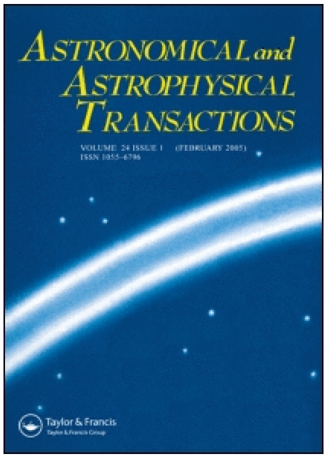

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 January 1994
To cite this Article: Ryabtsev, A. D. (1994) 'The flocculent spiral patternin a multicomponent rotating disc', Astronomical & Astrophysical Transactions, 5:1, 117 - 126
To link to this article: DOI: 10.1080/10556799408245865
URL: http://dx.doi.org/10.1080/10556799408245865

Astronomical and Astrophysical Transactions, 1994,
Vol. 5, pp. 117–126
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THE FLOCCULENT SPIRAL PATTERN IN A MULTICOMPONENT ROTATING DISC

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(4 February 1993)

A fluid-dynamical model of a multicomponent rotating disc is considered. The mass transfer and energy balance in the system of atomic and molecular gas and massive stars are taken into account. It is demonstrated that under certain conditions irregular spiral pattern can develop as a result of internal instability in the system.

Thus it is shown that cooperative effects of mass and energy transfer and hydrodynamic motions may be relevant for understanding the morphology of disc galaxies.

KEY WORDS Disc galaxies, spiral pattern, interstellar medium.

1. INTRODUCTION

Disc galaxies are complicated systems in which various physical phenomena are closely connected with each other. Thus star formation rate depends on the properties of the galactic disc such as the epicyclic frequency, and the density and velocity dispersion (Kennicutt, 1989). In turn, energy input from newborn massive stars affects the interstellar medium (ISM) and can change its properties relevant for fluid dynamics, i.e., temperature, velocity dispersion, gas density, etc. Therefore, mutual influence of processes of different nature taking place at different time and space scales should be taken into account in theoretical description of galaxies.

Korchagin and Ryabtsev (1993, hereafter KR) made a step towards the description of cooperative effects of star formation and phase transitions in the ISM on the one hand and long-range hydrodynamic motions on the other. In KR, star formation and mass exchange processes between the atomic and molecular phases of ISM are taken into account and are described in the framework of a model nonlinear system similar to that proposed by Bodifee & de Loore (1985). At the same time all the components are involved into hydrodynamic motions.

The main effect of star formation on fluid dynamics is due to the energy input of massive stars into the ISM. In KR, it was assumed that the main effects of energy input are the photodissociation of molecules and the maintenance of the velocity dispersions of components. The latter were assumed to be constant but different for atomic and molecular phases. The present paper presents the approach similar to that of KR, but an explicit equation of energy balance is included into the system. At the same time, the difference in the dispersions of the components are not taken into account. We confirm the result of KR that hydrodynamic motions, mass transfer and heating can lead to the formation of a flocculent wave pattern even in rigidly rotating disc. The waves perturb total density and velocity field in contrast to density wave patterns in the models of reaction–diffusion type (Nazakura and Ikeuchi, 1984, 1988; Korchagin and Ryabtsev, 1992).

2. THE MODEL AND BASIC EQUATIONS

To incorporate nonlinear exchange processes and hydrodynamic motions of the ISM we proceed from the set of nonlinear equations describing the evolution of the system consisting of massive stars and gas in atomic and molecular phases (Bodifee, 1986):

$$\frac{d\mu_s}{dt} = -\frac{1}{\tau}\mu_s + a\mu_s\mu_c, \tag{1}$$

$$\frac{d\mu_c}{dt} = -\alpha a\mu_s\mu_c + b\mu_c^2\mu_g, \tag{2}$$

$$\frac{d\mu_g}{dt} = \frac{1}{\tau}\mu_s - b\mu_c^2\mu_g + (\alpha - 1)a\mu_s\mu_c \tag{3}$$

Here μ_s, μ_c and μ_g are the densities of massive stars, molecular clouds and atomic gas, respectively. The coefficient a determines the rate of the induced star formation and the coefficient b determines the rate of transformation of the atomic gas into the molecular phase. The parameter α gives the fraction of the cloud phase evaporated by newborn massive stars. It should be noted that, in the case $\alpha = 1$, Eqs (1)–(3) coincide with those proposed by Ikeuchi and Tomita (1983) if μ_s, μ_c and μ_g are substituted for the densities of the warm (X_w), hot (X_h) and cold (X_c) gas respectively.

The key process determining the unstable dynamics of the system of Eqs (1)–(3) is the positive feedback loop of the molecular cloud generation from the gas phase in Eq. (2).

The production of H_2 molecules in the atomic phase is mainly a dust-catalyzed process and depends on the number densities of atoms and dust grains. The formation of dust grains is a multi-stage process sensitive to the intensity of the radiation flux of the stars causing the photodestruction of molecular chains (Vajner, Glukhov & Chuvenkov, 1991).

The increasing of the local gas density due to some process leads to the increasing of the number density of dust grains and therefore decreases the photodestruction of molecular chains by stellar radiation. One may expect therefore that the production rate of molecules is a nonlinear function of the densities of atomic and molecular phases of the form $\mu_g\mu_c^n$, where $n > 1$. In our paper it is assumed for simplicity that $n = 2$. Similar arguments were used by Bodifee and de Loore (1989) in their model of mass exchange in a molecular star-forming complex.

As shown below, this process leads to the generation of large-scale hydrodynamic motions in rotating discs, i.e., spiral density waves.

The positive feedback loop discussed above it not the only mechanism generating unstable oscillations in the ISM. Shore (1981), Scalo and Struck-Marcell (1986), Korchagin, Korchagin and Ryabtsev (1988) discussed the instabilities of the ISM caused by the processes with a time delay in the ISM. These instabilities could lead to the development of density waves as well.

The one-zone model can be generalized by introducing hydrodynamic and diffusion processes. Taking the transport processes into account the system of equations (1)–(3) can be generalized as follows:

$$\frac{d\mu_s}{dt} + \mu_s \operatorname{div} v = -\frac{1}{\tau}\mu_s + a\mu_s\mu_c, \tag{4}$$

$$\frac{d\mu_c}{dt} + \mu_c \operatorname{div} v = -\alpha a\mu_s\mu_c + b\mu_c^2\mu_g + \nabla\left[(\mu_c + \mu_g)D\nabla\frac{\mu_c}{\mu_c + \mu_g}\right], \tag{5}$$

$$\frac{d\mu_g}{dt} + \mu_g \operatorname{div} v = \frac{1}{\tau}\mu_s - b\mu_c^2\mu_g + (\alpha - 1)a\mu_s\mu_c + \nabla\left[(\mu_c + \mu_g)D\nabla\frac{\mu_g}{\mu_c + \mu_g}\right]. \tag{6}$$

Here v is the velocity in the multicomponent system determined as a total specific momentum of a multicomponent fluid (Landau and Lifschitz, 1959); D is the diffusion coefficient.

So, the equation of motion of a star-forming medium in a rigidly rotating disc in the reference frame rotating with the disc can be written in the form:

$$\frac{dv}{dt} = -\frac{1}{\mu}\nabla[\varepsilon(\mu - \mu_s)] + 2[v\Omega] + \nu\,\Delta v. \tag{7}$$

Here μ is the total density, ε is the specific energy of random motions in the ISM, ν is the specific energy of random motions in the ISM, ν is the kinematic viscosity, Ω is the angular velocity of the disc rotation. The centrifugal force is assumed to be compensated by the gravity of the bulge and halo of the galaxy. It should be noted that the diffusion, which is taken into account in Eqs (5) and (6), leads to the transport of the components even in the absence of the fluid motions as a whole. The motions of the whole multicomponent system could be caused by pressure gradients and Coriolis forces.

The Eqs (4)–(7) do not take into account the differential rotation of the disc. Therefore, our consideration is applicable only to inner rigidly rotating regions of disc galaxies. Such a treatment, however, could be of some astrophysical interest. The development of spiral structures in differentially rotating discs with self-propagating star formation is a well-known phenomenon (Seiden and Gerola, 1979, Seiden, Schulman and Feitzinger 1982). In contrast to this, we will demonstrate that even in a rigidly rotating disc spirals may develop as a result of nonlinear self-organization.

The ISM and collisionless stars are considered in the one-fluid approximation. This approximation is valid because the model incorporates rapidly evolving massive stars which do not travel far away from their parent clouds.

We assume that random motions in the ISM are maintained by energy input from massive stars and the dissipation is due to cloud collisions. Hence, the equation of energy balance for the random motions reads

$$\frac{d\varepsilon}{dt}+\varepsilon\,\mathrm{div}\,v=\chi\,\Delta\varepsilon+\Gamma\frac{\mu_s}{\mu_0}-\varepsilon^{3/2}L_c. \tag{8}$$

The cooling term is introduced similarly to Larson (1969); L_c is the mean free path of the clouds.

Assuming the absence of hydrodynamic motions one can obtain the stationary, spatially homogeneous solution of Eqs (4)–(8):

$$\mu_c^{(0)}=\frac{1}{\tau a}, \tag{9}$$

$$\mu_g^{(0)}=(\mu_0-\mu_c^{(0)})/(1+\xi/\alpha), \tag{10}$$

$$\mu_s^{(0)}=(\xi/\alpha)\mu_g^{(0)}, \tag{11}$$

$$\varepsilon_0=(\Gamma L\mu_s^{(0)}/\mu_0)^{2/3}, \tag{12}$$

where μ_0 is the total unperturbed surface density of the disc, $\xi=b/(a^2\tau)$ is a dimensionless parameter. We shall discuss the evolution of long-wave perturbations of the steady state. Therefore, it is convenient to present basic equations in terms of the deviations from the stationary solution:

$$\frac{d\tilde{\mu}_s}{dt}+(\tilde{\mu}_s+\mu_s^{(0)})\,\mathrm{div}\,v=\frac{\zeta}{\tau}\tilde{\mu}_c+a\tilde{\mu}_s\tilde{\mu}_c, \tag{13}$$

$$\begin{aligned}\frac{d\tilde{\mu}_c}{dt}+(\tilde{\mu}_c+\mu_c^{(0)})\,\mathrm{div}\,v=&\frac{\xi}{\tau}\tilde{\mu}+\frac{\alpha\zeta-\xi}{\tau}\tilde{\mu}_c-\frac{\alpha+\xi}{\tau}\tilde{\mu}_s\\&+a\xi\tilde{\mu}\tilde{\mu}_c-(\alpha+2\xi)a\tilde{\mu}_s\tilde{\mu}_c+(\alpha\zeta-2\xi)\tilde{\mu}_c^2+b\tilde{\mu}\tilde{\mu}_c^2-b\tilde{\mu}_c^3\\&-b\tilde{\mu}_s\tilde{\mu}_c^2+D\,\Delta\tilde{\mu}_c+D\frac{\xi}{\xi+\alpha\zeta}\Delta(\tilde{\mu}_s-\tilde{\mu}),\end{aligned} \tag{14}$$

$$\frac{d\tilde{\mu}}{dt}+(\mu_0+\tilde{\mu})\,\mathrm{div}\,v=0, \tag{15}$$

$$\frac{dv}{dt}=-\frac{\varepsilon_0}{\mu}\nabla(\mu-\tilde{\mu}_s)-(1-\mu_s^{(0)}/\mu_0)\,\nabla\tilde{\varepsilon}+2[\Omega v]+\nu\,\Delta v, \tag{16}$$

$$\frac{d\tilde{\varepsilon}}{dt}+\varepsilon_0\,\mathrm{div}\,v=\chi\,\Delta\tilde{\varepsilon}+\Gamma\frac{\mu_s^{(0)}}{\mu_0}-\frac{1}{\tau_\varepsilon}\tilde{\varepsilon}. \tag{17}$$

Here $\tilde{\mu}$, $\tilde{\mu}_s$, $\tilde{\mu}_c$ and $\tilde{\varepsilon}$ are the perturbations of the stationary values (9)–(12). The dimenisonless parameter ζ is related to ratio of the stationary densities as

$$\zeta=\mu_s^{(0)}/\mu_c^{(0)}. \tag{18}$$

The time scale of the dissipation of random motions is introduced as

$$\tau_\varepsilon=\tfrac{2}{3}\varepsilon_0^{-1/2}L_c. \tag{19}$$

3. THE DERIVATION OF THE AMPLITUDE EQUATION

The system discussed above is rather complicated and difficult to deal with. Fortunately, there is an important case when the system can be truncated and reduced to a single complex nonlinear equation, known as the Ginzburg–Landau equation. It is the case when solutions of the system are in a certain sense close to the solutions of the one-zone model.

A spatially homogeneous perturbation of the stationary state gives rise to the oscillations with the growth rate

$$\gamma = (\zeta\alpha - \xi)/2\tau \equiv \delta^2/2\tau, \tag{20}$$

and the frequency

$$\omega = \left[\omega_0^2 - \frac{\delta^4}{4\tau^2} - \frac{\delta^2(\xi+\alpha)}{\alpha\tau^2}\right]^{1/2}, \tag{21}$$

where $\omega_0 = [\xi(\xi+\alpha)/\alpha]^{1/2}/\tau$ is the frequency at the marginally stable point $\delta = 0$. For the unstable case, i.e., $\gamma > 0$, the development of the instability at nonlinear stage leads to the formation of a limit cycle, its amplitude being of order δ for small δ.

The Ginzburg–Landau equation can be derived if the two conditions are satisfied (KR): (i) The instability (20) is the only one in the system; (ii) Only sufficiently long-range perturbations are unstable, i.e. those with $\lambda > \mathrm{const} * \delta$, where λ is the wavelength. These conditions should be checked by linear analysis of the stability of the stationary state (9)–(12). This rather cumbersome analysis is analogous to that in (KR) and is omitted here.

Let us seek for a solution of Eqs (13)–(17) as a perturbation of spatially uniform oscillations with the marginal frequency ω_0:

$$\bar{\mu}_s = s_0 + (s_1(x,t)e^{i\omega_0 t} + s_2 e^{2i\omega_0 t} + \text{complex conjugate}), \tag{22}$$

$$\bar{\mu}_c = c_0 + (c_1(x,t)e^{i\omega_0 t} + c_2 e^{2i\omega_0 t} + \text{complex conjugate}). \tag{23}$$

If the deviation from the marginal stability δ is a small parameter, the spatial derivatives and amplitudes s_1 and c_1 are order δ and the amplitudes s_0, s_2, c_0 and c_2 are of order δ^2. Substituting expression (22) into the energy balance equation (17), one gets the relation between $\bar{\varepsilon}$ and s_1 in the leading order in δ as

$$\bar{\varepsilon} = \frac{\Gamma}{\mu_0(i\omega_0 + 1/\tau_\varepsilon)} s_1 e^{i\omega_0 t} + \text{complex conjugate}. \tag{24}$$

Similarly, one gets from the Euler equation (16) the velocity up to δ^3 terms:

$$v = \frac{i\omega_0}{\omega_0^2 - 4\Omega^2}\left(\frac{\Gamma}{i\omega_0 + 1/\tau_\varepsilon} - \varepsilon_0\right)\frac{\nabla s_1}{\mu_0} e^{i\omega_0 t} + \text{complex conjugate}. \tag{26}$$

The velocity divergency reads:

$$\operatorname{div} v = \frac{i\omega_0}{\omega_0^2 - 4\Omega^2}\left(\frac{\Gamma}{i\omega_0 + 1/\tau_\varepsilon} - \varepsilon_0\right)\frac{\Delta s_1}{\mu_0} e^{i\omega_0 t} + \text{complex conjugate}. \tag{26}$$

The perturbation of the total density can be expressed using Eq. (15) as

$$\bar{\mu} = -\frac{1}{\omega_0^2 - 4\Omega^2}\left(\frac{\Gamma}{i\omega_0 + 1/\tau_\varepsilon} - \varepsilon_0\right)\Delta s_1 e^{i\omega_0 t} + \text{complex conjugate.} \tag{27}$$

Substituting expressions (26) and (27) into Eqs (13) and (14) one can obtain two equations, for stars $\bar{\mu}_s$ and clouds $\bar{\mu}_c$ only.

The expressions for $\bar{\mu}$ and div v are of order δ^3 and do not contribute into the nonlinear terms. Therefore higher order harmonics can be expressed in terms of s_1 and c_1 using the one-zone model. After some calculations one gets the equation of Ginzburg–Landau type (some details of the calculations are given in Appendix):

$$\frac{\partial c_1}{\partial t} = \frac{\delta^2}{2\tau}\left[1 + \left(\frac{\alpha+\xi}{\alpha\xi}\right)^{1/2}\right]c_1 - b\frac{2\alpha+\xi}{\alpha+\xi}(1+i\Omega)\,|c_1|^2 c_1 + (D_1 + iD_2)\,\Delta c_1, \tag{28}$$

with

$$Q = \frac{\alpha^2 + (\alpha + 6\alpha^2)\xi + (6\alpha + 4\alpha^2)\xi^2 + (1+4\alpha)\xi^3}{3\xi(2\alpha+\xi)[\alpha\xi(\alpha+\xi)]^{1/2}}, \tag{29}$$

$$D_1 + iD_2 = \frac{1}{2}D + \frac{\xi\varepsilon_0}{2(2\alpha+\xi)\tau(\omega_0^2 - 4\Omega^2)}\left\{1 + \frac{\tilde{\Gamma}\left(\xi\frac{\tau_\varepsilon}{\tau} - 1\right)}{(1+\varepsilon_0^2\tau_\varepsilon^2)}\right\}$$
$$+ \frac{i}{2(2\alpha+\xi)}\left(\frac{\alpha\xi}{\alpha+\xi}\right)^{1/2}\left\{-D + \frac{\xi\varepsilon_0}{\tau(\omega_0^2 - 4\Omega^2)}\left[\frac{\tilde{\Gamma}\left(\frac{(\alpha+\xi)\tau_\varepsilon}{\alpha\tau} + 1\right)}{(1+\varepsilon_0^2\tau_\varepsilon^2)} - 1\right]\right\}, \tag{30}$$

where the dimensionless heat parameter is introduced as $\tilde{\Gamma} = \Gamma\tau_\varepsilon/\varepsilon_0$. For numerical simulations, it is convenient to rewrite Eq. (28) in dimensionless form introducing the time scale T, the length scale L and the dimensionless amplitude ψ by the relations:

$$T = \frac{2\tau}{\delta^2}, \tag{31}$$

$$L = \frac{1}{\delta}(2\tau D_1)^{1/2}, \tag{32}$$

$$c_1 = \mu_0\delta\alpha\left[\frac{\alpha+\xi}{2\xi(2\alpha+\xi)^3}\right]^{1/2}\psi \tag{33}$$

Equation (28) can be rewritten in dimensionless form as

$$\frac{\partial\psi}{\partial t} = \tilde{\gamma}(r)\left[1 + i\left(\frac{\alpha+\xi}{\alpha\xi}\right)^{1/2}\right]\psi + (1+i\beta)\,\Delta\psi - (1+iQ)\,|\psi|^2\psi, \tag{34}$$

where $\beta = D_1/D_2$. The slowly varying function $\tilde{\gamma}(r)$ gives the radial dependence of the growth rate of the instability.

4. THE PROPERTIES OF THE AMPLITUDE EQUATION AND NUMERICAL RESULTS

As it is seen from Eq. (30), the imaginary part of the effective diffusion coefficient is negative if $\omega_0 < 2\Omega$ and the terms related to the heating process dominate.

Therefore, β and Q have opposite signs. The behaviour of solutions of the one-dimensional Ginzburg–Landau equation in this case was investigated by Nozaki and Bekki (1983). These authors have shown that the Ginzburg–Landau equation has stable solutions if $\beta Q > -1$. If the opposite inequality takes place, i.e.,

$$\beta Q < -1, \tag{35}$$

all solutions are modulationally unstable, and numerical simulations performed by Nozaki & Bekki demonstrate the development of the one-dimensional chaos. In the two-dimensional case, similar results were obtained by Rogal'skii (1989).

It is demonstrated in what follows that the condition (35) may be satisfied for a set of astrophysically reasonable parameters. Thus, the system under consideration has nontrivial wave solutions. Let us assume that the ratio of the surface densities of the atomic and molecular phases is close to unity and the fraction of massive stars is about 0.1%. These assumptions give the following values for the dimensionless parameters α and ξ: α is of order 100 and ξ is of order 0.1.

The diffusion coefficient D can be estimated as

$$D = \tfrac{1}{3}\, l_t v_t.$$

where v_t and l_t are the velocity and length scales of the ISM turbulent pulsations. Adopting $v_t = 10$ km/s and $l_t = 100$ pc one obtains $D = 10^{26}\ \mathrm{cm^2/s}$ (Parker 1971). The kinematic viscosity ν and the temperature conductivity χ should be of the same order of magnitude.

We assume that the source of random motions in the ISM are SN shocks (Spitzer, 1978). The heating coefficient Γ can be estimated as follows:

$$\Gamma = \frac{\eta E_*}{\tau M_*},$$

where E_* is energy of an SN event, M_* is mass of the OB stars; η characterises the efficiency of transformation of the energy of an SN event into the energy of random motions of the ISM. Assuming $E_* = 10^{51}$ ergs, $\eta = 10^{-2}$ and $M^* = 10 M_\odot$, one obtains $\Gamma\tau/\varepsilon_0 = 10^3$, if the random velocity is $c = \varepsilon_0^{1/2} = 10$ km/s. The cooling time can be expressed as

$$\tau_\varepsilon = \frac{2}{3}\frac{\mu_0 \varepsilon_0}{\mu_s^{(0)}\Gamma},$$

and is of the same order as τ. A linear analysis shows that the conditions under which the Ginzburg–Landau equation is valid are satisfied for this set of parameters. The inequality (35) is also satisfied. Numerical simulations confirm that stochastic wave patterns develop if the system is unstable, i.e. $\gamma > 0$ in (20).

Let us seek for the solutions of Eq. (34) of the form

$$\psi = \psi_0(r, e) e^{im\theta}. \tag{36}$$

In order to perform numerical simulations let us assume that the growth rate of the instability $\bar{\gamma}(r)$ slowly varies with radius and it is positive in the inner parts of the disc and negative at its periphery. This assumption corresponds to the unperturbed total density decreasing with radius and leads to the perturbations vanishing at the periphery of the disc. In this case, the vanishing of the solution at the boundary is a natural boundary condition in numerical simulation.

The result of the evolution of small initial perturbation for $m = 2$ is plotted in Figure 1. In agreement with general theory discussed above, a flocculent pattern has developed.

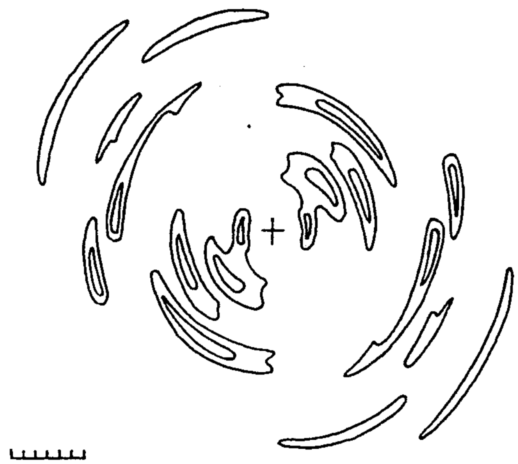

Figure 1 A well developed flocculent pattern. Contors correspond to Re ψ equal to 0.3 and 0.6; $t = 15$ in units of T; scale in units L is shown.

5. CONCLUDING REMARKS

In this paper an analytical model describing hydrodynamic motions in a multicomponent interstellar medium undergoing interchange processes between its components has been proposed.

The dynamics of perturbations depends on the deviation of the system from the marginal stability, as it can be seen from Eqs. (31)–(33). For example, the time scale is 2×10^8 yrs if $\delta^2 = 0.1$. The length unit is 180 pc, so the spatial scale of the perturbations is of a kiloparsec order.

Taking into consideration self-gravity gives a well-known possibility of the generation of a spiral structure due to the gravitational instability. It should be kept in mind, however, that galactic discs are usually close to the gravitational marginal stability and the time scale of the generation of the spiral structure by the gravitational instability is of the order of 10^9 years. Therefore, the 'mass-transfer' instability considered above could dominate even in self-gravitating discs. However, for a long time scales the self-gravity could lead to the coexistence of the gravitationally and non-gravitationally generated structures.

The magnitude of the wave velocity and the perturbation of the total density can be estimated with the help of Eqs (25) and (27). The estimation leads to the velocity magnitude about one kilometer per second and the total density perturbation about few per cent. These low values are not unexpected.

Indeed, our numerical estimates are limited by the validity of the perturbation theory. As shown in Section 3, the velocity is a quantity of the second order and the total density perturbation, of the third order in the expansion parameter δ. So, the above results should be regarded as a qualitative demonstration of the excitation mechanism of density waves by an internal instability in the ISM. In the case of a strong instability, the perturbations must be greater and achieve observationally detectable values.

Acknowledgments

The author thanks V. I. Korchagin and Yu. A. Schekinov for useful discussions.

APPENDIX

Substituting expressions (22) and (23) into Eqs (11) and (12) leads to the expressions for the higher harmonics of the form:

$$c_0 = 0, \tag{A1}$$

$$s_0 = -\frac{2\xi\tau a}{\alpha+\xi}|c_1|^2, \tag{A2}$$

$$c_2 = \frac{\tau a c_1^2}{3}\left\{2\frac{\alpha+2\xi}{\alpha+\xi} - i(1-2\xi)\left[\frac{\alpha}{\xi(\alpha+\xi)}\right]^{1/2}\right\}, \tag{A3}$$

$$s_2 = \frac{\tau a c_1^2}{3}\left\{\frac{\xi-2}{\alpha+\xi} - i(\alpha+2\xi)\left[\frac{\alpha}{\alpha(\alpha+\xi)}\right]^{1/2}\right\}, \tag{A4}$$

Substituting expressions (A1)–(A4) together with expressions (26) and (27) into Eqs (13) and (14) leads to the equations for the first harmonics:

$$\frac{\partial s_1}{\partial t} + i\omega_0 s_1 = \frac{\xi+\delta^2}{\alpha\tau}c_1 + (A_s + iB_s)|c_1|^2 c_1 + D_{ss}\,\Delta s_1 + D_{sc}\,\Delta c_1, \tag{A5}$$

$$\frac{\partial c_1}{\partial t} + i\omega_0 c_1 = -\frac{\alpha+\xi}{\tau}s_1 + \frac{\delta^2}{\tau}c_1 + (A_c + iB_c)|c_1|^2 c_1 + D_{cs}\Delta s_1 + D_{cc}\Delta c_1, \tag{A6}$$

where the coefficients have the form:

$$A_s + iB_s = b\left\{-\frac{1+7\xi}{3\xi(\alpha+\xi)} + i\frac{2\xi+\alpha}{3[\alpha\xi(\alpha+\xi)^3]^{1/2}}\right\}, \tag{A7}$$

$$A_c + iB_c = b\left\{\frac{\alpha+(2-6\alpha)\xi-3\xi^2}{3\xi(\alpha+\xi)} + i\frac{\alpha^2+\alpha(1-4\alpha)\xi-(1+4\alpha)\xi^2}{3[\alpha\xi(\alpha+\xi)^3]^{1/2}}\right\}, \tag{A8}$$

$$D_{ss} = -\frac{\xi}{2\alpha+\xi}\frac{i\omega_0\varepsilon_0}{\omega_0^2-4\Omega^2}\left\{i\left(\frac{\bar{\Gamma}}{1+\omega_0^2\tau_\varepsilon^2}-1\right)+\frac{\bar{\Gamma}\tau_\varepsilon\omega_0}{1+\omega_0^2\tau_\varepsilon^2}\right\}, \tag{A9}$$

$$D_{sc} = 0, \tag{A10}$$

$$D_{cs} = -\frac{\alpha}{2\alpha+\xi}\frac{i\omega_0\varepsilon_0}{\omega_0^2-4\Omega^2}\left\{i\left(\frac{\bar{\Gamma}}{1+\omega_0^2\tau_\varepsilon^2}-1\right)+\frac{\bar{\Gamma}\tau_\varepsilon\omega_0}{1+\omega_0^2\tau_\varepsilon^2}\right\}$$
$$+\frac{\xi\varepsilon_0}{(\omega_0^2-4\Omega^2)\tau}\left\{\left(1-\frac{\bar{\Gamma}}{1+\omega_0^2\tau_\varepsilon^2}\right)+i\frac{\bar{\Gamma}\tau_\varepsilon\omega_0}{1+\omega_0^2\tau_\varepsilon^2}\right\}+\frac{\alpha D}{2\alpha+\xi}, \quad \text{(A11)}$$

$$D_{cc} = D. \quad \text{(A12)}$$

Using the relation between s_1 and c_1,

$$s_1 = -i\left[\frac{\xi}{\alpha(\alpha+\xi)}\right]^{1/2} c_1 + D(\delta^2), \quad \text{(A13)}$$

one may express s_1 from Eq. (A5) with the δ^3 accuracy and substitute it into Eq. (A6). The result is Eq. (28).

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