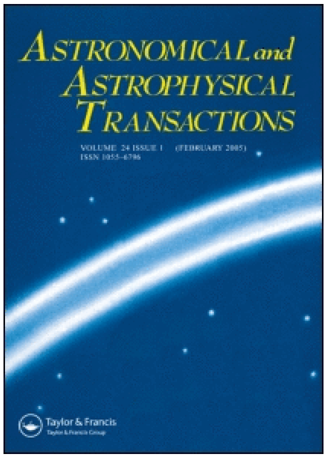

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NEUTRINO SYNCHROTRON EMISSION FROM THE NEUTRON STAR CRUST

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The neutrino energy loss rate Q is calculated and fitted by a simple expression for the synchrotron emission of $\nu\bar{\nu}$-pairs by electrons in a degenerate relativistic electron gas with magnetic field B (when the electrons populate many Landau levels). In a wide parameter range, Q is independent of density ρ: $Q \approx 9 \times 10^{14} B_{13}^2 T_9^5\,\mathrm{erg\,cm^{-3}\,s^{-1}}$. In the neutron star cores at $\rho = 10^7$–$10^{14}\,\mathrm{g/cm^3}$, $B = 10^{12}$–$10^{14}\,\mathrm{G}$, and at $T = 10^8$–$10^{10}\,\mathrm{K}$, the synchrotron losses are comparable to or exceed other neutrino energy losses.

KEY WORDS Neutron stars, neutrino, magnetic fields.

1. INTRODUCTION

It is well known that neutrino emission provides an effective mechanism of energy loss in neutron stars. Neutrinos are generated by many mechanisms that have been studied in detail for neutron star matter without magnetic field (see Soyer and Brown 1979; Itoh *et al.* 1989, and references therein). Huge magnetic fields, which exist in the neutron stars, can greatly modify these mechanisms. Moreover, the magnetic field opens another mechanism, the neutrino synchrotron radiation by electrons,

$$e^- \to e^- + \nu + \bar{\nu}, \tag{1}$$

which is forbidden in the field-free case.

The aim of this paper is to consider the neutrino synchrotron emission (1) in a degenerate relativistic electron gas of a neutron star crust under the typical conditions when the electrons populate many Landau levels. A detailed version of this paper has been published elsewhere (Kaminker *et al.* 1991, hereafter KLY). A fully correct general expression for the neutrino energy loss rate in the process (1) for a gas of electrons of any degeneracy in a magnetic field of arbitrary strength was derived by Kaminker *et al.* (1992). KLY applied this expression to relativistic degenerate electrons. A list and critical analysis of earlier works can be found in Kaminker *et al.* (1992) and KLY.

2. GENERAL EQUATIONS AND PARAMETER DOMAINS

According to Kaminker *et al.* (1992), the neutrino synchrotron energy loss rate Q [$\mathrm{erg\,cm^{-3}\,s^{-1}}$] can be written as

$$Q = \frac{Q_c b}{3(2\pi)^6} \sum_{n,n'=0}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{d}p_z \int \mathrm{d}\vec{q}\, A\, \omega f (1 - f'). \tag{2}$$

where

$$Q_c = \frac{G^2}{\hbar}\left(\frac{mc}{\hbar}\right)^9 \approx 1.02 \times 10^{23}\,\frac{\text{erg}}{\text{cm}^3\,\text{s}}, \qquad b = \frac{\hbar e B}{m^2 c^3} \approx \frac{B_{13}}{4.41},$$

G is the Fermi weak-coupling constant, m the electron mass, B the magnetic field ($B_{13} = B/(10^{13}\,\text{G})$); Q_c may be called the electron Compton neutrino energy loss rate unit. All quantities except Q_c on the right-hand side of (2) are assumed to be dimensionless ($\hbar = m = c = k = 1$). Equation (2) describes neutrino synchrotron emission in terms of electron transitions from an initial electron state with energy $\varepsilon = (1 + p_z^2 + 2bn)^{1/2}$ (p_z is the electron momentum along $\vec{B}$, and $n = 0, 1, 2, \ldots$ enumerates the electron Landau levels) to a final state with energy $\varepsilon' = (1 + p_z'^2 + 2bn')^{1/2}$. $\omega = \varepsilon - \varepsilon'$ and $\vec{q}$ denote, respectively, the energy and momentum carried away by a $\nu\bar{\nu}$ pair. $f = f(\varepsilon)$ and $f'(\varepsilon') = f(\varepsilon - \omega)$ are the Fermi–Dirac distributions of the initial and final electrons, respectively. A is proportional to the spin trace of the squared matrix element of the process. A is a complicated function of n, p_z, n', p_z', $\vec{q}$ (Kaminker *et al.* 1992). The integration region over $\vec{q}$ is restricted by the inequalities $\omega > 0$, $\omega^2 \geq \vec{q}^{\,2}$. One can show that $n' < n$ in Eq. (2).

Equation (2) has been derived from a quantum mechanical probability relevant to a 4-tail diagram of the process (1) in the framework of the Weinberg-Salam theory with exact wave functions of relativistic electrons in magnetic field (the Landau gauge; see, e.g., Kaminker and Yakovlev, 1981). The 4-tail approach is valid at $q \lesssim M$, where $M \sim 100$ GeV is the mass of the intermediate boson. This is an excellent approximation for the conditions under study.

Consider a relativistic degenerate electron gas in the most important case when the electrons populate many Landau levels. Then the electron Fermi momentum p_F is almost unaffected by magnetic field, so that $x \equiv p_F/mc \approx 1.01(\rho_6/\mu_e)^{1/3}$, where ρ is the density in units of $10^6\,\text{g/cm}^3$, and μ_e is the number of baryons per electron. The degeneracy temperature at $x \gg 1$ is $T_F \approx 5.93 \times 10^9 x\text{K}$ (Figure 1). Our consideration is restricted by the conditions $x \gg 1$, $T \ll T_F$, $x^2 \gg b$. The latter inequality is required to populate many Landau levels.

The main feature of neutrino synchrotron emission under the aforementioned conditions is a small energy and momentum transfer of the electrons: $\omega \lesssim T \ll \varepsilon$, $q_z^2 \ll p_z^2$, $q_\perp^2 \ll p_\perp^2 \equiv 2nb$ ($q_\perp$ is the momentum transfer across $\vec{B}$). The electrons lie within the thermal width of the Fermi surface ($\varepsilon \approx \varepsilon' \approx \sqrt{1 + p_F^2}$) both before and after the neutrino emission. These features result mostly from a strong degeneracy and allowed KLY to simplify Eq. (2) in a general form.

Now let us introduce two other temperatures (Figure 1):

$$T_B \approx \frac{mc^2 b}{xk} \approx \frac{1.35 \times 10^9}{x} B_{13}\,\text{K}, \qquad T_p = \frac{3mc^2 b x^2}{2k} \approx 2.02 \times 10^9 x^2 B_{13}\,\text{K}. \tag{3}$$

When $T \gtrsim T_B$, the separation of the Landau levels is smaller than the thermal width of the Fermi-Dirac distribution, and the magnetic field is nonquantizing. At $T \lesssim T_B$, the field becomes quantizing although many Landau levels are populated as long as $x^2 \gg b$. At $T \gtrsim T_p$, the Pauli exclusion principle does not restrict the synchrotron emission, while at $T \lesssim T_B$ this restriction is essential (see Sec. 3 for details). Accordingly, the $\rho - T$ range of study can be divided into three domains (Figure 1), where neutrino synchrotron emission has distinct properties. Domain

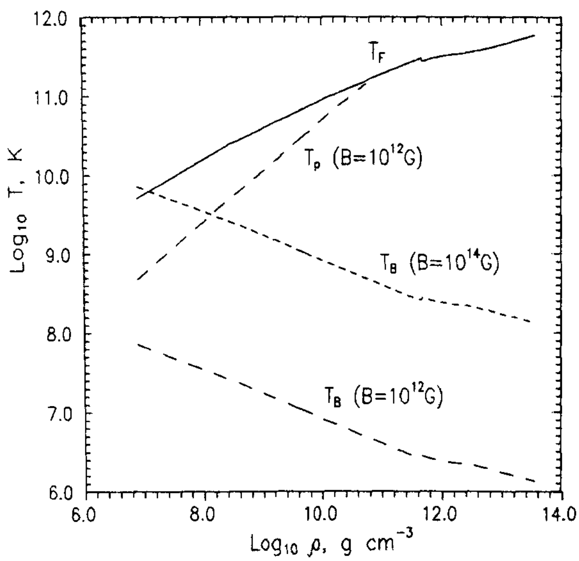

Figure 1 The domains of T and ρ where the neutrino synchrotron energy losses are of different character. The degeneracy temperature T_F does not depend on B in the nonquantizing magnetic field. T_B (given by (3)) is plotted for $B = 10^{12}$ G (long dashes) and 10^{14} G (short dashes), while T_p is given for $B = 10^{12}$ G only (it becomes larger than T_F for $B = 10^{14}$ G).

I corresponds to $T_p \ll T \ll T_F$; Domain II, to $T_B \ll T \ll T_p$; and Domain III, to $T \ll T_B$. For huge magnetic fields, $b \gg 1$ (and $T \ll T_F$), Domain I is absent. Consider neutrino synchrotron emission for these domains separately.

3. SYNCHROTRON RADIATION IN A NONQUANTIZING MAGNETIC FIELD (DOMAINS I AND II)

The synchrotron energy losses in Domains I and II were analyzed (KLY) using the quasiclassical approximation. It is convenient to introduce $s = n - n'$, which quantity can be called the number of cyclotron harmonics, in analogy with the electromagnetic synchrotron radiation. One can also define a classical electron momentum $p = (p_z^2 + p_\perp^2)^{1/2}$, and the pitch angle ϑ ($p_z = p\cos\vartheta$, $p_\perp = p\sin\vartheta$). In a nonquantizing magnetic field, relativistic degenerate electrons emit a quasi-continuum spectrum of cyclotron harmonics, $1 \ll s \ll n$. These features allowed KLY to reduce Eq. (2) to the expression valid equally in Domains I and II:

$$Q = \frac{4}{9}\left(\frac{3}{4\pi}\right)^7 Q_c b^6 y x^8 [C_+^2 F_+(\xi) - C_-^2 F_-(\xi)]$$
$$\approx 4.55 \times 10^{13} x^8 T_9 B_{13}^6 [C_+^2 F_+(\xi) - C_-^2 F_-(\xi)]\,\mathrm{erg\,cm^{-3}\,s^{-1}}. \quad (4)$$

Here $t = kT/(mc^2) = T_9/5.93$, $\xi = T_p/T$; $C_+^2 = C_V^2 + C_A^2 + N(C_V'^2 + C_A'^2) \approx 1.68$, $C_-^2 = C_V^2 - C_A^2 + N(C_V'^2 - C_A'^2) \approx 0.18$. $C_V = 2\sin^2\vartheta_W + 0.5$ and $C_A = 0.5$ are, respectively, the vector and axial-vector constants for emitting electron-type neutrinos; $C'_{V,A} = C_{V,A} - 1$ correspond to emitting muonic and taonic neutrinos ($N = 2$ is the number of neutrinos of non-electron type), and ϑ_W is the Weinberg angle ($\sin^2 \vartheta_W \approx 0.23$). Furthermore,

$$F_\pm(\xi) = \int_0^\pi \sin^7 \vartheta \, \mathrm{d}\vartheta \int_0^\infty \mathrm{d}y y^6 Q_\pm(y) \frac{z}{e^z - 1}, \qquad z = \frac{\omega}{T} = \xi y \sin \vartheta.$$

$$Q_+(y) = \tfrac{8}{3}R_0 + 2R_1 + \tfrac{4}{3}R_2, \qquad Q_-(y) = 4R_2 + 2R_1; \qquad R_0 = \frac{2\pi\sqrt{3}}{9y^3} \int_y^\infty K_{1/3}(r)\, \mathrm{d}r,$$

$$R_1 = \frac{2\pi\sqrt{3}}{9y^2} K_{1/3}(y), \qquad R_2 = \tfrac{3}{8}R_1 + \tfrac{9}{16}y^2 R_0 - \frac{\pi\sqrt{3}}{8y} K_{2/3}(y), \tag{5}$$

$y \equiv 2s/(3p_\perp^3)$, and $K_\alpha(r)$ is Mcdonald's function. The $\nu\bar{\nu}$-pair energy emitted for a given s is $\omega \approx bp_F s/p_\perp^2$.

Maxima of $y^6 Q_\pm(y)$ correspond to $y \sim 1$, i.e., the maximum of the neutrino emission spectrum (without allowance for the Pauli principle restriction) takes place at high cyclotron harmonics, $s_* \sim p_\perp^3 \sim p_F^3$ (for which $\omega = \omega_* \sim bp_F^2$), just as in the case of electromagnetic synchrotron emission (e.g., Sokolov and Ternov, 1983). This leads to two regimes of neutrino synchrotron radiation realized in Domains I and II at small and large values of $\xi \sim \omega_*/T$.

In Domain I, one has $\xi \ll 1$, and the Pauli principle imposes no restriction on $\nu\bar{\nu}$-pair emission. Typical cyclotron harmonics are s_*, and typical $\nu\bar{\nu}$-pair energies are ω_*. The appropriate asymptotic form of (4) and (5) is (KLY)

$$Q = \frac{2Q_c}{189\pi^5} tb^6 x^8 (25C_+^2 - 21C_-^2) \approx 3.06 \times 10^{15} B_{13}^6 T_9 x^8 \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}. \tag{6}$$

According to (6), Q grows rapidly with B and ρ which is an indication that synchrotron $\nu\bar{\nu}$ pairs are emitted freely.

In Domain II, one has $\xi \gg 1$ and the Pauli principle forbids emission of $\nu\bar{\nu}$ pairs with $\omega \gg T$. Then typical harmonics are $s \sim Tx/b \ll s_*$, and typical energies $\omega \sim T$. In this regime we have (KLY)

$$Q = \frac{2\zeta(5)}{9\pi^5} Q_c b^2 t^5 C_+^2 \approx 8.97 \times 10^{14} B_{13}^2 T_9^5 \,\mathrm{erg\,cm^{-3}\,s^{-1}}, \tag{7}$$

where $\xi(5) \approx 1.04$ is the Riemann zeta function. A remarkable feature is that, according to (7), Q depends neither on p_F (or density ρ) nor on the electron mass m. Accordingly, any charged relativistic degenerate fermions, for which $\xi \gg 1$, will produce the same neutrino synchrotron energy losses.

In addition to the asymptotics (6) and (7), we have computed $F_\pm(\xi)$ from Eqs. (5) at any ξ. With an error $\lesssim 1.5\%$ for any ξ, the results can be fitted by simple formulas:

$$F_+ = 44.0 \frac{(1 + c_1 y_1)^2}{(1 + a_1 y + b_1 y^2)^4}, \qquad F_- = 37.0 \frac{1 + c_2 y_2 + d_2 y_2^2 + e_2 y_2^3}{(1 + a_2 y + b_2 y^2)^5}, \tag{8}$$

where

$$y_1 = [(1 + \alpha_1 \xi^{2/3})^{2/3} - 1]^{3/2}, \qquad y_2 = [(1 + \alpha_1 \xi^{2/3})^{2/3} - 1]^{3/2};$$

$\alpha_1 = 3172$, $\alpha_2 = 172.2$; $a_1 = 2.04E(-4)$, $b_1 = 7.41E(-8)$, $c_1 = 3.68E(-4)$; $a_2 = 3.36E(-3)$, $b_2 = 1.54E(-5)$, $c_2 = 1.44E(-4)$, $d_2 = 1.02E(-5)$, $e_2 = 7.67E(-8)$.

4. SYNCHROTRON EMISSION IN A QUANTIZING MAGNETIC FIELD (DOMAIN III)

In Domain III ($T \ll T_B$), the distance between the Landau levels becomes larger than the thermal energy T. According to KLY, in this case the neutrino synchrotron energy loss is suppressed by an exponentially small factor $\exp(-\omega/T)$. The major contribution into Q comes from those electron transitions which correspond to the lowest ω. These transitions are appropriate to the lowest harmonics $s=1$ ($n \to n' = n-1$) and some specified momentum transfer $\vec{q}$ for which $\omega = b/(2p_F)$ due to the quantum recoil effect. In addition, when density increases, degenerate electrons populate new Landau levels and Q suffers quantum oscillations. Such oscillations are typical of many thermodynamic and kinetic properties of matter (e.g., Yakovlev 1984). KLY made use of Eq. (2) and derived the asymptotics of Q (averaged over quantum oscillations) at $T \ll T_B$. Their expression differs from Eq. (7) by a factor of $0.362 \exp[-T_B/(2T)]$. Thus to extend Eqs. (4) and (7) to Domain III for practical use, one may introduce in Eq. (4) an additional factor $U = \exp[-T_B/(2T)]\{0.362 + 0.638 \exp[-T_B/(2T)]\}$.

5. DISCUSSION AND CONCLUSIONS

Figure 2 shows the synchrotron energy loss rate versus density ρ for $T = 10^9$ K and $B = 10^{12}$, 10^{13}, 10^{14} G. For illustration, we also present the field-free energy loss rates for other neutrino production mechanisms (which have not been

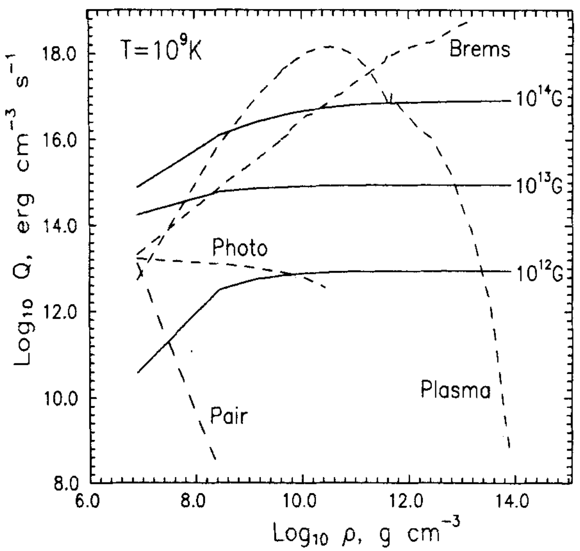

Figure 2 Neutrino energy losses versus ρ at $T = 10^9$ K. Solid lines show synchrotron losses at $B = 10^{12}$, 10^{13}, and 10^{14} G. Dashed lines correspond to the plasmon decay, pair annihilation, bremsstrahlung and photon decay at $B = 0$. The photon decay curve is shown at $\rho \lesssim 3 \times 10^{10}$ g/cm^3 in accordance with the data of Itoh *et al.* (1989).

considered at high magnetic fields under the above conditions). These mechanisms are: the electron bremsstrahlung on nuclei ($e^- + Z \rightarrow e^- + Z + \nu + \bar{\nu}$) (Soyer and Brown 1979), the photon decay ($e^- + \gamma \rightarrow e^- + \nu + \bar{\nu}$), the e^-e^+-pair annihilation ($e^- + e^+ \rightarrow \nu + \bar{\nu}$), and the plasmon decay ($\hbar\omega_p \rightarrow \nu + \bar{\nu}$) (Itoh *et al.* 1989). The density range displayed is appropriate to a neutron star crust. For $\rho \lesssim 4 \times 10^{11}$ g/cm^3, the composition of matter has been taken from Haensel and Zdunik (1990), and for higher ρ, from Negele and Vautherin (1973). As can be seen from Figure 2, neutrino synchrotron emission for $B \sim 10^{12}$–10^{14} G is quite comparable with other neutrino production mechanisms, especially at moderate ρ.

Neutrino energy losses in the neutron star crust are especially important during initial 10–100 years after the neutron star birth. In this period, thermal relaxation between stellar crust and core still has not been achieved, and the crust cools independently of the core. The fall of the surface temperature (the cooling curve) is determined by the cooling properties of the crust and, first of all, by the neutrino energy loss rates in different crust layers (see, e.g., Nomoto and Tsuruta 1987). Under these conditions, the synchrotron losses can be important. The most important seems to be the widest $\rho - T$ domain II, where Q is density independent (see (7)). Since synchrotron losses depend strongly on B, their distribution cannot be spherically symmetric within the crust. This can produce an asymmetric temperature distribution that may cause an intense mixing of matter.

Our results are valid also in neutron star cores, provided the core matter is non-superconductive. However the synchrotron energy losses in the core usually are much smaller than the neutrino energy losses due to the URCA process or the bremsstrahlung in nucleon-nucleon collisions.

Note that KLY formulated similarity rules which allow one to obtain simple order-of-magnitude estimates of the neutrino synchrotron energy losses from electromagnetic synchrotron energy losses.

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