface equations.

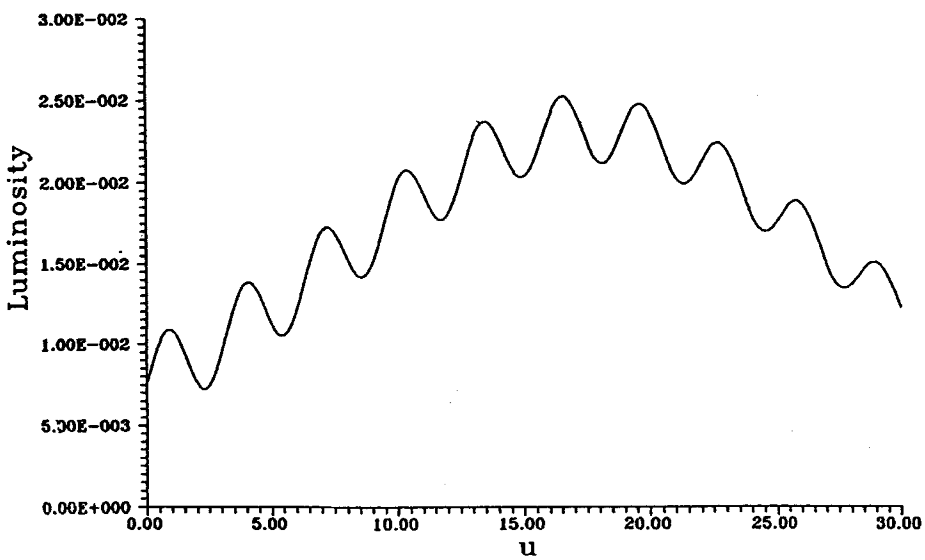

Figure 3 The oscillating luminosity profile for the Schwarzchild-like model.

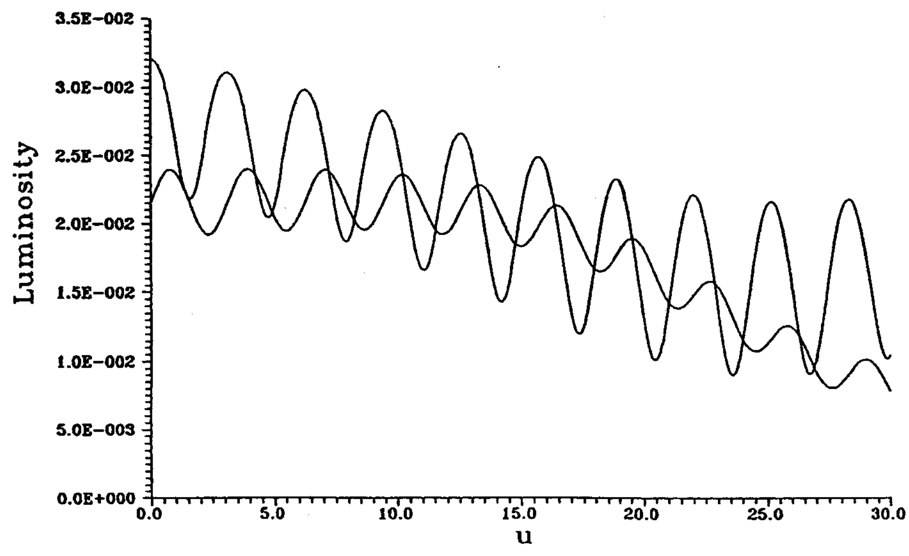

Figure 4 The oscillating luminosity profile for the Tolman-V and Tolman-VI-like models. The curve with a greater amplitude corresponds to the Tolman-V-like model.

4 CONCLUSIONS

Radiation flows freely only at early stages of the collapse, but for the present calculations we are interested to show that, even in the case when matter and radiation are slightly coupled, variations in luminosity may cause the oscillation on the surface of the distribution and vice versa.

It is clear from Figure 1 that the variations of the boundary radius are comparable for all the models considered. Figure 2 contains a typical adopted oscillatory luminosity profile. Again, Figures 3 and 4 show the same period for the variation of the luminosity in these models.

The possibility to treat oscillations in the "diffusion limit" is under a curse. Within this limit, radiation is considered to have a mean free path much smaller than the characteristic length of the system, and this limit is relevant at a later epoch of the collapse.

We would like to conclude to clear to the reader that of the procedure consist in avoiding a head-on integration of Einstein's equations. Instead, a heuristic relation among the matter variables is assumed.

This work has been carried out under the bilateral cooperation agreement CONICIT (Venezuela) – CONICET (Argentina). It has been also partially supported by the Consejo de Desarrollo Cientifico Humanistico y Tecnólogico de la Universidad de Los Andes, under project 559-92-05; by the Consejo de Investigaciones de la Universidad de Oriente under Project C.I. 5-022-00393-89; and by the Directorate General for Science, Research and Development of the European Communities under contract C-11-0540-M(TT).

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