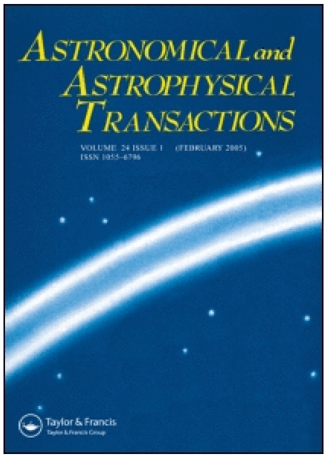

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THE PROBABILITY OF BLACK HOLE FORMATION FROM OSCILLATING COSMIC LOOPS

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A probability of Primordial Black Hole (PBH) formation from oscillating cosmic loops is estimated and some restrictions on this probability are obtained from observational upper limits on the PBH's abundance. Finally some possibility that the end point of any oscillating loop is the formation of a residual black hole of the Plank mass is considered.

INTRODUCTION

First predicted by Zeldovich and Novikov[1] in 1967 and then independently by Hawking[2] in 1971 Primordial Black Holes (PBHs), though never yet discovered, play an important role in exploring the very early Universe, as the unique tool for testing the exotic processes which took place just at the beginning of the Universe. The reasons for that are the following: 1) The PBHs can be formed very early, having arbitrary small masses up to the value of the Plank mass, $M_{Pl} \approx 10^{-5}$ g. This means that their abundance may depend drastically on physical conditions in the very early Universe.

2) The PBHs are almost stable objects with the lifetime scale considerably longer than the time scale of their formation. This means that information about the conditions at the moment of their formation may be stored by PBHs for a comparatively long time.

3) The fractional contribution of PBHs formed during the radiation dominated epoch to the mean energy density of the Universe increases with time as the Universe expands. This means that even a small initial abundance of PBHs could play rather an important role in the physical process in the later stages of the evolution of the Universe, and then PBHs might be observed or restricted by observational data.[1,2]

4) The PBHs of low mass evaporate through quantum processes discovered by Hawking in 1974.[3,4,63] Then not only the gravity of PBHs which is negligible at large distances, but the products of evaporation, e.g. gamma-rays,[5-8] might be a subject of analysis for their cosmological consequences (for other restrictions see[9-22,65,66]).

At present we have only upper limits on PBH density in the Universe. The strongest upper limits correspond to the PBHs of mass of the order of $5 \cdot 10^{14}$–10^{15} g, evaporating just in the present Universe. These restrictions have been obtained for the PBHs evaporating in the present Universe. Handling these upper limits, it is possible to put some constraint on the physical conditions in the very early Universe at the moment of formation of the PBHs[23–35,48,59–62,64] within the mass range mentioned above. In Refs 26–28, for example, some constraints were obtained on the parameters of exotic metastable super-massive particles which could exist in the very early Universe.

The present work is dedicated to the problem of black hole formation from oscillating cosmic loops,[36–39] considering this mechanism of PBH formation from the point of view of the possibility of putting some new independent constraints on the parameters of a cosmic string network (see for example[67–69]).

1. THE CRITERION OF BLACK HOLE FORMATION

The radius vector of mass elements on the loop is given as

$$\vec{R} = [R_0(t) + x(\phi, t)]\vec{n} + y(\phi, y)\vec{j}, \tag{1}$$

where $\vec{n}$ is the unit vector in radial direction, and $\vec{j}$ is the unit vector normal to the plane of the basic circle of radius R_0. The Fourier expansion of variables x and y has the form

$$x = \frac{1}{\sqrt{\pi}} \sum_{n=2}^{\infty} x_n e^{in\phi} + (\text{c.c.}), \tag{2}$$

and just the same for y (c.c. denotes complex conjugate). Then the criterion for black hole formation can be formulated in the most general case in the following way:

Absolute values of all Fourier components $|x_n|$ and $|y_n|$ (for $n = 2, 3, \ldots$) should satisfy the following inequality:

$$\sum_{n=2} (|x_n|^2 + |y_n|^2) < R_s, \tag{3}$$

where R_s is the Schwarzshild radius of the string at the moment when the value of R_0 given by the classic solution for the loop evolution is as small as R_S.

At the initial moment R_S is proportional to the length of the loop. After that, as this length decreases in the process of the loop contraction, the Schwarzshild radius of the loop remains approximately constant, because its value is determined by the total energy of the loop including the kinetic energy of different elements integrated over the loop. This total energy is a constant with time unless the loss of energy due to gravitational radiation[40–45] or friction[46,58] are taken into account.

2. BLACK HOLE FORMATION PROBABILITY AT AN ARBITRARY SPECTRUM OF OSCILLATIONS OF THE LOOP

The probability of black hole formation in the most general case can be given as a product of two factors corresponding to radial and transverse oscillations of the loop:[39]

$$P_{bh} = P_x P_y. \tag{4}$$

Taking into account that we deal with two independent random values for each projection x and y, which are the amplitude and the phase of the perturbation, we have

$$P_x \approx \prod_{n=2}^{n_x} \left(\frac{\sqrt{\pi}\, R_S}{\sigma_x(n)}\right)^2, \tag{5}$$

and similarly for P_y:

$$P_y \approx \prod_{n=2}^{n_y} \left(\frac{\sqrt{\pi}\, R_S}{\sigma_y(n)}\right)^2. \tag{6}$$

For an arbitrary power-law spectrum of initial oscillations along the loop with the exponent α at the moment when $R_0 \approx R_s$, we have

$$\sigma_x(n) = \sigma_0 n^{-(\alpha+1)}, \tag{7}$$

while

$$\sigma_y(n) = \sigma_0 n^{-\alpha}. \tag{8}$$

The difference in the slope of radial and normal oscillation spectra reflects the fact that radial oscillations shrink in the course of the contradiction of the loop[39] and the final amplitudes of radial oscillations at the moment of the maximum contraction is a factor n^{-1} less than their initial values. The values of the maximum wave numbers, n_x and n_y, in Eqs (5) and (6) are determined by the condition that the corresponding dispersions of σ_n for $n \gtrsim n_x$ and σ_n for $n \gtrsim n_y$ are less than R_S, then

$$n_x \approx q^{-1/(1+\alpha)}, \tag{9}$$

and

$$n_y \approx q^{-1/\alpha}, \tag{10}$$

where

$$q = \frac{4\pi^{3/2} G\mu}{\xi}. \tag{11}$$

Here $\xi = \sigma_0/R_0$ is the dimensionless parameter describing how strongly the loops are disturbed. Then from Eqs (4–11) by order of magnitude we have the following expression for the probability of black hole formation:

$$P_{bh} \approx q^{2(n_x+n_y-2)}(n_x!)^{2(\alpha+1)}(n_y!)^{2\alpha}. \tag{12}$$

Using Stirling's formula,

$$n! \approx \sqrt{2\pi}\, n^{n+1/2} e^{-n}, \tag{13}$$

we reduce Eq. (12) to

$$P_{bh} \approx (2\pi)^{2\alpha+1} q^{-6} \exp\left[-2(1+\alpha) q^{-2(1+\alpha)} - 2\alpha q^{-2/\alpha}\right]. \tag{14}$$

This general expression for P_{bh} is very useful for rough estimations. Evaluating the probability of PBH formation for different α and q one has: for $q = 1$, 10, 100, 1000, respectively,

$$P = 0.6,\ 2\cdot 10^{-6},\ \sim 0,\ \sim 0, \quad \text{if } \alpha = 1,$$
$$P = 0.5,\ 5\cdot 10^{-3},\ \sim 0,\ \sim 0, \quad \text{if } \alpha = 3/2,$$
$$P = 0.4,\ 8\cdot 10^{-2},\ 3\cdot 10^{-14},\ \sim 0, \quad \text{if } \alpha = 2,$$
$$P = 0.3,\ 0.3,\ 6\cdot 10^{-9},\ \sim 0, \quad \text{if } \alpha = 5/2.$$

[Here $P \sim 0$, means that the probability of black hole formation is too small to be compared with observations (see the next section)].

Assuming that the spectrum of oscillations along the loop is, to some extent, a continuation of the spectrum of individual separate loops within the global network of cosmic strings, and taking into account that for this case the contribution of the n-mode to the length of a loop is $\delta l \sim x_n^2 n^2 / R_0 \sim r \sim R_0/n$, we have $\alpha = 3/2$.

3. OBSERVATIONAL RESTRICTIONS

To relate the probability of PBH formation to observational data one should estimate the contribution of PBH to the dimensionless mean density of the Universe. If $t_{ev} + t_H > t_{eq}$ then

$$\Omega_{\rm BH} = \nu P_{\rm BH} \frac{M_{\rm BH}}{M_H} \left(\frac{t_{eq}}{t_H}\right)^{1/2}. \tag{15}$$

If $t_{ev} + t_H < t_{eq}$ then

$$\Omega_{\rm BH} = \nu P_{\rm BH} \frac{M_{\rm BH}}{M_H} \left(\frac{t_{ev} + t_H}{t_H}\right)^{1/2}. \tag{16}$$

Here $M_{\rm BH}$ is the mass of a black hole; M_H is the mass within the cosmological horizon at the moment of the black hole formation, which moment of time we denote as t_H, $t_{eq} \approx 4 \cdot 10^{10}$ s is the moment when the energy density of matter in the Universe is equal to the radiation energy density; $t_{ev} \approx t_{Pl}(M_{\rm BH}/M_{\rm Pl})^3$ is the time scale of black hole evaporation.[3] Up to the moment t_{eq}, the contribution of PBHs into the mean density of the Universe decreases more slowly than the mean density of ultrarelativistic matter and radiation surrounding PBHs.[70] For this reason we have the amplification factor $[(t_{ev} + t_H)/(t_H)]^{1/2}$ in Eq. (16).

PBHs of mass $M_{\rm BH} \approx 5 \cdot 10^{14}$ g are evaporating in the present Universe, therefore observational data put the strongest upper limit on the abundance of PBH of that mass:[5,6]

$$\Omega_{\rm BH} < 10^{-8}. \tag{17}$$

Then for the black holes formed after the first oscillation of the loop we have

$$P_{\rm BH} < 5 \cdot 10^{-18} \left(\frac{\nu}{0.1}\right)^{-1} \left(\frac{G\mu}{10^{-6}}\right)^{-3/2}, \tag{18}$$

If PBHs were formed after many oscillations of the loop when considerable mass-energy of the loop has been radiated away in the form of gravitational waves we would have

$$P_{\rm BH} < 5 \cdot 10^{-12} \left(\frac{\gamma}{100}\right)^{-3/2} \left(\frac{\nu}{0.1}\right)^{-1} \left(\frac{G\mu}{10^{-6}}\right)^{-3} \beta^{-3/2}, \tag{19}$$

where $\beta = M_{\rm BH}/M_L$ is the ratio of the black hole mass to the original mass of the loop, $M_L = 2\pi\mu R_0$, and $\gamma \approx 100$ is the dimensionless numerical factor taking into account a difference between actual gravitational radiation power and the corresponding value given by the quadrupole approximation:

$$\frac{dE}{dt} = -\gamma G\mu^2. \tag{20}$$

Using Eq. (14) and comparing the estimates obtained from this equation with the restrictions given by (18) and (19) we conclude that for any reasonable exponent α the parameter q should not exceed 100. This means that [see Eq. (11)]

$$\xi > 5 \cdot 10^{-3}(G\mu)/(10^{-6}). \tag{21}$$

Therefore loops should be highly disturbed to be compatible with the observational upper limits on the abundance of PBHs.

Another conclusion can be drawn for the case of PBHs formed after many oscillations (whether it is possible or not, is the subject of the next section). If it is assumed that the probability of late formation is close to unity, then we can put the following restriction on the fraction of the initial mass of the loops which goes to PBHs:

$$\beta < 10^{-10}\left(\frac{G\mu}{10^{-6}}\right)^{4/3}. \tag{22}$$

And vice versa, if assuming that this mass fraction is not too small, say $\beta \approx 0.1$, then we obtain the following restriction on the probability of PBH formation:

$$P_{\mathrm{BH}} < 10^{-10}\left(\frac{G\mu}{10^{-6}}\right)^{-3} \tag{23}$$

4. RESIDUAL PBHs OF THE PLANK MASS

In this section we discuss the following problem: whether it is possible or not to have somehow a higher probability of PBH formation from oscillating loops.

It is clear that if initial perturbations are not small enough then at the moment of maximum contraction of the loop, intersections between different parts of the loop are unavoidable. It is qualitatively clear that such intersections of the segments moving at relativistic velocities should result in the generation of very strong short wave oscillations. Gravitational radiation damping[10] is not able to suppress these short wave oscillations because the wavelength of the oscillations surviving up to the next half period of the loop oscillation is of the order of:

$$\lambda_{\min} \sim \gamma G\mu R_0(t), \tag{24}$$

i.e. this wavelength is proportional to the current radius of the basic loop. This means that the situation after each large-scale oscillation is quite similar to the initial situation. In other words, gravitational radiation damping would be able to smooth out the small-scale structure of the loop only if the generation of the small-scale perturbations with wavelengths as small as $\lambda_{\min}$, given by Eq. (24), were not taken into account, but due to the intersection of different segments of the loop the regeneration of a small scale-structure seems to be unavoidable. So let us discuss the following channel of PBH formation from oscillating loops, restricting our consideration by nonselfintersecting loops. As a loop oscillates without selfintersections for a long time, gravitational radiation results in smoothing out of small-scale perturbations. It would be attractive to hope that at the end the loop would be almost circular. Unfortunately, this possibility seems doubtful. The main argument against such a possibility is the existence of an exact two-parametric solution proposed by Turok.[12] According to this solution, two modes of oscillation proceeding on the main and the triple frequency seem to be able to maintain each other until the loop will completely disappear.

Nevertheless, taking into account the uncertainty principle we could conclude that the loop cannot disappear without a trace. Actually the radius of the loop decreases, according to Eq. (20), linearly with time,

$$r(t) = r_0 - \frac{\gamma G\mu}{2\pi} t, \tag{25}$$

then the energy of the loop also decreases linearly with time,

$$E(t) = 2\pi\mu r. \tag{26}$$

It is obvious that this energy should exceed the energy of the last graviton (the last quantum of gravitational radiation) emitted at the frequency

$$f = \frac{2\pi}{r}, \tag{27}$$

then

$$2\pi\mu r(t) > \frac{2\pi\hbar}{r(t)}. \tag{28}$$

This means that

$$r > \left(\frac{\hbar}{\mu}\right)^{1/2} \approx r_{\mathrm{Pl}}(G\mu)^{-1/2}. \tag{29}$$

Therefore we have the maximum mass of the string,

$$M_{\mathrm{min}} \approx M_{\mathrm{Pl}}(G\mu)^{1/2}. \tag{30}$$

(The Schwarzshild radius of these minimal loops is smaller than their radius, so these objects are not black holes. They are similar to the smile of the Cheshire cat from "Alice in Wonderland" by Lewis Carroll which remains after the cat itself has disappeared.) This naive speculation is only the hint on the possibility that after the decay of the loop due to gravitational radiation some residual mass is formed. According to Zeldovich's idea about the Plank mass at the lowest stable state of any object emitting energy (see also Refs 49–57) let us assume that taking into account all perturbations along the loop and applying rigorously quantum arguments (similar to those mentioned above in a naive form) we will obtain just the Plank value for the residual mass. The contribution of the resulting objects (PBHs of the Plank mass) to the dimensionless density of the Universe is:

$$\Omega_{pl} \approx \nu(G\mu)^{3/2}(t_{eq}/t_{\mathrm{Pl}})^{1/2}(M_{PL}/M_L)^3/2, \tag{31}$$

where M_L is a minimum initial mass of the loop. This dimensionless density should not considerably exceed unity (the well-known argument of Gerstein & Zeldovich:[16] in opposite case the age of the Earth would exceed the age of the Universe). Then

$$G\mu < 10^{-6}(\nu/10^{-10})^{2/3}(M_L/10^6\ \mathrm{g}). \tag{32}$$

Surely this restriction is based on the assumption about the residual Plank mass, nevertheless this restriction illustrates that analysis in this direction might be rather informative.

CONCLUSION

Finally, the following conclusion can be drawn. The probability of PBH formation from oscillating cosmic loops is too small to be used for obtaining model-independent decisive restrictions on the fundamental parameter of string theory $G\mu$. However even in this case observational upper limits on PBHs are very helpful in solving the problem of the cosmic loop final state.

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