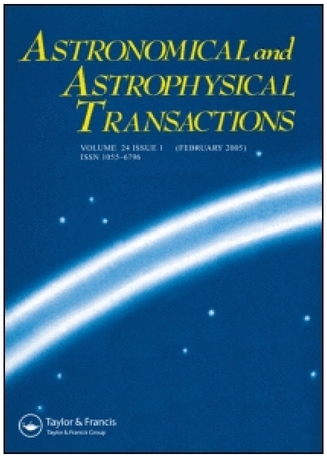

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K. V. Kholshevnikov [a]; V. A. Shor [b]
[a] Astronomical Institute of St-Petersburg University, St.-Petersburg, Russia
[b] Institute of Theoretical Astronomy, Russian Academy of Sciences, St.-Petersburg, Russia

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VELOCITY DISTRIBUTION OF METEOROIDS COLLIDING WITH PLANETS AND SATELLITES. I. THEORY

K. V. KHOLSHEVNIKOV[1] and V. A. SHOR[2]

[1] *Astronomical Institute of St.-Petersburg University, 198904, St.-Petersburg, Russia*
[2] *Institute of Theoretical Astronomy, Russian Academy of Sciences, 191187, St.-Petersburg, Russia*

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Algorithms for describing the velocity distribution of meteoroids colliding with planets and satellites are proposed providing a dense family of meteoroid orbits is known. The gravitation of the target body can be taken into account. The algorithms constructed here are semi-analytical, accomplishing operations with each known orbit via closed formulae mostly, whereas the average over all orbits of the family is to be calculated numerically. The characteristics of the distribution function will be published later. They may be obtained according to the above algorithms using Steel's idea to consider the family of asteroid orbits as representative for that of meteoroid orbits.

KEY WORDS Meteoroids, velocity distribution, collision, family of orbits

1 INTRODUCTION

Collisions in the Solar System play an important role in its history. The surfaces of planets and their satellites retain a memory of former collisions with bodies of asteroidal and cometary nature. Due to the typical collision velocities of the order of dozens kilometers per second, such an event resembles a powerful explosion. As a result, an impact crater emerges on the surface of the planet and a huge amount of matter (2–4 orders of magnitude greater than the mass of a projectile) is ejected.

Dimensions of such a cosmic catastrophe depend essentially on the mass and relative velocity of the impactor. There are several reasons to believe the mass and velocity distributions of meteoroids to be independent. The present paper is dedicated to the velocity distribution.

As all velocities are limited, we may use the Carleman theorem (see, e.g., Prokhorov and Rosanov, 1973, Sect. 4.3). According to it, the distribution function is uniquely determined by its momenta. So, it is sufficient to find a set of μ_k, the mathematical expectation of v^k, v being a collision velocity, $k = 1, 2, \ldots$. We propose algorithms for the estimations of the mathematical expectation of an arbitrary function of v, in particular that of v^k for any real k. For important cases $k = 2$ and $k = 4$, we indicate essential simplifications of the algorithms. Numerical results are to be published in the next paper.

The mean velocity of collision with one or other body is an important quantity when discussing a number of problems associated with impact events in the Solar System. Therefore, papers dedicated to this subject appear repeatedly (see, e.g., Steel *et al.*, 1985; Steel, 1985; Kholshevnikov *et al.*, 1991). In the present paper we utilize Steel's approach considering some of the elements of the known asteroids penetrating into the inner part of a selected planet orbit, as characteristic of the orbits of those particles which collide with the planet in reality. At the same time, we extend the subject of investigation applying the results not only to major planets, but also to asteroids and satellites. It is worth to consider comets also, but we suppose below the orbits of the colliding bodies to be stable, that is not true for almost all periodic comets.

Let us pass to the description of the main ideas. Fix the target planet (the distinction in the case of a satellite appears at the next stage only; see Sect. 4, 5 below). Now consider the set of potential projectiles. First of all, we note that the meteoroids of asteroidal and cometary nature differ by their physical and orbital properties. So they ought to be examined separately. We draw attention to the fact that collisions of the Earth with asteroids form the major portion of all events as compared with comets: 90–95% according to available estimations (Morrison, 1992). For Mars, the contrast is apparently further enhanced.

The minimal distances of the orbits of up-to-day known asteroids from those of the major planets are such that none of the asteroids can collide with major planets in the nearest future. On the other hand, due to secular perturbations the lines of nodes and apsides of the asteroids orbits rotate generally while their semi-major axes, eccentricities and inclinations undergo periodical changes only. If the asteroid orbit at the perihelion comes inside the orbit of the planet and at the aphelion it lies outside it, both orbits will, generally, intersect each other sooner or later. During the revolution of the line of apsides there are generally four instants when the orbits are intersecting in one and only one point. Of course, there exist degenerate cases: one can meet two or even one such an instant and two intersection points simultaneously.

The velocities at the intersection points can be calculated without difficulties and serve as a material for the μ_k determination. We expect that μ_k for the meteoroids of asteroidal nature differ slightly from those for asteroids themselves.

2 COLLISION WITH A NON-ATTRACTING PLANET

Let us fix a body-target, say the s-th major planet Q_s and a set of potential projectiles, say minor planets Q. Choose, among all the numbered minor planets

(Batrakov, 1992), those which have the semi-major axis a, and the eccentricity e, satisfying the inequalities

$$a(1-e) < a_s(1+e_s), \quad a(1+e) > a_s(1-e_s), \tag{1}$$

where the elements with index s refer to Q_s.

The elements a, e, and inclination, i, have no secular perturbations as a rule, whereas the longitude of the ascending node, Ω, and the argument of the perihelion, g, vary secularly, accomplishing one revolution during the time span of the order of several thousand years, that is short on the cosmogonic scale. So it seems to be reasonable to examine a family of orbits generated by each minor planet selected according to the criterion (1). Orbits belonging to the family have the same a, e, i and all possible Ω and g.

Finding intersection points of the orbits of the family and that of the major planet and calculating corresponding relative velocities v one can obtain an estimate μ_k as a mean value of v^k. The same is true for any function of v.

Let us introduce a right heliocentric coordinate system with the x-axis directed towards the perihelion of the orbit of Q_s, and the z-axis coinciding in direction with its area-vector. Position vectors of points Q, Q_s are (Subbotin, 1968):

$$\begin{aligned}
\mathbf{r} &= r(\cos\omega\cos\Omega - \cos i \sin\omega \sin\Omega, \\
&\qquad \cos\omega\sin\Omega + \cos i \sin\omega\cos\Omega, \quad \sin i \sin\omega), \\
\mathbf{r}_s &= r_s(\cos\theta_s, \quad \sin\theta_s, \quad 0),
\end{aligned} \tag{2}$$

ω being the argument of latitude, θ is the true anomaly and the angles i, Ω, g and ω are related to the above reference system. Supposing $\sin i \neq 0$, one finds $\sin\omega = 0$ from the condition $z = z_s$. We assume $\omega = 0$, the case $\omega = \pi$ reduces to the previous one by the substitution $\Omega \to \Omega + \pi$. The coincidence of the abscissas and ordinates of Q and Q_s gives

$$\theta_s = \Omega, \quad r = r_s = \frac{p_s}{1 + e_s\cos\Omega}, \quad \cos\theta = \frac{p(1 + e_s\cos\Omega) - p_s}{ep_s}, \tag{3}$$

Ω remaining a free parameter. Denote

$$\begin{aligned}
\Omega_1 &= \begin{cases} \arccos\dfrac{p_s - a(1-e)}{a(1-e)e_s}, & \text{if } a(1-e) \geq a_s(1-e_s), \\ 0, & \text{if } a(1-e) < a_s(1-e_s), \end{cases} \\
\Omega_2 &= \begin{cases} \arccos\dfrac{p_s - a(1+e)}{a(1+e)e_s}, & \text{if } a(1+e) \leq a_s(1+e_s), \\ \pi, & \text{if } a(1+e) > a_s(1+e_s). \end{cases}
\end{aligned} \tag{4}$$

The absolute values of the arguments of the arccos functions are less than 1 due to (1).

As the right-hand side of (3) is limited by ± 1, then Ω varies in symmetrical segments $[\Omega_1, \Omega_2]$ and $[-\Omega_2, -\Omega_1]$. In particular, Ω may receive any values within $[0, 2\pi]$ if more restrictive conditions in comparison with (1),

$$a(1-e) < a_s(1-e_s), \quad a(1+e) > a_s(1+e_s), \tag{5}$$

are valid.

For each value of Ω from the indicated set,

$$\sin\theta = l\frac{q}{e}, \quad q^2 = e^2 - \left[1 - \frac{p}{p_s}(1 + e_s\cos\Omega)\right]^2, \tag{6}$$

where $q > 0$, $l = \mp 1$.

Now let us proceed to the velocities (Battin, 1966):

$$\begin{aligned}
\dot{\mathbf{r}} &= v_s\sqrt{p_s/p}\big(-(\sin\omega + e\sin g)\cos\Omega - \cos i(\cos\omega + e\cos g)\sin\Omega,\\
&\quad -(\sin\omega + e\sin g)\sin\Omega + \cos i(\cos\omega + e\cos g)\cos\Omega,\\
&\quad \sin i(\cos\omega + e\cos g)\big),\\
\dot{\mathbf{r}}_s &= v_s(-\sin\theta_s, \quad e_s + \cos\theta_s, \quad 0),
\end{aligned}$$

v_s being the circular velocity at the distance p_s from the Sun. In the collision point we have

$$\begin{aligned}
\dot{\mathbf{r}}/v_s &= (lq\sqrt{p_s/p}\cos\Omega - h\cos i\sin\Omega,\\
&\qquad lq\sqrt{p_s/p}\sin\Omega + h\cos i\cos\Omega, \quad h\sin i),\\
\dot{\mathbf{r}}_s/v_s &= (-\sin\Omega, \quad e_s + \cos\Omega, \quad 0),
\end{aligned}$$

where

$$h = (1 + e_s\cos\Omega)\sqrt{p/p_s}$$

The planetocentric velocity is

$$\mathbf{v} = \dot{\mathbf{r}} - \dot{\mathbf{r}}_s = (v_x, v_y, v_z), \tag{7}$$

where

$$\begin{aligned}
v_x/v_s &= lq\sqrt{p_s/p}\cos\Omega - h\cos i\sin\Omega + \sin\Omega,\\
v_y/v_s &= lq\sqrt{p_s/p}\sin\Omega + h\cos i\cos\Omega - e_s - \cos\Omega,\\
v_z/v_s &= h\sin i.
\end{aligned}$$

The substitution $l \to -l$, $\Omega \to -\Omega$ preserves invariable the second and the third components of $\dot{\mathbf{r}}$, $\dot{\mathbf{r}}_s$ and $\mathbf{v}$ and changes the sign of the first one. So the moduli of heliocentric and planetocentric velocities do not change.

As a result, we obtain the planetocentric velocity v for each permissible value of Ω as

$$\left(\frac{v}{v_s}\right)^2 = \sum_{m=0}^{2} A_m e_s^m \cos m\Omega - 2le_s q\sqrt{\frac{p_s}{p}}\sin\Omega, \tag{8}$$

where

$$A_0 = 3 - \frac{p_s}{a} + e_s^2 - (2 + e_s^2)\cos i\sqrt{\frac{p}{p_s}},$$

$$A_1 = 4\left(1 - \cos i\sqrt{\frac{p}{p_s}}\right), \quad A_2 = -\cos i\sqrt{\frac{p}{p_s}},$$

and one can put $l = 1$.

Let us describe an algorithm of the μ_k estimation. For each selected asteroid we determine $\cos i$ according to

$$\cos i = \cos i' \cos i_s + \sin i' \sin i_s \cos(\Omega' - \Omega_s),$$

where i', Ω' and i_s, Ω_s are the inclination and the longitude of the ascending node of the orbits of the minor and major planets with respect to the ecliptic, respectively. If $s = 3$, such a procedure is unnecessary. For a set of admissible values of Ω we calculate v^k according to (6), (8) and find the mean value. Then we determine the mean over all selected asteroids.

The average over Ω can be fulfilled analytically for k even. By the symmetry of the set of the Ω values, the odd functions vanish.

For example, if $k = 2$ or 4:

$$\begin{aligned}\left[\frac{v^2}{v_s^2}\right] &= \sum_{m=0}^{2} A_m e_s^m [\cos m\Omega], \\ \left[\frac{v^4}{v_s^4}\right] &= \sum_{m=0}^{4} B_m e_s^m [\cos m\Omega]. \end{aligned} \tag{9}$$

Here

$$\begin{aligned} B_0 &= A_0^2 + e_s^2\left(\frac{1}{2}A_1^2 + 4 - 2\frac{p_s}{a} - 2\frac{p}{p_s}\right) + e_s^4\left(\frac{1}{2}A_2^2 - \frac{p}{2p_s}\right), \\ B_1 &= 2A_0A_1 + e_s^2\left(A_1A_2 + 2 - 2\frac{p}{p_s}\right), \\ B_2 &= \frac{1}{2}A_1^2 + 2A_0A_2 - 4 + 2\frac{p_s}{a} + 2\frac{p}{p_s}, \\ B_3 &= A_1A_2 + 2\frac{p}{p_s} - 2, \\ B_4 &= \frac{1}{2}A_2^2 + \frac{p}{2p_s}, \end{aligned}$$

brackets indicating the averaging

$$[\cos m\Omega] = \frac{\sin m\Omega_2 - \sin m\Omega_1}{m(\Omega_2 - \Omega_1)} \quad (m \neq 0).$$

Under the condition (5), $[\cos m\Omega] = 0$ if $m \neq 0$.

For μ_k with an odd integer subscript, the analytic averaging looks cumbersome. Numerical examples show $e_s\sqrt{A_1^2+4e^2p_s/p+e_s^2}|A_2| < A_0$ for Venus and the Earth. So a series expansion of v^k and then elementary integration is possible. But for Mercury and Mars, the situation is not so simple. There exist asteroids having the indicated quantity dozen times greater then A_0. And what is more, one can meet $A_0 < 0$ – of course in the case when not all values of Ω are permitted.

The algorithm simplifies drastically in the case of a negligible eccentricity e_s. The inequalities (1) and (5) coincide and become

$$a(1-e) < a_s < a(1+e). \tag{10}$$

The dependence on the longitude of the node in (8) vanishes as well as the necessity of averaging over Ω:

$$\left(\frac{v}{v_s}\right)^2 = 3 - \frac{a_s}{a} - 2\cos i\sqrt{\frac{p}{a_s}}. \tag{11}$$

3 COLLISION WITH AN ATTRACTING PLANET

The formulae (7, 8) represent the planetocentric velocity at the boundary of the planet action sphere. We may assume it as a velocity at infinity $\mathbf{v}_\infty$. The energy integral determines the velocity in the vicinity of a planet surface:

$$v^2 = v_\infty^2 + v_0^2, \tag{12}$$

v_0 being the parabolic velocity at the distance ρ from the planet centre. Here ρ is the radius of the planet (taking into account the atmosphere or not).

For k even the mean value of $[v^k]$ can be also calculated analytically. For example,

$$\begin{aligned} [v^2] &= v_0^2 + [v_\infty^2], \\ [v^4] &= v_0^4 + 2v_0^2[v_\infty^2] + [v_\infty^4], \end{aligned} \tag{13}$$

where $[v_\infty^2]$, $[v_\infty^4]$ are given by (9).

In the case $e_s = 0$,

$$v^2 = (3 - a_s/a - 2\cos i\sqrt{p/a_s})v_s^2 + v_0^2 \tag{14}$$

and we do not need the averaging over Ω again.

4 COLLISION WITH A NON-ATTRACTING SATELLITE

Four terrestrial planets possess three natural satellites. The plane of the lunar orbit is inclined at the angle of 5° to the ecliptic plane and its node line makes a full revolution during 19 years. The line of apses makes a full revolution in the orbital plane during 9 years. Thus, for our purposes, the lunar orbit may be assumed to be circular and lying in the ecliptic plane.

Similar orbital characteristics take place for both Phobos and Deimos except an essential reserve. Their orbits lie approximately in the Martian equatorial plane which precesses with the period of 200,000 years. If the velocity distribution depends on the position of the Martian equinox, then the estimates μ_k calculated for our epoch are valid for the time scale from dozens to dozens thousand years. If not, they are valid even for longer periods of time. Thus we assume that the orbits of Phobos and Deimos are circular and lie in the Martian equatorial plane.

Introduce a planetocentric Cartesian system with the axes parallel to the ones defined in Section 2. The satellite radius-vector $\mathbf{R}$ and its velocity $\mathbf{u}$ have the components

$$\frac{\mathbf{R}}{R} = (\cos\tilde{\Omega}\cos M - \cos\tilde{i}\sin\tilde{\Omega}\sin M, \\ \sin\tilde{\Omega}\cos M + \cos\tilde{i}\cos\tilde{\Omega}\sin M, \sin\tilde{i}\sin M), \tag{15}$$

$$\frac{\mathbf{u}}{u} = (-\cos\tilde{\Omega}\sin M - \cos\tilde{i}\sin\tilde{\Omega}\cos M, \\ -\sin\tilde{\Omega}\sin M + \cos\tilde{i}\cos\tilde{\Omega}\cos M, \sin\tilde{i}\cos M), \tag{16}$$

notations being clear. Assume the asteroid at $M = -\infty$ having the planetocentric velocity $\mathbf{v}_\infty$ according to (7), and at some finite moment M colliding with the satellite. The problem of hyperbolic orbit determination by a position at a fixed epoch and a velocity at infinity was solved by K. Sauer. Later R. Battin (1966, Sect. 4.7) improved the solution. Nevertheless he indicates one solution only, whereas in reality the problem has two solutions: the one corresponding to the retrograde motion has been omitted. Taking into account this remark, the relation

$$\mathbf{v} = \frac{v_\infty}{2R}(1 - l_1 q_1)\mathbf{R} + \frac{1}{2}(1 + l_1 q_1)\mathbf{v}_\infty - \mathbf{u} \tag{17}$$

determines the satellitocentric velocity of the asteroid at the collision epoch. Here $l_1 = \mp 1$, $q_1 > 0$,

$$q_1^2 = 1 + \frac{4Ru^2}{v_\infty(Rv_\infty - \mathbf{R}\mathbf{v}_\infty)}; \tag{18}$$

and $\mathbf{v}_\infty$ is given by (7).

When the vectors $\mathbf{R}$, $\mathbf{v}_\infty$ have the same direction, an indeterminate situation of the type $\infty - \infty$ emerges. It can be resolved without difficulties. But we do not dwell upon this matter. At practice it is easier to miss the corresponding value of M.

In accordance with (12), the squared sum of the first two addends on the right-hand side of (17) is equal to $v_\infty^2 + 2u^2$, so that

$$v^2 = v_\infty^2 + 3u^2 - (1 + l_1 q_1)\mathbf{u}\mathbf{v}_\infty. \tag{19}$$

The algorithm for estimating μ_k is as follows. For each selected asteroid and for each of four possible combinations $l = \mp 1$, $l_1 = \mp 1$, one calculates v^k using (19) on a set of admissible values of Ω and M from the segment $[0, 2\pi]$. One determines the v_∞ value using (8); q_1 using (18); and the scalar products $\mathbf{R}\mathbf{v}_\infty$ and $\mathbf{u}\mathbf{v}_\infty$, using (7), (15) and (16). For the Moon, $\tilde{i} = \tilde{\Omega} = 0$. For Phobos and Deimos, $\tilde{i} = 25.2°$ is the inclination of the equator of Mars to the orbit of the planet, $\tilde{\Omega} = 109.1°$ is the longitude of the ascending node of the equator on the orbit with respect to the axes introduced above, i.e., as measured from the perihelion of the planet.

Then one finds the v^k mean value over all admissible Ω, M and integer $l = \mp 1$, $l_1 = \mp 1$. At last one obtains μ_k as a mean over all selected asteroids.

The average over M and Ω can be evaluated analytically for $k = 2$. According to (7), (15) and (16),

$$\begin{aligned} \mathbf{R}\mathbf{v}_\infty / R v_\infty &= B_5 \cos M + B_6 \sin M, \\ \mathbf{u}\mathbf{v}_\infty / u v_\infty &= B_6 \cos M - B_5 \sin M, \end{aligned} \tag{20}$$

where

$$\begin{aligned} B_5 v_\infty &= v_x \cos\tilde{\Omega} + v_y \sin\tilde{\Omega}, \\ B_6 v_\infty &= \cos\tilde{i}(-v_x \sin\tilde{\Omega} + v_y \cos\tilde{\Omega}) + v_z \sin\tilde{i}, \end{aligned}$$

v_x, v_y, v_z being the components of $\mathbf{v}_\infty$ as given by (7). Represent the right-hand sides of (20) in the form $B_7 \cos\tilde{M}$, $-B_7 \sin\tilde{M}$, with $B_7 = \sqrt{B_5^2 + B_6^2}$, $\tilde{M} = M - M_0$, $\cos M_0 = B_5/B_7$ and $\sin M_0 = B_6/B_7$. According to (18), q_1 is an even function of $\tilde{M}$. So $(1 + l_1 q_1)\mathbf{u}\mathbf{v}_\infty$ is odd and

$$[v^2]_M = v_\infty^2 + 3u^2,$$

where $[v^2]_M$ denotes the average over M. Obviously,

$$[v^2] = [v_\infty^2] + 3u^2, \tag{21}$$

brackets indicating the average over M, Ω. As v_∞ does not depend on M, equation (9) is valid for $[v_\infty^2]$. We stress that (21) is valid for all $l = \mp 1$ and $l_1 = \mp 1$. So the only thing that has to be done is the averaging over all selected asteroids.

The average over M can be also evaluated analytically for $k = 4$. It follows from (19) that

$$[v^4]_M = (v_\infty^2 + 3u^2)^2 + [(1 + q_1^2 + 2l_1 q_1)(\mathbf{u}\mathbf{v}_\infty)^2]_M.$$

Taking into account (20) and averaging over $l_1 = \mp 1$, we obtain

$$[v^4]_M = (v_\infty^2 + 3u^2)^2 + \frac{1}{2} u^2 v_\infty^2 B_7^2 [(1 + q_1^2)(1 - \cos 2\tilde{M})]_M.$$

Obviously

$$v_\infty^2 (1 + q_1^2) = 2v_\infty^2 + \frac{4u^2}{1 - B_7 \cos\tilde{M}},$$

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{dM}{1-B_7\cos M}=\frac{1}{B_8},\quad \frac{1}{2\pi}\int_0^{2\pi}\frac{\cos 2M\,dM}{1-B_7\cos M}=\frac{(1-B_8)^2}{B_8B_7^2},$$

B_8 being the quantity $\sqrt{1-B_7^2}$. It follows that

$$[v^4]_M=(v_\infty^2+3u^2)^2+u^2\{B_7^2v_\infty^2+4(1-B_8)u^2\}. \tag{22}$$

When averaging over Ω and $l=\mp1$, it is better to proceed numerically due to the irrationality of B_8.

The case $e_s=0$ brings no essential simplifications.

5 COLLISION WITH AN ATTRACTING SATELLITE

The quantity (19) represents now the velocity at infinity with respect to the satellite. According to (12) the collision velocity is equal to

$$v^2=v_\infty^2+3u^2+u_0^2-(1+l_1q_1)\mathrm{u}\mathrm{v}_\infty, \tag{23}$$

u_0 being the parabolic velocity on the satellite surface. For Phebos and Deimos, u_0^2 is negligible. One has to use (23) in case of the Moon only.

The algorithm of estimating μ_k differs from that described above only by using (23) instead of (19).

If $k=2$, the formula

$$[v^2]=[v_\infty^2]+3u^2+u_0^2 \tag{24}$$

substitutes (21).

If $k=4$, the formula

$$[v^4]_M=(v_\infty^2+3u^2+u_0^2)^2+u^2\{B_7^2v_\infty^2+4(1-B_8)u^2\} \tag{25}$$

has to be used instead of (22).

Numerical results and their discussion will be presented in the next paper.

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