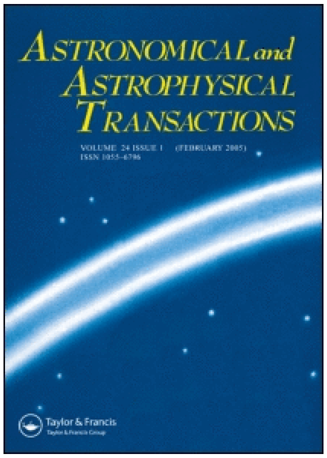

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 January 1994
To cite this Article: Popova, A. D. (1994) 'Cosmological tests for determining the real spatial dimensionality of the universe', Astronomical & Astrophysical Transactions, 5:1, 31 - 33
To link to this article: DOI: 10.1080/10556799408245851
URL: http://dx.doi.org/10.1080/10556799408245851

Astronomical and Astrophysical Transactions, 1994,
Vol. 5, pp. 31–33
Reprints available directly from the publisher.

COSMOLOGICAL TESTS FOR DETERMINING THE REAL SPATIAL DIMENSIONALITY OF THE UNIVERSE

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(25 November 1992)

The description of the modern state of the Universe is based on the paradigm of the 3-dimensional space and on that of the constancy of this dimensionality. However, only a habit can prevent one from considering other possibilities. Moreover, it seems that there exist convincing arguments to refuse the above paradigms. Passing to greater and greater scales, from galaxies to clusters and superclusters of galaxies, the discrepancy between the luminous matter and its dynamics becomes stronger and stronger. To resolve this difficulty, one usually introduces the concept of dark matter that is required in increasing amounts when passing to larger scales.[1] The nature and the origin of the dark matter is, until now, a subject of debate; the most relevant model now is that of a mixed cold and hot dark matter.[2,3] The existing modifications of the Newtonian dynamics[4] are unsatisfactory due to certain reasons, in particular, owing to the difficulties concerning a covariant relativistic extension. Another set of arguments comes from the discrepancy between the age of galaxies[5] and globular clusters,[6] on one hand, and the relatively small age of the Universe in the standard cosmology, on the other hand.

Our concept which can help us to resolve the above two discrepancies is that, first, the space dimension n, that is not necessarily integer, may be a function of the relative distance between the bodies. It is worth noting that non-integer dimensions can be incorporated into a somewhat generalized Riemannian space of general relativity.[7] Second, it can be shown that in order to obtain a correct law for the gravitational force and to entail the age of the Universe, the dimensionality which is 3 at the laboratory scales should be less than 3 at larger (relative) distances; probably, it tends to 2 in the limit of the cosmological scales.

In fact, in the special case of $n = 2$, there exists a stationary solution of the Einstein equations for closed dust-filled Universe in which the scale factor a is connected with the energy density ε:

$$a = c(\kappa^{(2)}\varepsilon)^{-1/2}$$

with c the speed of light, κ with the superscript (2) is a certain Einstein-type constant in 2 dimensions which is proposed to be finite. From some preliminary investigations, it seems that the solution is stable: small fluctuations against the background of this solution do not grow in time. Formally, the age of the Universe is infinite, however the solution can be viewed as an asymptotic form for large times.

For all the signs of the Gauss curvature (k), there exists also, in $n=2$, the solution with a linear growth of the scale factor: $a \propto t$. The age of the Universe is H_0^{-1} where H_0 is the modern value of the Hubble parameter (cf. the standard age $2/3(H_0)^{-1}$ in $n=3$ for $k=0$). It is interesting to note that, for any k, the cosmological solutions for pure radiation in $n=2$ (in the given case, $p=\varepsilon/2$) coincide with the known solution for dust ($p=0$) in $n=3$ for the same k.[8]

Below we present three traditional cosmological tests which can help us to determine, in principle, the real space dimensionality. They are based on homogeneous and isotropic solutions to the Einstein equations for any constant, arbitrary (non-integer) n. The analysis is done following Ref. 8. The tests are given in the first two orders in the redshift (z); although they have different analytic forms for different values of the density parameter $\Omega_0^{(n)}$, the above two orders surprisingly coincide analytically (like in the case $n=3$).

The first test is the visual magnitude vs. redshift relation. This test is very sensitive to the possible change of the dimensionality because it depends on the volume of the region between a source and the observer where the luminosity is being measured. Assume conditionally that the dimensionality is 3 if the distance to any body is less than R_0 and it is n otherwise. Thus, the source and the observer possess their own R_0-spheres. If z is small and the ratio R_0H_0/cz is small, then

$$m-M=5z\left[\frac{5-n}{4}-\frac{(n-1)(n-2)}{8}\,\Omega_0^{(n)}\right]\lg e-5-2.5\lg\beta^{(n)}$$
$$+2.5(n-1)\lg\frac{cz}{H_0}+2.5(3-n)\lg\frac{R_0}{\sqrt{2}}-\frac{5}{16}\frac{(n-1)(3-n)}{(n+1)}\times\frac{R_0H_0}{cz},$$

where

$$\beta^{(n)}=2^{(n+1)/2}\pi^{-1/2}(n-1)^{-1}\Gamma\left(\frac{n}{2}\right)\left[\Gamma\left(\frac{n-1}{2}\right)\right]^{-1},$$

with Γ the gamma function.

The second test is the angular size vs. redshift relation. In defining $\tilde{R}$ as the ratio of the source linear size l to its angular size θ, we obtain

$$\tilde{R}=\frac{l}{\theta}=\frac{c}{H_0}\left[z-\left(\frac{3}{2}+\frac{n-2}{4}\,\Omega_0^{(n)}\right)z^2\right]=\frac{c}{H_0}\left[z-\left(\frac{3}{2}+\frac{1}{2}q_0^{(n)}\right)z^2\right],$$

where $q_0^{(n)}$ is the deceleration parameter in n dimensions:

$$q_0^{(n)}=\frac{n-2}{2}\,\Omega_0^{(n)}.$$

This test is insensitive to a possible change of the dimensionality inside a given z. This is because we should only suppose that in the space of an arbitrary non-integer dimensionality one can construct a geodesic 2-surface that passes through the linear diameter of the source and through the observer. Unfortunately, in the above orders, the effects of n and $\Omega_0^{(n)}$ are undistinguishable one from another. For example, the case $n=2$ gives the same result as the vanishing density ($\Omega_0^{(n)} = 0$). However, the expression of $\tilde{R}$ via the deceleration parameter is formally independent of n.

The third test is the number of sources vs. redshift relation. The differential of the sources at a given z is

$$dN = n_0 s^{(n)} \left(\frac{c}{H}\right)^n z^{n-1}\left[1 - \frac{n+1}{2}\left(1 + \frac{n-2}{2}\Omega_0^{(n)}\right)z\right],$$

where n_0 is the density of the sources, its physical dimensionality is (length)$^{-n}$, $s^{(n)}$ is the area of an n-dimensional sphere:

$$s^{(n)} = 2\pi^{n/2}\left[\Gamma\left(\frac{n}{2}\right)\right]^{-1}.$$

This test could be the most powerful one unless it was mostly contaminated by evolution and selection effects.

In conclusion, we add that we hardly make easier the tasks for observers because the above tests involve one or more additional parameters: n and/or R_0. Evidently, all the tests become familiar in the case $n = 3$.

The author is very grateful to V. E. Yakimov for stimulating this work and for helpful critical discussions; the author is also very grateful to A. V. Zasov and M. E. Prokhorov for numerous discussions on the underlying conceptions.

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