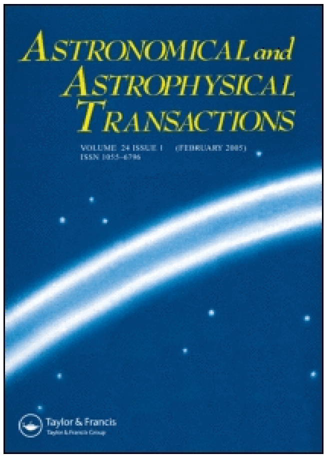

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Interacting universes and their quantum birth from nothing

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INTERACTING UNIVERSES AND THEIR QUANTUM BIRTH FROM NOTHING

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A non-linear form of the Wheeler–DeWitt equation is obtained from a viewpoint of the interacting many universes. The non-linear parts can be classified into two types; (i) one is reducible to the linear term, whose coefficient is absorbed into the Hamiltonian of the original linear equation, (ii) and the one which still remained in the non-linear form. It is shown that the second type term survives only in the microscopic region of the Planck size. Due to this second type term, it would be possible to obtain the double exponential factor with respect to the cosmological constant in the expectation value of the number of created Universes.

KEY WORDS Topology-change, Creation of universe.

1. INTRODUCTION

Recently, an instanton solution called the wormhole has been found by adding an appropriate matter field[1,2] or higher curvature terms[3] to the pure Einstein gravity. The discovery of this euclidean configuration has opened a new viewpoint that the topology of a space can be changed quantum mechanically by the tunnelling effect. Then many disconnected spaces (= universes) could interact with each other via wormholes. It should be noticed that this interaction is a pure quantum gravitational effect, and any contact between different universes cannot occur as a classical phenomenon. It would be astonishing that the small value of the cosmological constant (Λ) and the values of other fundamental parameters in our world could be determined if we take into account this effect. This has been first pointed out by Coleman.[4]

The viewpoint of many universes has also been obtained from the second quantization framework of the Wheeler–DeWitt equation.[5] The situation is similar to the case of the field equation of a scalar field in a curved space.[6] Similarly to the particle creation in the example in,[6] we are led to the viewpoint that a great number of universes could be created from nothing. So it is reasonable that we consider the many body problem of universe by applying the second quantization formalism of the Wheeler–DeWitt equation to an interacting field theory of universe.[7,8] We call it the "universal" field theory. It is formulated here in a special mini-superspace, where only the size $a(t)$ of the de Sitter space is dynamical, and the universal field is denoted by $\Psi(a)$.

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The field theory is composed of the free part (the quadratic term with respect to $\Psi(a)$), which gives the usual Wheeler–DeWitt equation, and the interaction term. As for the interaction, it is given in the form of the Ψ^3-type, which represents the bifurcation of a universe into two parts. This term is constructed so that it reproduces the wormhole induced vertex, which can be derived in a path integral formalism.[9] Due to the introduction of this interaction term, the non-linear terms with respect to $\Psi(a)$ appear in the classical equation, and the resultant Wheeler–DeWitt equation has a modified non-linear form.

At large a, we get however the usual linear form of the Wheeler–DeWitt equation, because the universal interactions can be effectively rewritten in a linearized form for large a. But parameters in the Hamiltonian are modified after this linearization. While at small a, the interactions, which cannot be reduced to the linear Wheeler–DeWitt equation, survive. Then it is necessary to solve non-linear equation in order to get the value of $\Psi(a)$ at small a. The surviving non-linear terms are classified into two types by the arguments (a's) of the three $\Psi(a)$s, i.e., (i) all the arguments are small, and (ii) only one of them is small and the other two are large. In the calculation of the number of created universes from nothing, it is necessary to connect the solutions at small and large a[5,8,10]. So the non-linear term which was neglected in the case of the usual Wheeler–DeWitt equation, can be expected to give a new effect to this problem. The same effect would be expected for any case in which the information of $\Psi(a)$ in the small a region is necessary. Our purpose is to study the effect of this non-linear term in the calculation of the number of created universes from nothing. It is shown that the vertex of type (ii), mentioned above, could lead to the double exponential factor of $1/\Lambda$ in the expectation value of the number operator of created universes. This result is consistent with the one obtained in the euclidean path integral approach[4]. But this is not a unique solution. In fact, we also get a result of a single exponential of $1/\Lambda$ by using another simple solution of the non-linear equation. The problem remained here is to find a principle to determine which solution should we choose. This is an open question.

2. *Quantum Birth of Universes and Initial Conditions*

Here we briefly review how to calculate the number of the universes created from nothing in terms of the solutions of the Wheeler–DeWitt equation[5,10]. For the sake of brevity, consider the pure Einstein gravity with the lagrangian,

$$\mathscr{L} = \sqrt{-g}(\kappa^2 R + \Lambda), \tag{1}$$

where κ denotes the gravitational constant. Hereafter, the metric is restricted to the following Robertson–Walker type:

$$ds^2 = -dt^2 + a^2(t)\, d\Omega_3^2, \tag{2}$$

where $d\Omega_3^2$ denotes the metric of S^3. Then the Wheeler–DeWitt equation in this mini-superspace is written as

$$[\partial_a^2 - v^2(a)]\Psi(a) = 0, \tag{3}$$

where

$$v^2(a) = a^2(1 - \tfrac{1}{3}\Lambda a^2) - \varepsilon, \tag{4}$$

and $\bar{\Lambda} = \Lambda/(48\pi^2\kappa^4)$, ε is a small positive number introduced for the convenience[5]. In Eqs. (3) and (4), we should use $a' = 2\sqrt{6}\,\pi\kappa a$ instead of a, but the prime is omitted here for brevity.

The solutions of Eq. (3) are given separately in three regions; (i) $a < a_S$, (ii) $a > a_L$ and (iii) $a_S < a < a_L$, where $a_S(<a_L)$ denotes the zero point of $v^2(a)$. In the regions (i) and (ii), oscillating solutions are allowed and they can be written in the operator form in terms of the creation and annihilation operators of the universes defined in each regions. Solutions obtained in the region (i) can be connected to the one given in the region (ii) through non-oscillating solutions of the region (iii) by analytic continuation.

The expectation value of the number of universes (denoted by N) created from nothing in the region (ii) can be obtained by taking the expectation value of the number operator defined in the region (ii) between the vacuum state of the region (i). The result can be written in terms of the coefficients of the Bogoliubov transformation between the states in the regions (i) and (ii). The result is obtained within the WKB approximation, as follows:

$$N = e^{2I},$$

where

$$I = \int_{a_S}^{a_L} v(a')\, da'.$$

It can be seen that $I \doteq 1/\bar{\Lambda}$ for $\varepsilon = 0$. Then we obtain

$$N = e^{2/\bar{\Lambda}}. \tag{5}$$

This result means that the cosmological constant is very small for almost all created Universes. However this result is different from the double exponential behavior, $N \sim \exp\exp(1/\bar{\Lambda})$, which has been obtained in the path integral approach[4] as the probability to find universe whose cosmological constant is $\bar{\Lambda}$.

Similar calculations can be performed when matter fields exist. Consider a scalar field, φ, with the following lagrangian:

$$\mathscr{L}_s = \sqrt{-g}\{\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi_\nu\varphi + V(\varphi)\}. \tag{6}$$

For $\varphi = \varphi(t)$, the Wheeler–DeWitt equation is written as follows:

$$\left\{\partial_a^2 - \frac{1}{a^2}\partial_\varphi^2 - v^2(a,\varphi)\right\}\Psi(a,\varphi) = 0, \tag{7}$$

where

$$v^2(a,\varphi) = a^2(1 - \tfrac{1}{3}\Lambda_{\text{eff}}a^2) - \varepsilon, \tag{8}$$

and $\Lambda_{eff} = (\Lambda + V(\varphi))/(48\pi^2\kappa^4)$. We must note that φ and a in Eqs. (7) and (8) are rescaled by $2\sqrt{6}\,\pi\kappa$, but we used the original notations used in Eq. (6). For the case of $\partial_\varphi V \ll 1$, we can get the following result:

$$N \sim \exp\left(\frac{1}{\Lambda_{\text{eff}}}\right). \tag{9}$$

This result means that φ should be at the minimum of $V(\varphi)$ just after the creation of the universes. Then an inflational explosion of the Universe after the quantum creation cannot be expected. The initial condition of the universes created from nothing is therefore unfavourable for the inflation. In order to improve this situation, second quantization of inhomogeneous universes has been proposed [11]. In this approach, it has been expected that a hot part of a large Universe should be excited quantum mechanically to create an inflational Universe.

However we here restrict our attention only to the homogeneous space, and we examine the effect of the universal interaction on the initial condition mentioned above. The initial condition just after its creation is largely dependent on the dynamics in the microscopic region, where the equation cannot be reduced to a linear form. Then it is important to solve a complicated non-linear differential equation for $\Psi(a)$. In the following, we show two typical solutions. However, the problem with respect to the inflation is not resolved. And the situation is more serious than the previous result obtained by the linear equation. But our result is consistent with the path integral approach of ref. [4].

3. NON-LINEAR WHEELER–DEWITT EQUATION

3.1. *Universal Field and Interactions*

First, we define the universal fields of $\Psi(a)$, which means the field of the Universes, and their interaction, $\Psi(a)$ is defined here by the so-called Hartle–Hawking wave function,

$$\Psi(a) = \int_{a=0}^{a} D^{(4)}g e^{-S_E}, \tag{10}$$

where the lower and upper limits of the integral denote the de Sitter spaces with $a=0$, the state of nothing, and a finite a, respectively. S_E is the euclidean action of the gravitational theory. Among the various configurations of the paths, which connect two limits of the path-integral in Eq. (10), there exists the one representing the topology changing. Namely a universe can bifurcate to two universes and return to the state of a single universe. This process implies an extended form of the Wheeler–DeWitt equation that includes the non-linear term of $\Psi(a)$. To obtain the extended form, we introduce a universal interaction by extending a simple form of vertex, which can be obtained according to a usual canonical approach in the path-integral, of the Universe into a general form.

Taking into account the wormhole configuration in the path-integral in Eq. (10) is equivalent to considering the interactions with a baby universe whose size is very small. Due to this interaction, S_E is modified so that the values of all the parameters in S_E are shifted from their original values to the new ones. From this observation, we can find the interaction vertex of the three universal fields at a special limit where one of them has a negligibly small size, a. It would be obtained as follows.

Consider Eq. (10) in the Lorentz metric, then rewrite its action into a canonical form according to ref. [9] and expand $\exp(iS)$ by a power series of a part of the Hamiltonian, which is denoted here by $V(a_3)$. Here a_3 denotes the radius of the space between $a_1=0$ and $a_2=a$. If we set $a_2=0$, the linear term of $V(a_3)$ in this expansion can be considered as an elementary vertex function with a very small universe. We denote it as $S_3^{(0)}$. It can be written, with the notations used in ref. [9], as follows:

$$S_s^{(0)} = \int dN(t_2 - t_1) \int_{t_1}^{t_2} dt_3 \int da_3 NV(a_3) K(2, 3; N(t_2 - t_3)) K(3, 1; N(t_3 - t_1)) \quad (11)$$

$$= \int dT_{23} \int dT_{31} \int da_3 V(a_3) K(2, 3; T_{23}) K(3, 1; T_{31}), \quad (12)$$

where $T_{13}(\equiv T(t_1 - t_3))$, $T_{32}(\equiv T(t_3 - t_2))$. $K(1, 2; N(t_1 - t_2))$ denotes the path integral kernel and N is the lapse function. Using the definition of the wave function, Eq. (10), we obtain

$$S_3^{(0)} = \int da V(a) \Psi^2(a). \quad (13)$$

We should note here that the above simple form can be obtained for the mini-superspace only. The generalization of $S_3^{(0)}$ to the vertex of three finite-sized universes is given in the next subsection so that it is consistent with the above special limit

3.2. *The Generalized Form of the Wheeler–DeWitt Equation*

Next, we extend the limited form of the universal interaction obtained above. According to the formalism of the universal field theory[7,8], we write the action of the universal field as follows,

$$S = S_2 + S_3, \quad (14)$$

$$S_2 = \int da \tfrac{1}{2} \Psi(a) H \Psi(a), \quad (15)$$

$$S_3 = \frac{1}{3} \int [\Pi_{i=1}^3 da_i \Psi(a_i)] \rho(a_1, a_2 \mid a_3), \quad (16)$$

where H in Eq. (15) is the Wheeler–DeWitt Hamiltonian, which will be given by the usual field theory defined in a four dimensional space-time manifold. By the variational principle with respect to $\Psi(a)$, the usual Wheeler–DeWitt equation is obtained from S_2.

S_3 is the generalized form of $S_3^{(0)}$. The generalization is performed by introducing a local vertex function ρ. Since S_3 corresponds to the extension of the wormhole effect and the effective size of the wormholes is of the order of the Planck size, M_{pl}^{-1}, we require that only small-sized universes can be emitted from the other small or large one. Then we impose the following restrictions (Eqs. (18, 19)) on the vertex function $\rho(a_1, a_2 \mid a_3)$:

$$\rho(a_1, a_2 \mid a_3) = \rho(a_2, a_1 \mid a_3) \quad (17)$$

$$\rho(a_1, a_2 \mid a_3)_{a_1 \to \infty} \to \mu(a_1, a_2) \delta(a_1 - a_3) \quad (18)$$

$$\rho(a_1, a_2 \mid a_3)_{a_3 \to \infty} \to \tfrac{1}{2}[\mu(a_1, a_2) \delta(a_1 - a_3) + \mu(a_2, a_1) \delta(a_2 - a_3)], \quad (19)$$

where the symmetry with respect to a_1 and a_2 given by Eq. (17) is imposed for the sake of convenience. The function $\mu(a_1, a_2)$ was introduced to denote the limiting form of ρ at large a_1. As shown below, μ is related to the Hamiltonian operators by Eqs. (16) and (13). This function should decrease rapidly with respect to the second argument, a_2, because a_2 denotes the size of the small universe coupled to the larger one. So we require the following behavior:

$$\mu(a_1, a_2) \propto \exp(-r^2 a_2^2), \tag{20}$$

where r is some typical scale factor of the order of M_{pl}. Eq. (19) is symmetrized so that it is consistent with Eq. (17).

Using these equations, we get

$$\frac{\delta S_3}{\delta\Psi(a)} = \frac{1}{3}\int da_1\, da_2 \Psi(a_1)\Psi(a_2)[2\rho(a, a_1 \mid a_2) + \rho(a_1, a_2 \mid a)] \tag{21}$$

$$= \Psi(a)\int da_1 \mu(a, a_1)\Psi(a_1) + \int da_1\, da_2 \Psi(a_1)\Psi(a_2)\xi(a, a_1, a_2), \tag{22}$$

where ξ is given as follows:

$$\xi(a, a_1, a_2) = \tfrac{1}{3}[2\delta\rho(a, a_1 \mid a_2) + \delta\rho(a_1, a_2 \mid a) + 2\mu(a_1, a)\delta(a_1 - a_2)], \tag{23}$$

$$\delta\rho(a_1, a_2 \mid a) = \rho(a_1, a_2 \mid a) - \tfrac{1}{2}[\mu(a, a_1)\delta(a - a_2) + \mu(a, a_2)\delta(a - a_1)], \tag{24}$$

$$\delta\rho(a, a_1 \mid a_2) = \rho(a, a_1 \mid a_2) - \mu(a, a_1)\delta(a - a_2) - \mu(a_1, a)\delta(a_1 - a_2). \tag{25}$$

For large a, the first term in Eq. (22) represents the contribution of the small universes to the larger one, because $\Psi(a_1)$ in the integrand survives only at small a_1 due to the function $\mu(a, a_1)$. And it causes the corresponding wormhole effect, i.e., the shift of the parameters like cosmological constant and other fundamental constants. After the integration over a_1 in this scheme, we will get the term which can be obtained from Eq. (13). Then the function μ should be constrained as follows,

$$\int da_1 \mu(a, a_1)\Psi(a_1) = \sum_i \alpha^i V_i(a), \tag{26}$$

where $V_i(a)$ are the operators in the Hamiltonian H. Then the first term of Eq. (22) can be rewritten as $\sum_i \alpha^i V_i(a)\Psi(a)$, and they are absorbed into $H\Psi(a)$.

This reduction does not work for the second term, and it remains to be a non-linear form. In Eq. (25), we defined $\delta\rho(a, a_1 \mid a_2)$ by subtracting the asymptotic form $\mu(a_1, a)\delta(a_1 - a_2)$ from $\rho(a, a_1 \mid a_2)$. So, ξ is defined in Eq. (23) by adding this term to retain the consistency. Then the functions $\delta\rho$ survive only in the region where all three arguments of $\delta\rho$ are small, and they can be neglected if one of the arguments is large. On the other hand, the separated term, $\mu(a_1, a)\delta(a_1 - a_2)$, remains to be finite for large a_1 and a_2, but it vanishes at large a, which is the second argument of $\mu(a_1, a)$. This represents the influence of the large universes on the tiny one. This is in a prominent contrast to the case of Eq. (26), where the effect of the small universes on the larger one is represented in terms of the same function μ.

Finally, we obtain the following modified form of the Wheeler–DeWitt equation,

$$\bar{H}\Psi(a) = -\int da_1\, da_2 \Psi(a_1)\Psi(a_2)\xi(a, a_1, a_2), \tag{27}$$

$$\bar{H} = H + \sum_i \alpha^i V_i(a). \tag{28}$$

We should note here the following point. At large a, ξ approaches zero and we have the usual linear form of the Wheeler-DeWitt equation with a modified Hamiltonian $\bar{H}$,

$$\bar{H}\Psi(a)=0. \tag{29}$$

In the next section, we try to solve Eq. (27).

4. THE EFFECT OF THE UNIVERSAL INTERACTION ON N

As seen in §2, the expectation value of the number of created universe, N, is determined in terms of the solution of the Wheeler–DeWitt equation. Here we try to solve Eq. (27) in the region including small a.

The r.h.s. of Eq. (27) is separated as follows:

$$-(\text{r.h.s.})=G(a)+K(a), \tag{30}$$

$$G(a)=\frac{2}{3}\int da'\Psi^2(a')\mu(a',a), \tag{31}$$

$$K(a)=\int da_1\, da_2\Psi(a_1)\Psi(a_2)\xi(a,a_1,a_2), \tag{32}$$

where

$$\xi(a,a_1,a_2)=\tfrac{1}{3}[2\delta\rho(a,a_1\mid a_2)+\delta\rho(a_1,a_2\mid a)]. \tag{33}$$

Here we should note that the dominant part of the solution for $\Psi(a)$ is exponentially increasing a in the tunnelling region as seen in §2. Further, we note that (i) $\mu(a',a)$ shows a gaussian damping with respect to a, but it does not necessarily decrease with a', and (ii) $\xi(a,a_1,a_2)$ decreases rapidly with both a_1 and a_2, which are the integration variables in Eq. (32). From these facts, it can be said that

$$G(a)\gg K(a). \tag{34}$$

Then Eq. (27) can be approximated by

$$\bar{H}\Psi(a)=-G(a). \tag{27'}$$

Then it is not necessary to know the detailed form of ξ, about which we do not have any clue to get its functional form. However, Eq. (27′) is still complicated, and it is difficult to solve generally. We give two typical solutions in the following, one of which leads to the double exponential behavior of N.

4.1. *A Simple Solution*

We first give a simple but exact solution of Eq. (27′). Multiply both sides of Eq. (31) by $\Psi(a)$ and integrate it over a, then we obtain

$$\int daG(a)\Psi(a)=\frac{2}{3}\int da\Psi^2(a)\sum_i \alpha^i V_i(a), \tag{35}$$

where we used Eq. (26). Now we introduce

$$G(a)=\frac{2}{3}\sum_i \alpha^i V_i(a)\Psi(a). \tag{36}$$

By substituting this expression to Eq. (27′), we obtain

$$H_{\text{mod}}\Psi(a)=0, \qquad \bar{H}_{\text{mod}}=H+\frac{4}{3}\sum_i \alpha^i V_i(a). \tag{37, 38}$$

In this case, we would obtain a result similar to that obtained in §2,

$$N\sim\exp\left(\frac{1}{\Lambda_{\text{mod}}}\right), \tag{9′}$$

where A_{mod} denotes the cosmological constant in $\bar{H}_{\text{mod}}$. This result is not new. However, Eq. (36) is not a unique solution which satisfies Eq. (35), and we search for another non-trivial solution in the next sub-section.

4.2. *Self-consistent Solution*:

We can get a self-consistent solution by imposing a reasonable ansatz on the solution. For a while, we forget the fact that $G(a)$ is a functional of $\Psi(a)$. Then the solution of Eq. (27) can be written formally as

$$\Psi(a)=A^+\Psi_0^+(a)+A^-\Psi_0^-(a)+\frac{1}{2}\int_0^a da'G(a')[\Psi_0^-(a')\Psi_0^+(a)-\Psi_0^+(a')\Psi_0^-(a)]. \tag{39}$$

where $\Psi_0^{\pm}(a)$ are the solutions of Eq. (29), and $A^{\pm}$ are the constants depending on the boundary condition. From this solution, we can observe the following two points: (i) At small a, the third term is negligible, so the solution can be approximated by the solution of the linearized equation (29). (ii) At large a, the solution is approximated by

$$\Psi(a)=\left[A^++\frac{1}{2}\int_0^a da'G(a')\Psi_0^-(a')\right]\Psi_0^+(a). \tag{40}$$

Since $G(a)$ decreases rapidly with a, the integral in the prefactor of $\Psi_0^+(a)$ in Eq. (40) becomes to be independent of a for large enough a. Then the full solution at large a has the same form with the one of the linearized equation (29). However, we recall that $G(a)$ depends on $\psi(a)$'s, so the above prefactor should be determined self-consistently with the form of the formal solution, Eq. (39).

In order to estimate this prefactor, we consider another formal solution. Eq. (27) may be rewritten formally as

$$[\bar{H}+f(a)]\Psi(a)=0, \tag{41}$$

where

$$f(a)=G(a)/\Psi(a). \tag{42}$$

As in the previous case, we can obtain a solution of Eq. (41) by the WKB approximation if $f(a)$ is regarded as a function which is independent on $\Psi(a)$. In this case, the increasing solution can be written as

$$\Psi(a) = Bv_{\rm eff}^{-1/2} \exp\left(\int_0^a da' v_{\rm eff}(a')\right), \tag{43}$$

$$v_{\rm eff}^2(a) = \bar{v}^2(a) + f(a), \tag{44}$$

where B denotes the normalization constant, and $\bar{v}^2$ is defined by the equation, $\bar{H} = -\partial_a^2 + \bar{v}^2(a)$. Here we notice that $f(a)$ should decrease very rapidly with a because of its form, Eq. (42). This is seen from the fact that the a-dependence of $G(a)$ is controlled by $\mu(a', a)$ and $1/\Psi(a)$ rapidly decreases with a. Then we can consider that $f(a)$ appears only at small α, where $\bar{v}^2(a)$ can be neglected on the other hand. Then Eq. (43) can be approximated at large a as follows:

$$\Psi(a) = B \exp(f^{1/2}(0)\sigma)\Psi_0^+(a), \tag{45}$$

where $\Psi_0^+(a)$ is the WKB approximated form,

$$\Psi_0^+(a) = \bar{v}^{-1/2} \exp\left(\int_0^a da' \bar{v}(a')\right), \tag{39'}$$

and σ is a small constant given by

$$\int_0^a da' f^{1/2}(a') = f^{1/2}(0)\sigma. \tag{46}$$

Then it is necessary to obtain the value of $f(0)$ for the evaluation of the number operator. In order to estimate $f(0)$ by Eq. (42), we use Eq. (39) to get the value of $\Psi(a)$ at small a. Further, we use both expressions Eq. (40) and Eq. (45) to see the consistency of the two formal solutions at large a.

Here it should be noticed that the above Eqs. (43)–(45) are obtained for the case of smooth and positive $f(a)$. For negative $f(0)$, the solution of the form of Eq. (44) cannot be obtained and we get another solution which is similar to the one given for $f(a) = 0$. The difference between the solutions of the two cases, $f(a) < 0$ and $f(a) = 0$, is the value of the smaller zero point, a_s, of $v_{\rm eff}^2(a)$. In the case of $f(a) < 0$, a_s is larger than the one for the case of $f(a) = 0$. Since this difference does not produce any qualitative difference of the solutions compared to the case of $f(a) = 0$, we get a solution which has been obtained in ref. [10], for $f(a) < 0$. So we consider hereafter the case of $f(a) > 0$.

4.3. *Determination of* $f(0)$

For the sake of brevity, we assume the following factorization property for $\mu(a', a)$[8],

$$\mu(a', a) = \zeta(a')\mu(0, a). \tag{47}$$

The function $\zeta(a)$ can be written as follows if we consider Eq. (26):

$$\zeta(a) = \sum_i \alpha_i V'(a)/\alpha_0, \tag{48}$$

where

$$\alpha_0 = \int da \mu(0, a)\Psi(a). \tag{49}$$

Since $\mu(0, a)$ fall off with a rapidly (see Eq. (20)), α_0 can be approximated as,

$$\alpha_0 = \Psi(0)\sigma_1, \tag{50}$$

where σ_1 is a number of order $1/r$.

Then we get from Eqs. (39), (42) and (45),

$$f(0) = \sigma_2\left(\frac{B}{\phi(0)}\right)^2 \sum_i \alpha_i \int da' \Psi_0^+(a')^2 V^i(a'), \tag{51}$$

where $\sigma_2 = 2\mu(0, 0)/(3\sigma_1)$ and $\phi(0)$ is defined as $\Psi(0) = \phi(0)\exp(f^{1/2}(0)\sigma)$. Further,

$$\phi(0) = \bar{A}^+\Psi_0^+(0) + \bar{A}^-\Psi_0^-(0), \tag{52}$$

where $A^\pm$ in Eq. (38) is replaced by $\bar{A}^\pm \exp(f^{1/2}(0))$ for the later convenience. As for $\bar{A}^\perp$, they are constrained by the consistency of Eqs. (40) and (45),

$$1 = \left[\Psi_0^+(0) + \frac{\bar{A}^-}{\bar{A}^+}\Psi_0^-(0)\right]\left[\sigma_3 - \frac{2}{3\sigma_2}\Psi_0^-(0)f(0)\right], \tag{53}$$

where $\sigma_3 = B/\phi(0)$.

Here we assume σ_3 being independent of $f(0)$, this assumption is consistent with the Eqs. (40) and (45). This is the most simple case where we can get a nontrivial form of $f(0)$. In this case, $f(0)$ can be obtained from Eq. (51) if $V_i(a)$ were given. On the other hand, Eq. (53) gives a relation between $\bar{A}^+$ and $\bar{A}^-$, and the determination of this relation is equivalent to the prescription of an initial condition of the Universe.

We evaluate $f(0)$ for the dominant and most interesting case, $V_i(a) = a^4$. The coefficient of this operator corresponds to the effective cosmological constant, which is denoted by Λ for simplicity. $\Psi_0^+(a)$ is given by Eq. (39′), and the range of integration in Eq. (51), is taken from $a = 0$ to a_L, which is defined in §2, namely in the tunnelling region. Near the turning points, $a = a_L$ and 0, Eq. (39′) is not valid, and we should estimate the integral using another approximate form for $\Psi_0^+(a)$, which is given as

$$\Psi_0^+ \sim \text{const.} + |x|,$$

where x denotes the deviation of a from the turning point. However we can see that the main contribution to the integral comes from the form of Eq. (39′), and it is given as

$$I_4 = \int_0^{a_2} da a^4 \Psi_0^+(a) = (A_1\Lambda^{-5/3} - A_2\Lambda^{-1})\exp\left(\frac{2}{\Lambda}\right), \tag{54}$$

where

$$A_1 = \int_0^{\Lambda^{-1/3}} dx \exp(-\tfrac{2}{3}x^3), \qquad A_2 = \int_0^{\Lambda^{-1/3}} dx x^2 \exp(-\tfrac{2}{3}x^3).$$

For $\Lambda \ll 2/3$, A_1 and A_2 are considered as numbers independent of Λ, because the integrand decreases rapidly with x. Since the created universes should develop according to the classical Einstein equation after the quantum birth, Λ should be fairly small and we consider A_1 and A_2 as Λ-independent numbers. Since the factor in Eq. (51), $\sigma_2(B/\phi(0))^2\alpha_i = \sigma_2\sigma_3^2\alpha_i$, can be replaced by const. $(A +$

const.), we obtain,

$$f(0) = f_1(\Lambda)\exp\left(\frac{2}{\Lambda}\right), \tag{55}$$

where $f_1(\Lambda)$ is a finite power series of Λ, which is given as $(A_1\Lambda^{-5/3} - A_2\Lambda^{-1})(\Lambda + \text{const.})$. Then the dominant Λ-dependence of $f(0)$ is given by the exponential factor for small Λ.

Finally, the expectation value of the number operator N is given as $N = \exp(2\bar{I})$ and

$$\bar{I} = \exp\left(\frac{1}{\Lambda} + \frac{1}{2}\ln(\sigma^2 f_1)\right) + \frac{1}{\Lambda}, \tag{56}$$

where the first exponential term corresponds to $f^{1/2}(0)\sigma$, which cannot be obtained from the usual lines: Wheeler–DeWitt equation. In order to get this factor, the interaction term in the small a region should be taken into account under an appropriate boundary condition.

As in 2, we can perform the same analysis given above in the case where some matter field, which is responsible for inflation, was added. Then we could get the result, Eq. (56), by replacing Λ by $\Lambda_{\rm eff} = \Lambda / V(\varphi)$ under the same condition given in 2 for $V(\varphi)$. So it would become more difficult to get an appropriate initial condition for the inflation than in the case of the linear Wheeler–DeWitt equation.

5. CONCLUSIONS AND DISCUSSION

Second quantization of the Wheeler–DeWitt equation, which is often called the third quantization, leads to the concept of many universes, and this formulation can be extended to the interacting system of universes which are regarded as fields defined in the superspace. The interaction of the universes can be intimately related to the topology changing effects, the wormhole effect in the euclidean approach. Then a special limit of the vertex, where the size of one universe is very small, can be derived from the path integral formulation. This vertex can be generalized so that it includes the above special limit.

From the universal field theory, which is formulated here by introducing the vertex mentioned above, we obtained an extended form of the Wheeler–DeWitt equation, which is non-linear with respect to the universal field. The non-linear terms are separated into two groups, (i) one is reducible to a linear form and (ii) the other is irreducible. Both terms are determined by the vertex function $\mu(a', a)$, which is an asymptotic form of *three-point* vertex being realized when the size of one of them is very small compared to the other two. For the type (i), this function μ is responsible for the shift of the parameters in the Wheeler–DeWitt Hamiltonian. The same effects are seen also in the path integral approach as the wormhole effect. So we constrain $\mu(a', a)$ so that it reproduces the vertex which is derived from the path integral approach. We have calculated the expectation value of the number of the universes created from nothing by solving the non-linear equation. The second type of non-linear term is essential to obtain a double exponential factor with respect to $1/\Lambda$ in the expectation value of the number of created universes. In the usual analysis using the linear Wheeler–DeWitt equation, we cannot get this factor.

It should be noted that the result given in Eq. (56) depends on the assumption that σ_3 is independent of $f(0)$, which is related to the boundary condition for the Universe. So this result is not unique. In fact we can get a simple other result given in §4-1, where a modified form of the linear equation is obtained. If we consider the case of $f(0)$-dependent σ_3 in solving Eq. (51), it may be possible to find other interesting form of $f(0)$. It is still an open problem to find such a solution which is consistent with the creation of inflationary universes.

Another important assumption go get Eq. (56) is the inequality $f(0) > 0$, or equivalently $\mu(0, 0)\alpha_i > 0$, where α_i is the coefficient of $V_i(a) = a^4$. In the case of $\mu(0, 0)\alpha_i < 0$, we cannot however obtain Eq. (56) and we obtain only an extra phase of N due to the universal interactions, because $f(0)$ is negative and $f^{1/2}(0)$ is pure imaginary. This situation corresponds to the case where the smaller zero point (a_s) of $v_{\text{eff}}^2(a)$ moves to a slightly larger point because of the negative $f(0)$ (see Eq. (43)), and we obtain in this case almost the same result as that given by the linear Wheeler–deWitt equation. Therefore, our result given in Eq. (56) is sensitive to the sign of $\mu(0, 0)\alpha_i$.

The remaining problem is to give a principle to choose the most preferable solution among many ones. This will be done by developing the universal field theory proposed here. We can say that the framework of the universal field theory could give a useful tool to study both the quantum cosmological and gravitational problems.

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