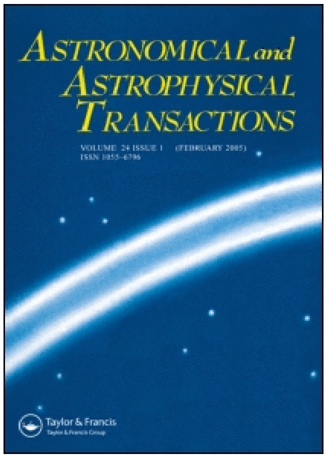

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ON THE FRACTAL NATURE OF THE LARGE-SCALE STRUCTURE OF THE UNIVERSE

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The observational evidences of the fractality of the large-scale distribution of galaxies are reviewed. A perfect linearity of the redshift-distance relation deeply inside the inhomogeneity cell in the fractal structure are stressed upon being contradictory to the traditional interpretation of the Hubble law as a consequence of homogeneity. It is shown that this contradiction could be resolved with allowance for the effect of gravitational cosmological redshift within a fractal structure with the fractal dimension $D_F \simeq 2$.

KEY WORDS Cosmology, fractals, Hubble law.

1. INTRODUCTION

The language of the theory of fractals has already been used in describing the large-scale distribution of galaxies for several years. The term "fractal" first introduced by Mandelbrot (1967; 1977; 1982) has appeared to be very fruitful in many branches of modern physics (see, e.g., Zel'dovich and Sokolov, 1985; Pietronero and Tosatti, 1986; Feder, 1988; Aharony and Feder, 1989). The first symposium completely devoted to fractals in astronomy was held in 1990 (Hech, 1990).

There are three main problems arising when one considers the large-scale structure as a fractal. First, this is a cut-off problem, i.e., one of the observational determination of the fractal structure limits, of its crossover into a homogeneous distribution. Second, this is the problem of dark matter: we need to obtain the parameters of the fractal distribution not only for the luminous matter (observed galaxies) but for the whole mass. And the third is a problem of origin and evolution: what physical processes are responsible for the fractal structure arising, its stability and development (evolution) with time.

This paper discusses some aspects of the problems. A review of observational evidences for fractality of the large-scale distribution of galaxies is given in Section 2. In Section 3 an "inhomogeneity paradox" is formulated which is connected with the strict linearity of the Hubble law deep within the fractal structure. A possible solution of the paradox is proposed in Section 4.

2. OBSERVATIONAL EVIDENCES FOR A FRACTAL STRUCTURE

The idea of fractality of the large-scale distribution of matter in the Universe has a quite long history. One of the simplest examples of a regular fractal is the usual hierarchy when some initial elements (e.g. stars) form first-level clusters that, in turn, are the elements of the second-level clusters and so on (to infinity). Hierarchical cosmological models were discussed as long ago as in the eighteenth century by Thomas Wright, Immanuel Kant and Johann Lambert (see a historical review in Harrison, 1981, Chapter 4: "Location and the Cosmic Center").

The most interesting history of the large-scale galaxy distribution investigation in the twentieth century is excellently described by Peebles (1980; Chapter 1: "Homogeneity and Clustering"), see also Baryshev (1981).

According to Mandelbrot (1967, 1977, 1982), the main feature of the fractals of discrete-mass clusters in three-dimensional Euclidean space are the statistical self-similarity at different scales and the power-law dependence between mass (or the number of objects) and radius of a sphere containing the objects:

$$M(R) \propto R^{D_F}, \tag{1}$$

where D_F is the fractal dimension of the cluster. Unlike a classical inhomogeneous sphere with a distinguished center, the fractal sphere has a "distinguished center" at any point mass of the structure. So, (1) is true for any observer at any point mass of the fractal structure (the "observer-homogeneous structure").

Galaxy counts in the 13^m–20^m interval are usually believed to be the main argument for their homogeneous space distribution. In the 30's, Hubble found that $\log N(m)$ is proportional to $0.6m$ in this magnitude interval. In the case of a fractal galaxy distribution (1), the Zeeliger theorem generalization is given by

$$\log N(m) = 0.20 D_F m + \text{const}, \tag{2}$$

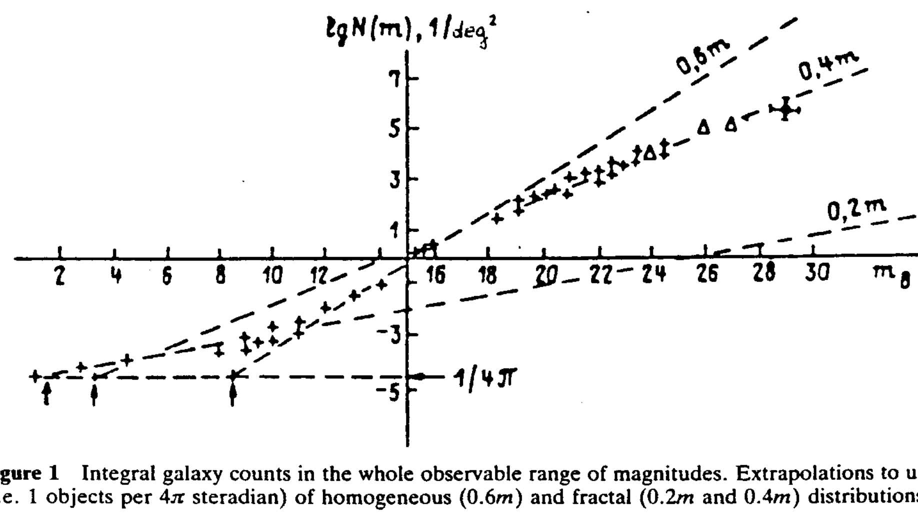

Figure 1 Integral galaxy counts in the whole observable range of magnitudes. Extrapolations to unity (i.e. 1 objects per 4π steradian) of homogeneous ($0.6m$) and fractal ($0.2m$ and $0.4m$) distributions.

where D_F is the fractal dimension of the galaxy distribution. The homogeneity takes place when $D_F = 3$.

Figure 1 taken from Baryshev (1992) shows galaxy counts in the whole observable magnitude range. $N(m)$ has three different slopes: 1) $0.2m$ for $m_B < 12$; 2) 0.6m for $12 < m_B < 19$; and 3) $0.4m$ for $m_M > 19$. Such complexity of the dependence means at least that the homogeneity is not so evident from galaxy counts as was thought earlier. In any case, different interpretations of the $N(m)$ behavior are possible. One of them, for example, is that there exists a multifractal structure of the large-scale galaxy distribution. Also, it is possible that the luminous mass (galaxies) has the space distribution different from that of the hidden mass whose fractal dimension is hidden too.

The slope 0.2 at the bright end of $N(m)$ is naturally interpreted as our being within a fractal structure with $D_F \approx 1$ for the luminous mass. The slope corresponds to the density-radius dependence of the form $\rho(R) \propto R^{-2}$ which is

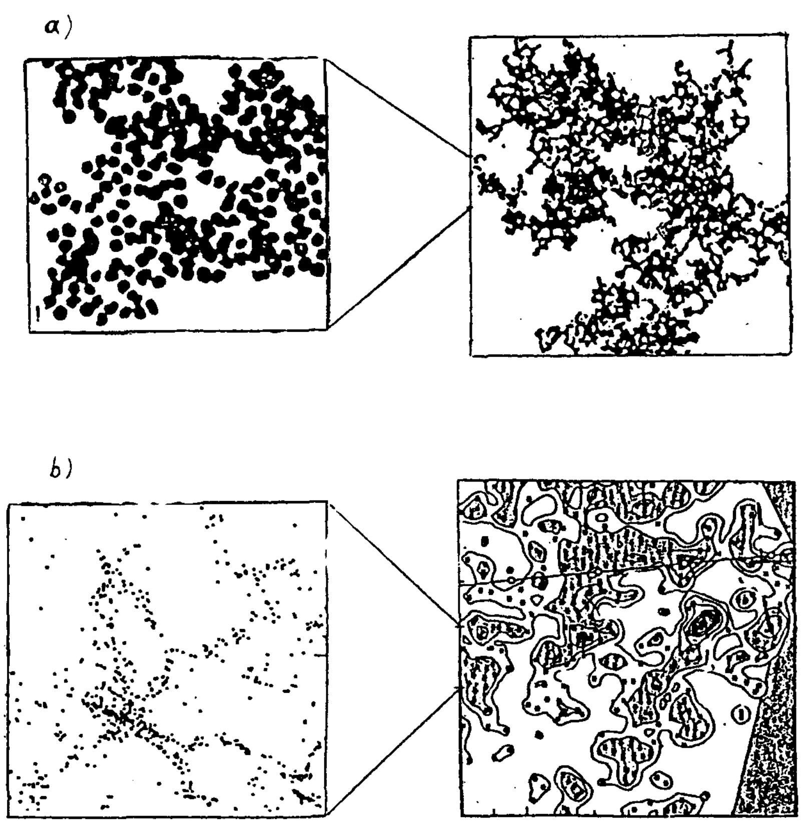

Figure 2 Statistical self-similarity of two examples of fractals: a). Fragments of the photographs of a fractal structure of colloid gold (Weitz, Huang, 1984). Sizes are 200 × 200 nm and 1000 × 1000 nm. b). Fragments of the distributions of galaxies, 100 × 100 Mpc (Lapparent *et al.*, 1986) and of rich clusters of galaxies, 500 × 500 Mpc (Tully *et al.*, 1992).

close to the well-known relation $\rho(R) \propto R^{-1.8}$ for various systems of galaxies (de Vaucouleurs, 1970).

We can consider the statistical self-similarity of the structures at various scales as another argument for the fractality. Figure 2a presents photographs of the fractal structures of colloid gold taken with an electron microscope. There sizes are 200×200 nm and 1000×1000 nm. Figure 2b shows the distribution of galaxies in a 100×100 Mpc square (Lapparent *et al.*, 1986) and the distribution of rich galaxy clusters (Tully *et al.*, 1992) in a 500×500 Mpc square. It is clearly seen in both cases that, in the clusters, there are holes of all sizes consistent with the cluster size. For the colloid gold clusters, $D_F = 1.7$, while for galaxy clusters D_F lies possibly between 1 and 2.

The power-law behavior of the two-point correlation function $\xi(R)$ (Peebles, 1980) is a strong evidence for the fractality of the large-scale galaxy

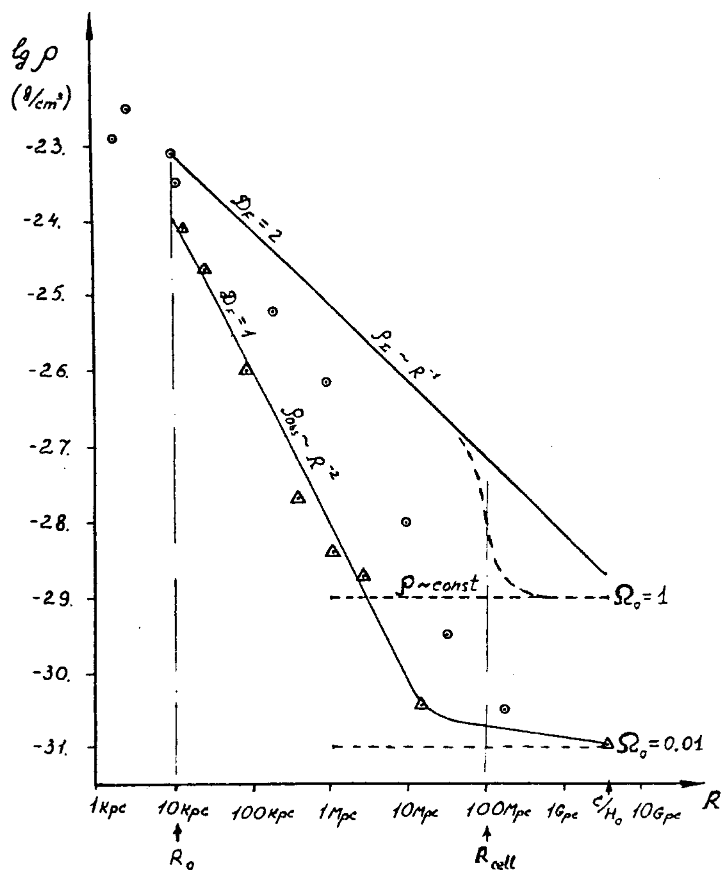

Figure 3 Dependence of mass density ρ on radius R for various galaxy systems (Vaucouleurs, 1970). Δ is the density estimate for the luminous mass; O is the virial estimate of the density. Straight lines correspond to fractal distributions with $D_F = 1$ and $D_F = 2$.

distribution together with the dependence of its amplitude A on the survey depth R_s (Pietronero, 1987; Calzetti *et al.*, 1988):

$$1 + \xi(R) = \frac{n_d(R)}{\langle n \rangle} = A \cdot R^{-\gamma}. \tag{3}$$

Here $n_d(R) = (1/4\pi R^2)/(dN(R)/dR)$ is the differential density of galaxies in a sphere of radius R, $\langle n \rangle$ is the average density of the sample within R_s,

$$\gamma = 3 - D_F, \tag{4}$$

$$A = \left(1 - \frac{\gamma}{3}\right) R_s^{\gamma}. \tag{5}$$

According to (4), γ is expressed through the fractal dimension D_F. Observations give the universal slope $\gamma = 1.8$ for the correlation function for galaxies, galaxy clusters and superclusters (Bahcall, 1988) and their amplitudes follow the relation (5) (Calzetti *et al.*, 1988; Luo and Schramm, 1992). This value of γ agrees with the fractal dimension of the luminous mass, $D_F \approx 1.2$.

A well-known relation between the mass density ρ and radius R for various galaxy systems (Karachentsev, 1968; Vaucouleurs, 1970) is shown in Figure 3. Solid straight lines are the expected $\rho(R)$ for fractal galaxy distributions with $D_F = 1$ and $D_F = 2$. The density of the luminous mass is close to $\rho \propto R^{-2}$ but then the virial mass density will be between R^{-2} and R^{-1}.

It is important that the galaxy counts for $m < 12^{\mathrm{m}}$, the power-law two-point correlation functions and the luminous mass density are all in a natural agreement with a fractal distribution of the luminous mass with $D_F \approx 1$, at least in the range 10 kpc–10 Mpc. At the same time, virial correlations indicate that the hidden mass seems to have $D_F = 1$ to 2 at scales 10 Mpc–100 Mpc. As for the inhomogeneity cell where the galaxies are distributed according to the fractal law and have a homogeneous distribution outside the cell, its size is, at least, more than 100 Mpc and there are indications that it can be as large as 200–300 Mpc (Lebedev and Lebedeva, 1988; Tully *et al.*, 1992).

3. THE INHOMOGENEITY PARADOX

It is well known that Hubble law, i.e., the linearity at small scales

$$z = \frac{\nu_s - \nu_{\mathrm{obs}}}{\nu_{\mathrm{obs}}} \approx \frac{H_0}{c} R, \tag{6}$$

is a consequence of homogeneity and isotropy of the Universe expansion, in terms of standard Friedmann models. However, one can consider the homogeneity only outside an inhomogeneity cell. Density fluctuations inside the cell will lead to a disturbance of pure Friedmann expansions and, hence, to a deviation from the linearity in the $z - R$ relation.

Deviation of the Hubble ratio $H(R)/H_0$ from unity within a fractal inhomogeneity cell was calculated by Fang *et al.*, (1991) in the framework of a generalized Robertson-Walker model taking into account the gravitational condensation of matter within a spherically symmetric inhomogeneity. Their

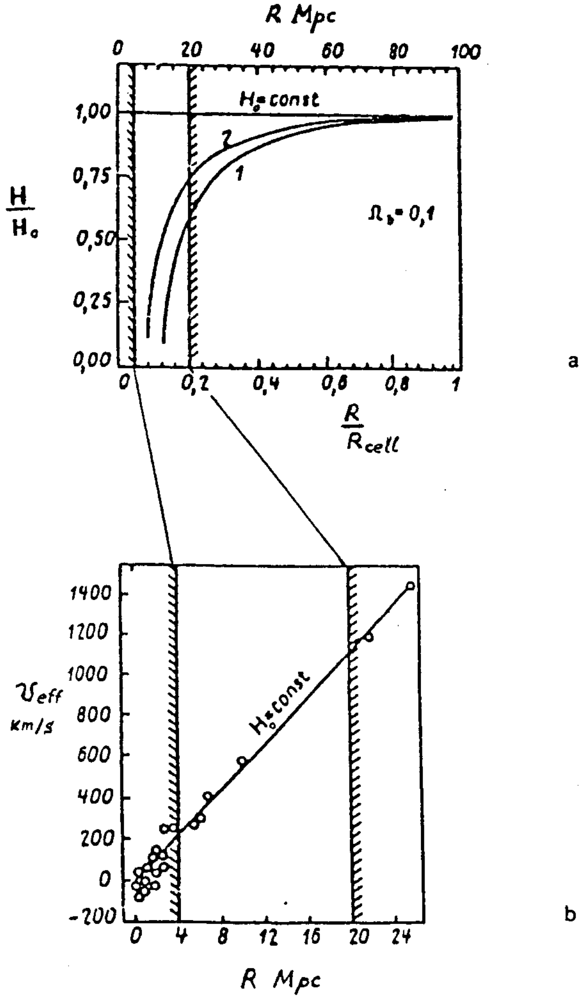

Figure 4 a). Calculated deviation from the linear Hubble law inside a fractal inhomogeneity cell with $D_F = 1.2$ (curve 1) and $D_F = 1.5$ (curve 2) (Feng, *et al.*, 1991). b). Observed relation between the effective radial velocity and distance inside the inhomogeneity cell (Sandage, 1986).

results are shown in Figure 4a. It follows from these calculations that at distances $R \leq 0.2R_{\text{cell}}$ (where R_{cell} is the inhomogeneity cell size) the Hubble constant can vary by several times! Moreover, the lesser is the fractal dimension, the stronger is the deviation from the linearity.

As mentioned above, the inhomogeneity size reaches at least 100 Mpc (Lebedev and Lebedeva, 1988; Tully *et al.*, 1992; Luo and Schramm, 1992). So, at distances less than 10–20 Mpc there must be a strong non-linearity in the redshift-distance relation.

However, observations suggest the opposite conclusion. According to Sandage

(1986), striking linearity of the $z - R$ relation is observed down to the distances of several Mpc. The observed $z(R)$ for 0–25 Mpc is presented in Figure 4b. Shaded are the coinciding scales in Figures 4a and 4b.

Strict linearity in the 4–20 Mpc range contradicts the theoretical prediction. This is called the inhomogeneity paradox: a highly inhomogeneous galaxy distribution at small scales, together with the linearity of (6) at these distances mean that the Hubble law linearity (discovered by Hubble at small scales, by the way) is not a consequence of the homogeneity.

One may assume that our Galaxy is in some "local void" outside the fractal structure and thus we have (see) a linear Hubble law for small distances. But this "good luck" explanation is difficult to reconcile with bright galaxy counts, $N(m) \approx 0.2m + \text{const}$, demonstrating that the Galaxy lies deep within the fractal structure or in a strong density fluctuation.

The next section proposes another explanation of the paradox.

4. POSSIBLE SOLUTION OF THE "INHOMOGENEITY PARADOX"

So far as we are concerned with cosmologically small scales ($R < 100$ Mpc), the Newtonian theory of gravitation is adequate for our purposes.

Let us consider a fractal distribution of galaxies with the dimension D_F in the distance range from the galaxy radius R_0 to R_{cell}, the inhomogeneity cell radius. As mentioned above, in this case the Universe looks equally inhomogeneous from any galaxy (observer-homogeneous structure), i.e., spherically symmetric mass distribution with any galaxy as a center will be given by $M(R) \propto R^{D_F}$, where $R \in (R_0, R_{\text{cell}})$.

Following Bondi (1947), let us choose a spherical coordinate system centered on an arbitrary galaxy emitting radiation. Also, let the observer be at the distance R from the source. As shown by Bondi (1947), in this case the cosmological redshift of spectral lines observed at small R is given by

$$z_{\cos}(R) \approx \frac{v(R)}{c} + \frac{1}{2} \cdot \frac{v^2(R)}{c^2} + \frac{\delta \Phi_N(R)}{c^2}. \tag{7}$$

The first two terms on the right-hand side of Eq. (7) correspond to the cosmological Doppler shift because of the relative recession velocity v. The third term is the cosmological gravitational spectral shift arising from the gravitational potential difference $\delta\Phi_N(R) = \Phi_N(R) - \Phi_N(0)$ between the source and the observer. Thus the spectral shift does not only depend on conditions at the source and at the observer's location but also on the distribution of matter within the whole sphere of radius R around the source. It is essential to note that the choice of the frame with the origin at the source is dictated by causality and isotropy principles. That is why the cosmological gravitational shift is the red shift.

It was shown by Bisnovatyi-Kogan (1972) that there is a principal possibility of the construction of a stationary hierarchical stellar cluster with a high gravitational redshift. Here we consider the gravitational redshift by a fractal galaxy distribution with the fractal dimension D_F. Let us define the differential mass density in the sphere of radius R around the source as

$$\rho(r) = \rho_0 \left(\frac{R_0}{r}\right)^{3-D_F}, \tag{8}$$

where ρ_0 and R_0 are the density and the radius of the basic "point-like" object of the structure (galaxies in our case). Solving the Poisson equation for the sphere we obtain the following expression for the Newtonian gravitational potential within the sphere (for $1 < D_F \leq 3$):

$$\Phi_N(r) = -\frac{4\pi G\rho_0 R_0^2}{D_F(D_F-1)}\left[D_F\left(\frac{R}{R_0}\right)^{D_F-1} - \left(\frac{r}{R_0}\right)^{D_F-1}\right]. \quad (9)$$

For a static structure, $v = 0$, then it follows from (7):

$$z_{\cos}(R) = \frac{\delta\Phi_N(R)}{c^2} = \frac{1}{c^2}(\Phi_N(R) - \Phi_N(0)), \quad (10)$$

and using (9) we obtain

$$z_{\cos}(R) = \frac{4\pi G\rho_0 R_0^2}{c^2 D_F(D_F-1)}\left(\frac{R}{R_0}\right)^{D_F-1}. \quad (11)$$

For the fractal structure with $D_F = 2$ an interesting conclusion follows immediately that the cosmological gravitational redshift is a linear function of distance (Baryshev, 1981):

$$z_{\cos}(R) = \frac{2\pi G\rho_0 R_0 R}{c^2} = \frac{H_g}{c} R,$$

where H_g may be called the gravitational Hubble constant expressed by

$$H_g = \frac{2\pi G\rho_0 R_0}{c} = 68.6\left(\frac{\rho_0}{5.2\cdot 10^{-24}\,\mathrm{g/cm^3}}\right)\left(\frac{R_0}{10\,\mathrm{kpc}}\right)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}.$$

Here, numerical values of ρ_0 and R_0 correspond to characteristic galaxy parameters and were chosen so that $\rho_0 R_0 = 1/2\pi$, i.e., H_g numerically equals to G/c.

Thus, a possible solution of the inhomogeneity paradox is provided by taking into account the cosmological gravitational redshift. In the case of $D_F = 2$ it just provides the linearity of the Hubble law inside the inhomogeneity cell.

However, one should explain why the fractal dimension is equal to 2. Are there now any observational or theoretical evidences for the fundamental nature of this value of D_F?

As already noted, the luminous matter inside the cell is fractally distributed with $D_F \approx 1.2$. Consequently, the only one possibility for the gravitational explanation of the paradox is to assume that the hidden mass has the fractal distribution with $D_F \approx 2$. Virial estimates of the hidden mass in galaxy systems of various scales are in accordance with $D_F \approx 2$ (see a review by Baryshev, 1981). It is important to note that, for a fractal structure, the estimation of the total mass via observations of peculiar galaxy velocities is a difficult problem because each galaxy participates in many motions at different structure levels (scales).

Theoretical arguments for fractals with $D_F = 2$ are as follows. First, there exists a special class of robust Brownian fractals with $D_F = 2$ (Mandelbrot, 1977). Second, Perdang (1990) showed that self-gravitating fractal configurations have critical dimension $D_F \approx 2$, below which these configurations become stable to the gravitational phase transition. And third, Lou and Schramm (1992) considered a theoretical restriction on a possible fractal structure arising by diffusion-limited aggregation. From Ball and Witten's causality bound they concluded that the observed fractal dimension $D_F = 1.2$ implies that the dimension d_F of the growth space must be less then 2.2. That is, the background growth space (it can be some sort of dark matter) should involve a two-dimensional fractal structure.

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