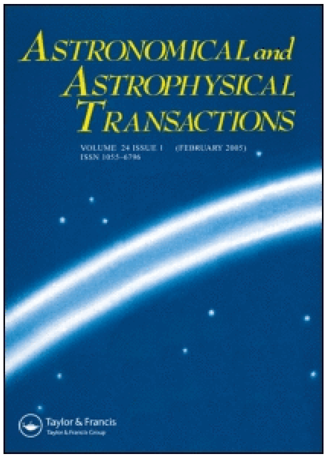


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# MAGNETOROTATIONAL EVOLUTION OF ISOLATED NEUTRON STARS

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We consider a decay of the magnetic field, originally confined to a moderately deep layer of the neutron star crust with the density $\rho \lesssim 4 \cdot 10^{11}$ g/cm$^3$. The effect of neutron star cooling on the conductivity and the rate of the magnetic field dissipation is taken into account. The main goal of this paper is the investigation of the influence of the field decay on the neutron star spinning down in the magnetic dipole braking model. Evolutionary tracks of pulsars on the $B_s - P$ diagram, where $B_s$ is the magnetic field at the surface and $P$ is the spin period, have been calculated. It is shown that the crustal decay model is in a qualitative agreement with observational data if a significant fraction of the outer crust was originally occupied by the magnetic field.

KEY WORDS Neutron stars, magnetic field decay.

## 1. INTRODUCTION

Neutron stars are among the most strongly magnetized objects known in nature. Their magnetic fields span a wide range of values from $\sim 10^8$ to $\sim 10^{13}$ G (see, e.g., Chanmugam 1992). These fields can play a fundamental role in a wide variety of phenomena both inside neutron stars and in their vicinities. The origin and evolution of these huge magnetic fields has been a subject of considerable interest and debate ever since the discovery of pulsars. The discovery of millisecond pulsars in the last few years has generated a new wave of interest in this subject, in view of the role the magnetic fields play in the spin evolution of neutron stars, especially in a binary system. As far as the field evolution is concerned, it is determined by the location and configuration of the field intimately tied to its origin.

Regarding the origin of neutron star magnetic fields there are two different points of view: one maintains that these strong fields are inherited from their progenitors and amplified during the collapse to the neutron state, and the other contends that these fields are generated after the neutron star was born. The magnetic flux conservation during the collapse may lead to a strong field of the order of $\sim 10^{13}$ G in a neutron star. In this case the magnetic field probably occupies the whole star including the central regions with very high conductivity. It was pointed out by Baym *et al.* (1969) that the magnetic flux passing through the interior will not decay significantly during the neutron star lifetime. Recently, Haensel *et al.* (1991), considering the field evolution in a neutron star with a magnetized core, cast doubts on this conclusion. In more detail the evolution of the magnetic field which occupies the neutron star core with normal npe-matter has been discussed by Urpin and Shalybkov (1992).

However, one cannot exclude the possibility that electric currents and magnetic fields are mainly concentrated in the neutron star surface layers. For instance, magnetic fields can be generated by the thermomagnetic instability in the crust after the star was born (Blandford *et al.*, 1983, Urpin *et al.*, 1986). This instability transforms some fraction of the energy carried by the thermal flux into the energy of magnetic fields. In neutron stars, the thermomagnetic instability may be effective at the early stage of evolution when the surface temperature $T_e$ is greater than $\approx 3 \cdot 10^6$ K. The instability develops within a layer of the depth ~100–150 m from the surface, which corresponds to the density $\rho \sim 10^8$ g/cm$^3$. Probably, the hydrodynamic motions induced by nonlinear effects may drag the field deeper in the crust. It is also possible that some other mechanisms can provide the field generation in the crust. The present paper analyzes the evolution of the magnetic field originally confined to moderately deep layers in the crust. We will not specify the mechanism that results in a field generation at the very early stage of evolution. The available constraints from the observed properties of a few young pulsars suggest that most of the field generation should occur within a few hundred years after the neutron star was born, in which case it becomes observationally almost indistinguishable from the fields being present since their birth.

The crustal magnetic field evolution has been a subject of study in recent years. The calculations of Ohmic decay of the dipolar magnetic field have been first performed by Sang and Chanmugam (1987) who demonstrated that the field does not decay exponentially. They also emphasized that the field diffusion into the interior can strongly slow down the decay due to a conductivity increase in the deep layers. Later, Sang and Chanmugam (1990) (see also Chanmugam 1992) took into account the neutron star cooling neglected in their previous paper. Cooling can also strongly slow down the decay since the conductivity depends on temperature in the crust. However they restricted themselves to a single model in which the field initially occupies the outer layers with the density $\rho \lesssim 4 \cdot 10^{11}$ g/cm$^3$. These computations confirmed a nonexponential decay of the crustal field.

A simple model of the crustal field evolution has been proposed by Romani (1990), who suggested that the magnetic field is generated soon after the neutron star birth by thermomagnetic effects. After a short generation phase the field undergoes a slow Ohmic dissipation. In this model, isolated neutron stars can maintain large magnetic fields for more than $10^{10}$ yrs.

The decay of the field initially confined to the surface layers of the crust has also been considered by Urpin (1992). Using a simplified model, it was argued that the neutron star cooling may be responsible for the existence of two different stages of decay. At the initial stage, when the conductivity $\sigma$ in the crust is determined by the electron–phonon scattering, the field decreases comparatively rapidly (with the characteristic time scale ~$10^6$–$10^7$ yrs depending on the depth penetrated by the initial field). Later, the conductivity increases due to the cooling and the decay slows down. At this stage, $\sigma$ is mainly determined by the scattering on impurities and the decay time may be $\gtrsim 10^9$ yrs. Detailed numerical computations by Urpin and Muslimov (1992) have confirmed the hypothesis of a two-stage decay. These authors considered the decay of the field for the neutron star model with normal npe-matter in the core. The main results of this paper will be briefly discussed in Sec. 3.

The evolution of the field in the neutron star crust was also analyzed by Jones (1988), who pointed out that, under certain conditions, the Hall drift can accelerate the decay. This effect was studied in detail by Urpin and Shalybkov (1991) who consider the dissipation of electric currents in a strongly magnetized plasma and obtain that the Hall drift can strongly enhance the field dissipation.

In recent years, a wide variety of observational data concerning the pulsar field evolution has also been obtained. For instance, a statistical analysis of pulsars (see, e.g., Lyne *et al.*, 1985) shows that the magnetic fields decay approximately exponentially on the time scale $\tau_B$ of the order of $(5\text{–}9) \cdot 10^6$ yrs. Pulsar proper motion measurements (see Lyne *et al.*, 1982) also support the field decay hypothesis, with $\tau_B \sim 9 \cdot 10^6$ yrs. A detailed statistical analysis performed by Narayan and Ostriker (1991) for 301 pulsars led the authors to a conclusion that magnetic fields decay on the time scale of about $10^7$ yrs, but the form of the decay (exponential versus a power law) cannot be determined from available observational data. However, it should be noted that previous statistical analyses have been a subject of criticism by Curtis Michel (1990), who believes that the statistical quality of the data seems to be insufficient to show confidently fine details of pulsar evolution. Analyzing the binary millisecond pulsars, Kulkarni (1986) and van den Heuvel *et al.* (1986) argued that the pulsar magnetic fields do not decay indefinitely to zero. According to the scenario suggested by van den Heuvel *et al.* (1986), the fields of such objects decrease only at an initial evolutionary stage, and the decay practically stops when the field strength reaches about $10^{-2}\text{–}10^{-3}$ of its original value. Qualitatively similar conclusions have also been obtained by Srinivasan (1989). Recent optical and radio observations of the millisecond pulsar PSR 1855 + 09, reported by Kulkarni, Djorgovski and Klemola (1991), enable one to estimate the characteristic age of this object at $5 \cdot 10^9$ yrs. This estimate supports the hypothesis that magnetic field strengths of millisecond pulsars are essentially constant and that millisecond pulsars are long-lived objects. Analyzing hard X-ray spectra of $\gamma$-ray bursts, van Paradijs (1989) argued that properties of these sources are consistent with the assumption that the magnetic field decays on a time scale $\sim 10^7$ yrs.

On the contrary, some papers cast doubts on the arguments regarding the field decay in isolated neutron stars (see, e.g., Chanmugam and Brecher 1987, Bhattacharya 1991). For instance, Bhattacharya (1991) concludes that the field decay can occur only when the neutron star has evolved into an interacting binary. Probably, the nonuniformity of observational data sets reflects various magnetic evolutionary histories of different types of objects.

The present paper considers the decay of the magnetic field originally confined to the outer crust of a neutron star. Computations have been performed for various values of the depth penetrated by the initial field. The effect of neutron star cooling is taken into account. The influence of the field decay on the spinning down is examined for the magnetic dipole braking model. Pulsar evolutionary tracks have been calculated on the $B_s - P$ diagram where $B_s$ is the magnetic field at the surface and $P$ is the spin period. The results of calculations have been compared with the observational data.

## 2. BASIC EQUATIONS

In the present work we study the Ohmic dissipation of electric currents originally concentrated within the outer crust which consists mainly of free degenerate electrons and fully ionized ions having the atomic number $A$ and charge number $Z$ (for simplicity we assume that one species of ions is predominant and all other ones can be considered as impurities). The thickness of the layer with non-degenerate electrons does not exceed few meters, and the influence of this layer on the field decay can be neglected. If hydrodynamic motions are negligible and the anisotropy of the conductivity is small, the induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c^2}{4\pi}\nabla \times \left(\frac{1}{\sigma}\nabla \times \mathbf{B}\right), \tag{1}$$

where $\sigma$ is the electrical conductivity. Except during a very early stage of neutron star evolution, the outer crust is mainly solidified. Therefore, the neglect of hydrodynamic motions seems to be a reasonable assumption. The anisotropy of conductivity is characterized by the parameter $\Omega_B \tau_e$, where $\tau_e$ is the relaxation time of electrons and $\Omega_B = eBc/\varepsilon_F$ is the gyrofrequency, with $\varepsilon_F$ being the Fermi energy. The anisotropy is important if $\Omega_B \tau_e > 1$. Near the surface, the magnetic fields typical of neutron stars can strongly magnetize electrons. However, in deeper layers with density $\rho \gtrsim 10^{10}\,\text{g/cm}^3$, the electron gas may be non-magnetized practically during the whole lifetime of a star. In the present paper, we deal with magnetic configurations for which currents are concentrated at $\rho > 10^{10}\,\text{g/cm}^3$ and the effect of magnetization is negligible.

We restrict ourselves to the dipolar field decay. Let us untroduce vector potential $\mathbf{A} = (0, 0, A_\varphi)$ and suppose $A_\varphi = s(r, t)\sin\theta/r$, where $r$ and $\theta$ are the spherical radius and polar angle, respectively. Then $s(r, t)$ obeys the following equation:

$$\frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2} = \frac{4\pi\sigma}{c^2}\frac{\partial s}{\partial t}. \tag{2}$$

At the stellar surface $r = R$ the following boundary condition should be fulfilled:

$$r\frac{\partial s}{\partial r} + s = 0. \tag{3}$$

Since we study the decay of the surface field, the function $s(r, t)$ should evidently vanish in deep layers.

To solve Eq. (2) one should know the dependence of $\sigma$ on the depth measured from the surface, $H$. If the chemical composition of the envelope is fixed, this dependence is determined by the surface gravity $g = GMR^{-2}(1 - R_g/R)^{-1/2}$ alone ($M$ being the neutron star mass and $R_g = 2GM/c^2$ the gravitational radius). Simple analytical formulae describing the growth of $\rho$ with depth $H$ were derived by Urpin and Yakovlev (1979) as

$$\begin{gathered} x = \sqrt{z(z+2)}, \qquad x = (\rho/10^6\mu_e\,\text{g}\cdot\text{cm}^{-3})^{1/2}, \qquad z = H/H_R, \\ H_R = \frac{mc^2}{g\mu_e m_p} = 17.5\left(\frac{2}{\mu_e}\right)\cdot\frac{R_6^2}{M_1}\left(1 - 0.3\frac{M_1}{R_6}\right)\ \text{m}, \end{gathered} \tag{4}$$

where $m$ and $m_p$ are the electron and proton masses, respectively; $\mu_e = A/Z$, $R_6 = R/10^6\,\text{cm}$, and $M_1 = M/M$.

In the outer layers we have $T > T_m$, where $T_m$ is the melting temperature, and ions are in a liquid or gaseous phase. In deeper layers, where $T_m > T$, ions are crystallized. The melting temperature $T_m$ is (see, e.g., Slattery *et al.*, 1980)

$$T_m = 3.04 \cdot 10^7 x Z_{26}^{-5/3}\left(\frac{170}{\Gamma_m}\right), \tag{5}$$

where $Z_{26} = Z/26$; $\Gamma_m \approx 170$ is the melting value of the ion coupling parameter, $\Gamma = Z^2e^2/k_BTa$, $a = (3/4\pi n_i)^{1/3}$ is the mean inter-ion distance; $n_i$ is the ion number density and $k_B$ is the Boltzmann constant. Using Eqs. (4) and (5), one can easily obtain that crystallization of matter at the temperature $T$ occurs at the depth $z_m$,

$$z_m = \sqrt{1 \times x_m^2} - 1, \qquad x_m = 0.329 Z_{26}^{-5/3}\left(\frac{T}{10^7\,\mathrm{K}}\right)\left(\frac{\Gamma_m}{170}\right). \tag{6}$$

In the melted layer near the surface, where ions form a Coulomb liquid, electrons scatter mainly off ions and the conductivity is given by

$$\sigma_{\mathrm{ei}} = \frac{8.53 \cdot 10^{21} x^3}{Z\Lambda_{\mathrm{ei}}(1+x^2)}\ \mathrm{s}^{-1}, \tag{7}$$

where $\Lambda_{\mathrm{ei}}$ is the Coulomb logarithm. A simple interpolation formula for $\Lambda_{\mathrm{ei}}$ was proposed by Yakovlev and Urpin (1980),

$$\Lambda_{\mathrm{ei}} = \ln\left[\left(\frac{2\pi Z}{3}\right)^{1/3}\left(1.5 + \frac{3}{\Gamma}\right)^{1/2}\right] + \frac{x^2}{2(1+x^2)}. \tag{8}$$

It should be noted that in the liquid phase $\delta_{\mathrm{ei}}$ is practically independent of $T$.

In the solid crust, at $z > z_m$, one of the most important scattering mechanisms of electrons is the scattering off phonons. With an exception of a very low temperature case, the Umklapp processes play a major role under astrophysical conditions. These processes are rather unusual for metals under laboratory conditions since they are mainly due to the absence of the longitudinal acoustic mode in the phonon spectrum of astrophysical crystal. The conductivity $\sigma_{\mathrm{ph}}$ for such scattering mechanism was derived by Yakovlev and Urpin (1980),

$$\sigma_{\mathrm{ph}} = \frac{1.57 \cdot 10^{23} x^4}{T_6(1+x^2)} \cdot \frac{1}{F}\quad \mathrm{s}^{-1}, \qquad F = \frac{2+x^2}{1+x^2} \cdot \frac{13u}{\sqrt{0.017+u^2}}, \tag{9}$$

where $T_6 = T/10^6$ K, $u = 0.45T/T_D$, with $T_D$ being the Debye temperature,

$$T_D = 0.45\frac{\hbar\omega_p}{k_B} = 2.4 \cdot 10^6\left(\frac{2}{\mu_e}\right)^{1/2} x^{3/2}\,\mathrm{K}, \qquad \omega_p = \left(\frac{4\pi Z^2 e^2 n_i}{Am_p}\right)^{1/2}, \tag{10}$$

where $\omega_p$ is the ion plasma frequency. The conductivity is proportional to $T^{-1}$ at $T > T_D$ and to $T^{-2}$ at $T_D > T$.

The Umklapp processes in astrophysical crystals become less important at low $T$. On the contrary, the so-called normal processes accompanied by the small-angle scattering of electrons play a dominant role just as under laboratory conditions. The normal processes contribute mainly to the electron scattering at $T_u > T$ (see, e.g., Raikh and Yakovlev 1982), where

$$T_u = 2.22\frac{Z^{1/3}e^2}{hv_F} = 3.9 \cdot 10^4 Z^{1/3} x^{1/2}\sqrt{1+x^2}\left(\frac{2}{\mu_e}\right)^{1/2}\mathrm{K}, \tag{11}$$

and $v_f$ is the electron Fermi velocity. In this case the conductivity $\sigma_u$ is given by

$$\sigma_u = \frac{1.12 \cdot 10^{26} x^{11}}{T_6^5 (1+x^2)^{3/2}} \left(\frac{2}{\mu_e}\right) \quad \mathrm{s}^{-1}. \tag{12}$$

At $T_u > T$, the neutron star cooling leads to a strong increase of $\sigma_u$ which is proportional to $T^{-5}$.

However, astrophysical crystals are unlikely to be perfect. Perhaps, there is a lot of structural defects in such crystals: impurities, dislocations, cracks and so on. At low temperatures, the electron scattering by defects can be more important than that by phonons. Most likely, the scattering by impurities dominates at low $T$ since the number density of impurities can be very large in astrophysical crystals. The conductivity for such scattering is (see Yakovlev and Urpin 1980)

$$\sigma_{\mathrm{imp}} = \frac{8.53 \cdot 10^{21} x^3}{\Lambda_{\mathrm{imp}}(1+x^2)} \cdot \frac{Z}{Q} \quad \mathrm{s}^{-1}. \tag{13}$$

Here $\Lambda_{\mathrm{imp}}$ is the Coulomb logarithm (at $\rho \gtrsim 10^5\ \mathrm{g/cm^3}$ one has $\Lambda_{\mathrm{imp}} \approx 2$); $Q$ is the parameter which characterizes the number densities and charges of impurities,

$$Q = \frac{1}{b_i} \sum_{n'} n'(Z - Z')^2, \tag{14}$$

where $n'$ is the number density of the impurity having the charge $Z'$; in Eq. (14), summation is carried over all the species of impurities. It was mentioned by Urpin (1992) that the scattering on impurities is more significant than for normal processes for not very pure crystals with comparatively large values of $Q$ and, therefore, the regime (12) may be unimportant in many cases.

With the exception of the low-temperature case in which the scattering on impurities dominates, the conductivity of the crust depends on $T$, and, hence, on the neutron star age. In our computations of the field decay we use the cooling models by Van Riper (1990). The cooling rate of neutron stars is strongly dependent on the properties of matter in a neutron star core. That is why the crustal field evolution depends on the state of matter in a core where $\rho$ can be above the nuclear density, $\rho_n = 2.8\,10^{14}\ \mathrm{g/cm^3}$. The present paper considers the field decay for a neutron star model with a normal npe-matter in the core and without accounting for the magnetic field influence on the cooling (the model 1.3 M. BPS, N, B = 0 in notation of Van Riper 1990). In this model, a neutron star cools down most slowly and, therefore, the field decays most rapidly.

Solving Eq. (2) we assume that the temperature in the crust is uniform and equal to the central temperature $T_c(t)$. In fact, the thermal flux through the crust is non-zero and, hence, $\nabla T \neq 0$. However, excluding a very early evolution stage, the main temperature growth takes place in the surface layers of neutron stars where thermal conductivity is not high (see, e.g., Urpin and Yakovlev 1979, Gudmundsson *et al.*, 1983). For instance, at $\rho \gtrsim 10^9\ \mathrm{g/cm^3}$ large variations of $T$ occur only in very hot stars with the surface temperature $T_e > 4 \cdot 10^6$ K. Neutron stars can be so hot during an extremely short period of duration $\lesssim 10$ yrs after their birth. After this time one can suppose the layers with $\rho \gtrsim 10^9\ \mathrm{g/cm^3}$ to be isothermal, $T \approx T_c(t)$. Since the present paper considers magnetic configurations with the electric current concentrated at $\rho > 10^9\ \mathrm{g/cm^3}$, the assumption $T \approx T_c$ seems to be quite reasonable.

The computations of Van Riper (1990) provide us with the dependence $T_e(t)$, but according to our simplified treatment the conductivity in Eq. (2) depends on $T_c$. Therefore one needs the relationship between $T_c$ and $T_e$. We use the approximate formula suggested by Gudmundsson *et al.* (1983),

$$T_c = 1.288 \cdot 10^8 (T_{e6}^4/g_{14})^{0.455}\ \mathrm{K}, \tag{15}$$

where $T_{e6} = T_e/10^6$ K and $g_{14} = g/10^{14}$ cm/s$^2$. This formula is applicable in the temperature domain $10^{6.5}$ K $> T_e > 10^5$ K, but at lower $T_e$ its accuracy is not high. However, at low $T_e$ the specific form of the dependence $T_c(T_e)$ is not important for the field evolution since $\sigma$ in the crust is determined by impurities and does not depend on $T$.

Equations (1)–(15) with an appropriate initial condition determine the evolution of magnetic field in the neutron star crust. The dependences of $B_s(t)$ for various original magnetic configurations have been analyzed in detail by Urpin and Muslimov (1992). However, our knowledge of the magnetic fields of neutron stars come mainly from radio pulsars with known spindown rates. Therefore, in many cases it is more convenient to analyze the dependence of $B_s$ not on the age $t$ but on the spin period of the neutron star $P$, or on the "spindown" age $\tau = P/2\dot{P}$, where $\dot{P}$ is the spindown rate. With the assumption that the spindown torque on a pulsar is determined by the magnetic dipole radiation, the observed spin period and the spindown rate can be combined to yield a measure of the dipole field strength at the surface (Ostriker and Gunn 1969):

$$B_s = \left(\frac{3c^3 IP\dot{P}}{8\pi^2 R^6}\right)^{1/2}, \tag{16}$$

where $I$ is the moment of inertia of the star and $B_s$ is the magnetic field strength at the magnetic pole. We assume the magnetic and spin axes to be perpendicular. This equation can be rewritten in a simple dimensionless form:

$$P\dot{P} = \alpha B_{s12}^2, \qquad \alpha = 9.75 \cdot 10^{-16} R_6^6/I_{45}, \tag{17}$$

where $B_{s12} = b_s/10^{12}$ G and $I_{45} = I/10^{45}$ g $\cdot$ cm$^2$. The magnetic field decay can change the spindown rate during a neutron star life and, hence, alter a form of the evolutionary tracks of pulsars. Using the magnetic decay curves $B_s(t)$ the dependencies $p(t)$ or $r(t)$ can be easily calculated from Eq. (17). After that, eliminating the age from the couples of equations $\{B_s(t), P(t)\}$ or $\{B_s(t), \tau(t)\}$ one can obtain the pulsar evolutionary tracks in diagrams $B_s - P$ or $B_s - \tau$.

## 3. THE MAIN RESULTS

Equation (2) with the corresponding boundary conditions was solved numerically for different initial field distributions given by the functions $s(z, 0)$. The calculations have been performed by making use of the standard Crank-Nicholson difference scheme. Since $\sigma$ is discontinuous at the liquid-solid interface $z = z_m$, one should, strictly speaking, solve Eq. (2) for the liquid and solid regions separately, and then match the solutions at $z = z_m$ using standard conditions at the interface. To simplify computations we have assumed that the transition between the liquid and crystal phases is spread over a sufficiently thin transition layer where $\sigma$ varies smoothly. For these purposes the expression for $\sigma$ within the crust was taken in the form

$$\sigma^{-1} = \sigma_{ei}^{-1}\psi(z) + (\sigma_{ph}^{-1} + \sigma_{imp}^{-1})[1 - \psi(z)],$$
$$\psi(z) = \left[\exp\left(\frac{z - z_m}{\Delta z}\right) + 1\right]^{-1}, \tag{18}$$

where $\Delta z = \Delta H/H_R$ with $\Delta H$ being the thickness of the transition layer. At $z_m - z \gg \Delta z$ (liquid region) one has $\psi(z) = 1$ and $\sigma = \sigma_{ei}$; on the contrary, at $z - z_m \gg \Delta z$ (crystal crust) $\psi(z) = 0$ and $\sigma^{-1} = \delta_{ph}^{-1} + \delta_{imp}^{-1}$. In computations we supposed $\Delta H = H_R$. It should be noted that such simplification does not essentially influence the numerical results because the solid–liquid interface is localized in the layers of relatively low densities practically during the whole course of evolution. In the region of the interface, the currents are absent and the value of $\sigma$ there cannot influence the dissipation of currents concentrated in deeper layers.

The calculations have been performed for the model of a neutron star of $M = 1.4_{\odot}$ and $R = 1.06 \cdot 10^6$ cm ($g = 2.72 \cdot 10^{14}$ cm/s$^2$ and $H_R = 8.38$ cm) with the outer crust consisting of $^{56}Fe$ ions. The parameter $Q$ characterizing the impurity content in the crust has been assumed to be independent of the depth and time and varied within the interval 0.001–0.1.

We have considered the decay of the field which initially occupies the surface layers of the crust down to some depth, $z = z_0$. It should be remarked that at the early stage of evolution (at $T_e > 3 \cdot 10^6$ K) Eq. (15) is not fulfilled. However, the duration of this stage is so short that the specific form of the dependence $T_c(T_e)$ for $T_e > 3 \cdot 10^6$ K does not practically influence the field evolution. The computations have been performed for the following values of $z_0$: 30.33, 35.96 and 57.09. These values correspond to the densities $\rho_0 = 6 \cdot 10^{10}$, $10^{11}$ and $4 \cdot 10^{11}$ g/cm$^3$, respectively. The initial configuration of the field was taken the same for all values of $z_0$, $s(z, 0) = s(z/z_0)$.

Figure 1 shows the evolution of $s(z, t)$ within the crust for $z_0 = 57.09$ ($\rho_0 = 4 \cdot 10^{11}$ g/cm$^3$) and $Q = 0.01$. The original configuration is chosen to be practically constant near the surface at $z/z_0 \leqslant 0.4$. In the configuration under consideration, electric currents are mostly concentated at $1 \gtrsim z/z_0 > 0.6$, with the maximum current being reached at $z/z_0 \approx 1$. It should be noted that the main features of the field decay are qualitatively independent of the form of the original configuration while numerical results can differ for various choices of $s(z/z_0)$. During the evolution, the field not only decreases but also diffuses deeper into the crust. For instance, due to a diffusion, $s(z, t)$ reaches the value $\approx 0.01 s(0, 0)$ at the depth $2z_0$ after $\sim 10^9$ yrs. At the same time, the surface value of $s$ decreases by a factor of $\sim 50$. In deeper layers, at $z/z_0 > 2$, the value of $s$ is much smaller. At $t \approx 10^9$ yrs the conductivity of the layer with $z = 2z_0$ is determined by the impurity scattering and is twice greater than at $z = z_0$. Due to the higher conductivity of the deep layers and increasing field length scale, the decay slows down. Sang and Chanmugam (1987) were the first to emphasize the importance of this effect for the field decay in a neutron star crust. Diffusion into deeper layers can change qualitatively the field decay (for details, see Urpin 1992).

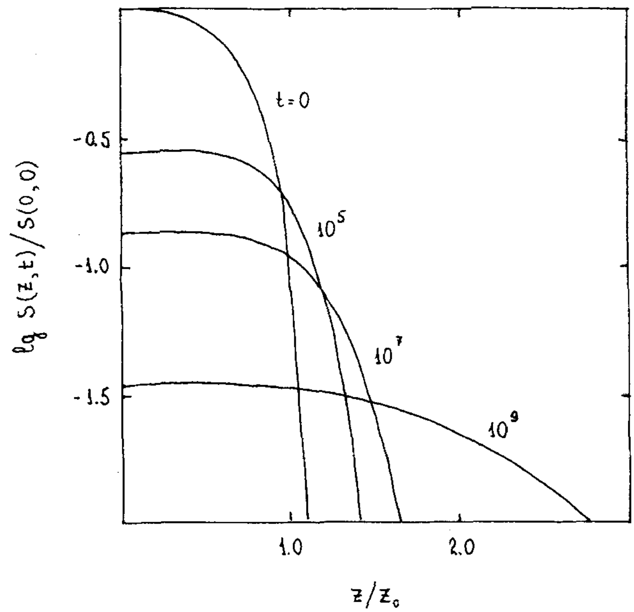


**Figure 1** The function $S(z, t)/s(0, 0)$ versus the dimensionless depth $z/z_0$. The numbers near the curves indicate $t$ in years; $\rho_0 = 4 \cdot 10^{11}\,\mathrm{g/cm^3}$, $z_0 = 57.09$, $Q = 0.01$.

Figure 2 shows the evolution of the surface magnetic field $B_s$ for various values of $z_0$ and $Q$. One can see that for a large impurity content (Fig. 2a) the field decays comparatively rapidly if it was originally confined to the layers of low density ($\rho_0 = 6 \cdot 10^{10}$–$10^{11}\,\mathrm{g/cm^3}$). In these cases the surface magnetic field decreases approximately by a factor of 100 in 1–10 Myrs. Then the dissipation slows down: the field weakens up to 0.001 of its original value in subsequent $\sim 10^9$ yrs. If the initial field configuration extends down to the density $\rho_0 = 4 \cdot 10^{11}\,\mathrm{g/cm^3}$ ($z_0 = 57.09$), then the decay is much slower. The field becomes lower by a factor of 10 in the first $\sim 10$ Myrs, but only a factor of 100 in the subsequent $\sim 10^{10}$ yrs. In all the cases considered, the magnetic field does not decay appreciably at the initial stage, when the matter is melted in the layers of a maximum current density. This is due to a very short duration of such stage. A signficant decrease of the field takes place later, after a solidification of the outer crust, particularly if $T_D > T$ in the region occupied by the currents. Note that the conductivity at $z \sim z_0$ is determined by the electron-phonon scattering during the first few Myrs when the field decays rapidly. Later on, the conductivity is mainly due to the impurity scattering. That is why the behaviour of decay curves at $t \gtrsim 10^7$ yrs differs from that at $t \gtrsim 10^7$ yrs. Due to both factors, the change of the conductivity and diffusion deeper into the crust, the field decay is essentially nonexponential. In the case of a small impurity content (Figure 2b, 2c), the field evolution is slightly more complicated. In these cases, as well as at $Q = 0.1$ (see Figure 2a), the decay mainly occurs if $T_D > T$ in the region occupied by the currents. At the initial stage, when $t \lesssim 3$ Myrs, the decay is determined by the conductivity due to the scattering on phonons. For $t \lesssim 3$ Myrs, the curves in Figure 2b and 2c are quite similar to those in Figure 2a. At the late stage

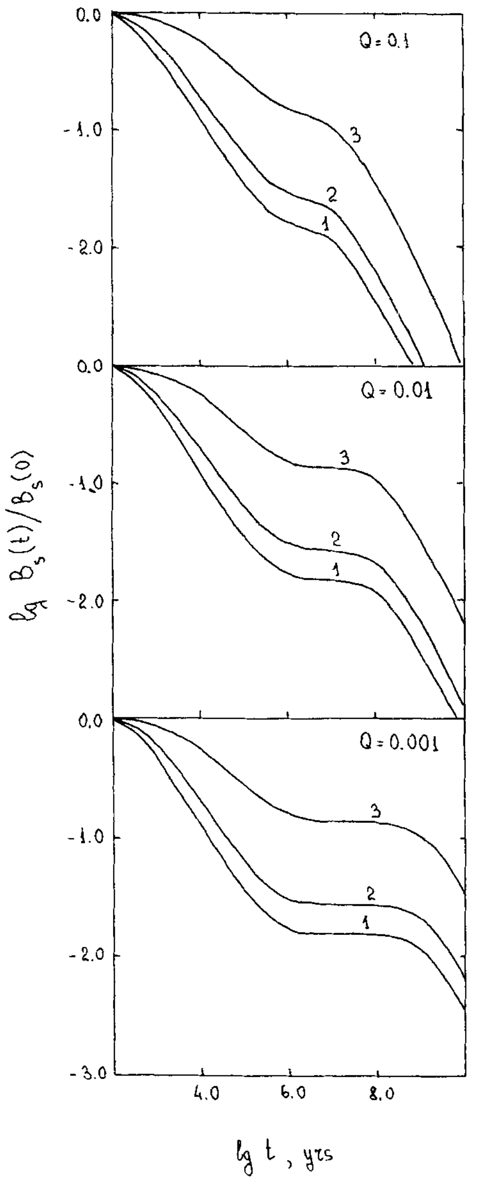


**Figure 2** The evolution of the surface magnetic field normalized by its initial value, for different values of $Q$. The numbers near the curves correspond to the initial depth $z_o$ penetrated by the field: 30.33 (curves 1), 35.96 (curves 2) and 57.09 (curves 3).

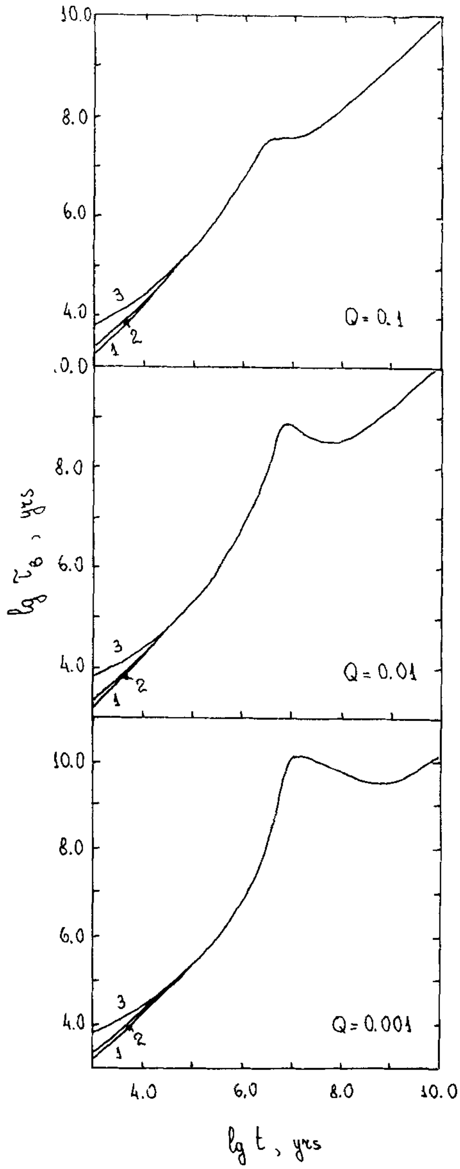


**Figure 3** The evolution of the decay time $\tau_B = -B_s(t)/\dot{B}_s(t)$ for different values of $Q$. The numbers indicate the same initial depths penetrated by the field as in Figure 2.

($t \gtrsim 3$ Myrs), $\sigma$ is determined by the impurity scattering, and the behaviour of the decay curves becomes to be different for different values of $Q$. The decrease of the impurity content results in the increase of the conductivity and slowing down of decay at the late stage of evolution. Basically, the character of evolution for $Q = 0.01$ and 0.001 is qualitatively similar to that for $Q = 0.1$. The field comparatively rapidly dissipates at the early stage: after 1–10 Myrs, it can decrease down to 0.1–0.01 of its original value depending on the initial localization. At the late stages, when the neutron star becomes cool, the field decays very slowly: at $t > 10^7$ yrs, the characteristic decay time can be as large as $\sim 10^9$–$10^{10}$ yrs. Even if the magnetic field was initially concentrated within moderately deep layers with $\rho_0 = (2\text{–}4) \cdot 10^{11}$ g/cm$^3$, it can only weaken about 300 times for $Q = 0.01$ and $\sim 100$ times for $Q = 0.001$, after $\sim 10^{10}$ yrs. Obviously, the curves in Figure 2 cannot be described by a simple exponential or power laws.

Figure 3 shows the dependence of the characteristic field decay time, $\tau_B = -B_s(t)/\dot{B}_s(t)$, on the real time for various values of $Q$. At a given $Q$ the decay time is slightly different for different initial localizations only at the very early stage. After about $10^5$ yrs, the curves begin to describe some self-similar regime which is intrinsically determined by the value of $Q$. In the case $Q = 0.1$ at $t \lesssim 3 \cdot 10^6$ yrs electrons scatter mainly on phonons, while at $t > 3 \cdot 10^6$ yrs, on impurities. The change of the conductivity regime in the region of maximum current density at $t \approx 3 \cdot 10^6$ yrs leads to a change in the behaviour of $\tau_B(t)$. Note that, in perfect crystals with small values of $Q$, the change of the conductivity regime takes place at the later moments of time. For instance, the scattering on impurities dominates at $t > 6 \cdot 10^6$ yrs if $Q = 0.01$, and at $t > 10^7$ yrs if $Q = 0.001$. At the early stage, the slowing down of the decay is caused mainly by the neutron star cooling, while at the late stage, by the diffusion into the deeper layers having a higher conductivity. From our computations, it can be seen that the decay time $\tau_B$ can be very large at late evolutionary stages for all values of $Q$.

Figure 4 presents the surface magnetic field versus the "spin-down" age of a neutron star, $\tau = P/2\dot{P}$. Computations have been performed for the initial field $B_s(o) = 10^{12}$ G. Curves 1, 2 and 3 correspond to the same initial depths penetrated by the field as in Figure 1. The solid lines show the field decay for a star with $Q = 0.1$ in the outer crust, the dashed lines, with $Q = 0.01$ and dash-and-dotted lines, with $Q = 0.001$. Panels *a, b* and *c* display the pulsar tracks for the initial spin periods $P(0) = 0.1$, 0.03 and 0.01, respectively. The rate of magnetic field dissipation is comparatively high at the initial stage (at $\tau \lesssim 10^9$ yrs) when the conductivity $\sigma$ in the crust is determined by the electron-phonon scattering. However, the rate of dissipation decreases with age due to the increase of $\sigma$. Later on (at $\tau > 10^9$ yrs), the decay slows down significantly because of a high conductivity. At this stage, $\sigma$ is determined by the scattering of electrons on impurities, and the evolution of the field depends on $Q$. It is worth mentioning that the decay in terms of the "spin-down" time is much slower than in the real time $t$. This is due to the fact that the spindown rate decreases strongly with the decrease of $B_s$ and, as a result, one has $\tau(\tau) > t$. Initial values of the spin period count for the form of the evolutionary tracks at very short "spin-down" ages ($\tau \lesssim 10^6$ yrs). If the stars possess identical magnetic configurations at $t = 0$ but rotate with different periods, then the star with a smaller period loses its angular momentum more rapidly. Due to the more effective slowing down, this star "overtakes" the star with a long initial period in the course of evolution.

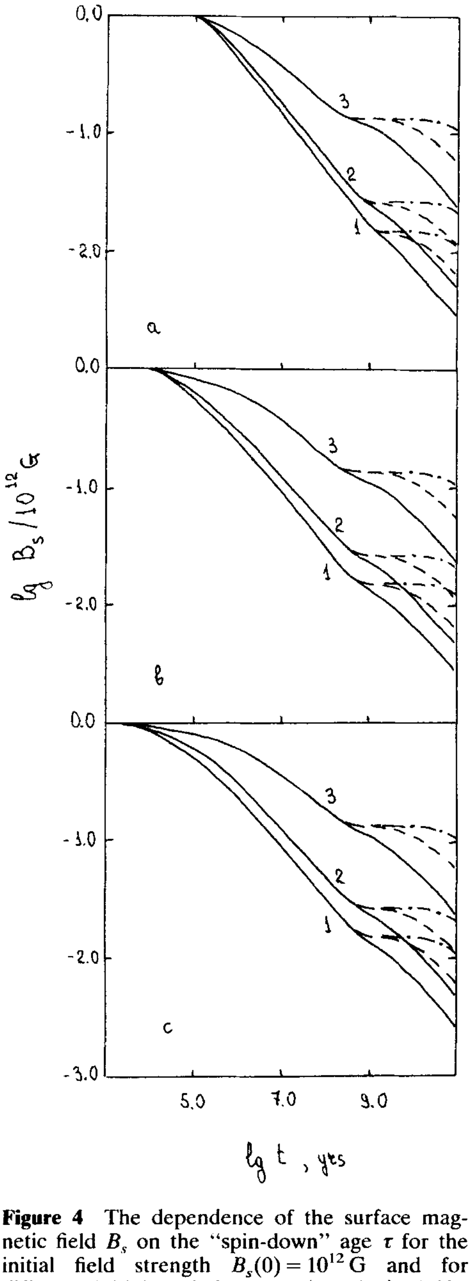


**Figure 4** The dependence of the surface magnetic field $B_s$ on the "spin-down" age $\tau$ for the initial field strength $B_s(0) = 10^{12}$ G and for different initial periods: 0.1 s (panel a), 0.03 s (panel b) and 0.01 s (panel c). Solid lines show the field decay for $Q = 0.1$, Dashed lines, for $Q = 0.01$ and dash-and-dotted lines, for $Q = 0.001$. Curves 1, 2 and 3 corresponds to the same initial depths penetrated by the field as in Figure 2.

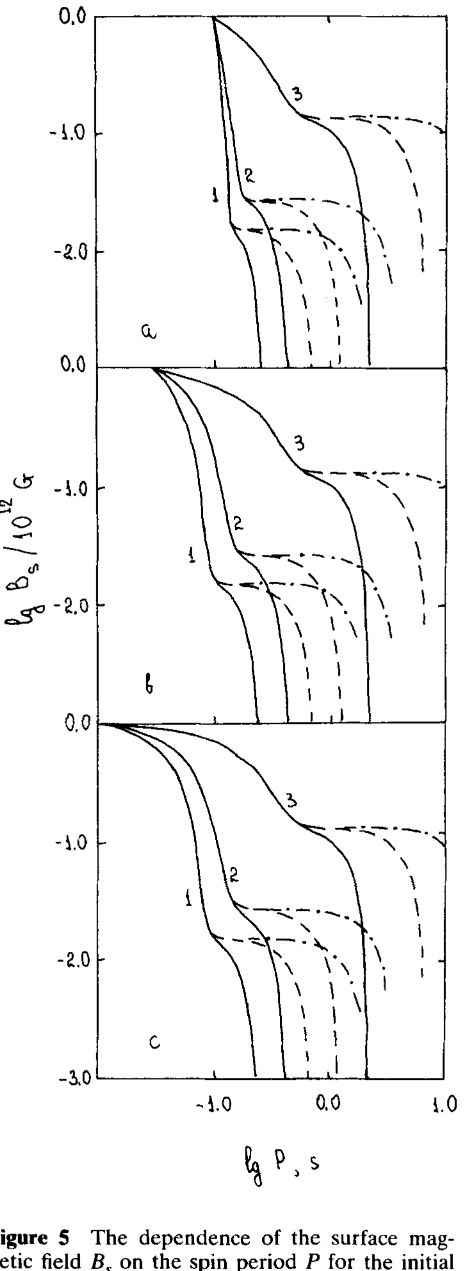


**Figure 5** The dependence of the surface magnetic field $B_s$ on the spin period $P$ for the initial field $B_s(0) = 10^{12}$ G. Notation is the same as in Figure 4.

Therefore, soon after the birth the pulsars reach some self-consistent regimes independent of the initial period. It should be noted that the curves in panels *a, b* and *c* originate from the different point because the values of $\tau$, corresponding to the initial moment $t = 0$, are different for different initial periods.

Figure 5 shows the evolutionary tracks of pulsars in the $B_s - P$ diagram. As in the previous case, computations have been performed for the initial field strength at the surface equal to $B_s(0) = 10^{12}$ G. Curves 1, 2, and 3 correspond to the same values of $z_0$ as in Figures 2–4. Solid lines present the field decay for $Q = 0.1$, dashed lines, for $Q = 0.1$ and dash-and-dotted lines, for $Q = 0.001$. Panels *a, b* and *c* correspond to various initial spin periods. At initial segments of the tracks ($0.1\,\mathrm{s} > P \gtrsim 0.01\,\mathrm{s}$), the magnetic field falls significantly even for a moderate increase of the period. These segments reflect a rapid decay of the field at the initial stage with $t \lesssim 10^7$ yrs (Figure 2) when the conductivity is determined by the electron-phonon scattering. At $t < 10^7$ yrs, the spinning down (proportional to $B_s^2$) is comparatively slow for a modest initial field $B_s(0) = 10^{12}$ G. It leads to a rapid fall of the tracks at $P \leqslant 0.1$–$0.3$ s. Later on, when $\sigma$ is determined by the scattering on impurities and becomes high, the decay slows down. At this stage the field becomes rather weak and, hence, the spin-down rate is also very small. The period $P$ increases slower than the magnetic field decreases. The nearly horizontal segments of the tracks correspond to this evolutionary stage. Evidently, the smaller the content of the impurities (and, hence, the higher $\sigma$ in the crust), the longer these horizontal segments. At the final evolutionary stage, the field decreases strongly again with the increase of $P$. The magnetic field weakens very slowly at this stage (see Figure 2) but, due to a small strength of the field and a large period, the spin-down is slower than the field dissipation. That is the reason of the abrupt behaviour of the decay curves at the late stage. It should be noted that the dependencies $B_s(P)$ reach rapidly some self-consistent regimes that do not depend on the initial value of $P$. In the parameter domain considered, the evolutionary tracks coincide with the self-consistent regimes practically at $P \gtrsim 0.1$ s.

In Figure 6, the resulting magnetic fields ($\alpha\sqrt{P\dot{P}}$) of 403 radio pulsars are plotted against their spin periods. Ten pulsars, entering binary systems, are shown encircled. The dashed line presents the so-called "death line" below which the pulsar activity is likely to switch off. Solid lines show the evolutionary tracks calculated for the initial field strengths $B_s(0) = 10^{12}$ and $10^{13.5}$ G, initial period $P(0) = 0.01$ s and $Q = 0.1$, dash-and-dotted lines, for $Q = 0.01$ and the same values of $B_s(0)$ and $P(0)$. The depths penetrated by the original field have been assumed to be equal to $\approx$300 m (this corresponds to the density $\rho_0 = 10^{11}\,\mathrm{g/cm^3}$). Majority of points corresponding to the observed radio pulsars lie between the evolutionary tracks considered. Probably, this fact indicates that the magnetic fields of neutron stars at their birth can be confined to moderately deep layers of the depth ~300–400 m (or with the density $\rho_0 \lesssim (1\text{–}4) \cdot 10^{11}\,\mathrm{g/cm^3}$). If so, then the original magnetic field of neutron stars lies in the range from $\sim 10^{12}$ to $4 \cdot 10^{13}$ G. Certainly, a more detailed statistical analysis is necessary to conclude more definitely about the magnetic field localization.

## 4. CONCLUSIONS

In the present paper, we have examined the hypothesis that the neutron star magnetic field is initially confined to moderately deep layers of the crust. At all

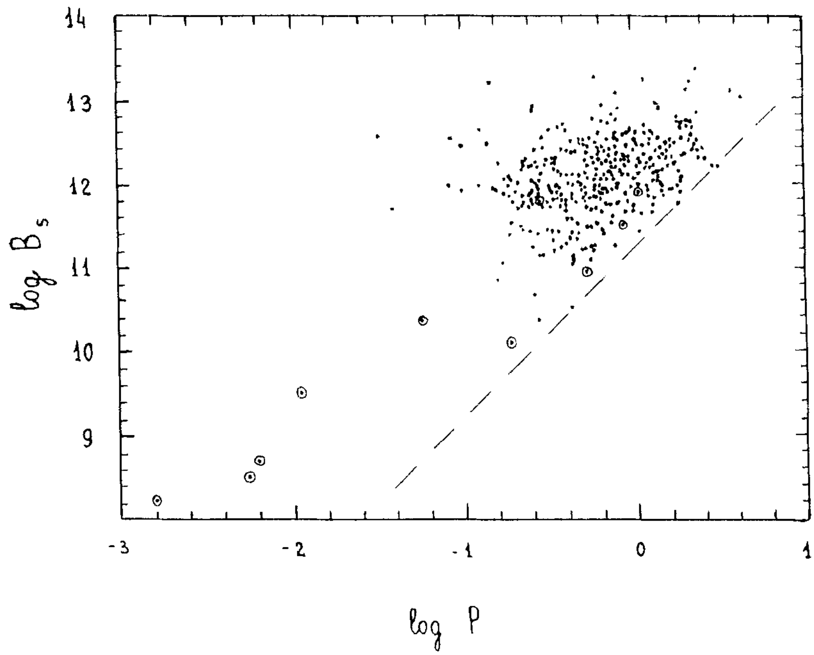


**Figure 6** The $B_s - P$ diagram for pulsars. The points show 403 observed radio pulsars. Pulsars in binary systems are shown by dotted circles. Solid lines present the evolutionary tracks for $B_S(0) = 10^{12}$ G and $10^{13.5}$ G, $Q = 0.1$, the initial period $P(0) = 0.01$ s and the initial depth penetrated by the field $z_0 = 35.96$ ($\rho_0 = 10^{11}$ g/cm$^3$). Dash-and-dotted lines show the tracks for $Q = 0.01$ and the same values of $B_s(0)$, $P(0)$ and $z_0$. A dashed line shows the so-called "death line" for pulsars.

values of parameters considered, characterizing the initial field distribution and the conductivity of the crust ($\rho_0 = 6 \cdot 10^{10} - 4 \cdot 10^{11}$ g/cm$^3$, $Q = 0.1 - 0.001$), the behaviour of the decay curves is qualitatively the same. At the initial stage, the field decreases comparatively rapidly (with the average characteristic time $\sim$ a few Myrs) but the decay continuously slows down. At this stage, the slowing down is mainly due to the the increase of the crustal conductivity because of the neutron star cooling. Later on, the cooling leads to a conductivity regime with $\sigma$ determined by the scattering on impurities. In comparatively pure astrophysical crystals, the conductivity in this regime may be very high. Therefore, the decay time may reach a very large value at the late stage. The slow-down of the field dissipation takes place at the late stage as well but, during this period, it is due to the field diffusion into deeper layers of the crust where the conductivity is higher.

The slowing down of the dissipation leads to the fact that the field can retain a substantial fraction of its initial strength even at age $t \gtrsim 10^9$ yrs. For instance, in the case $\rho_0 \sim (1\text{–}4) \cdot 10^{11}$ g/cm$^3$ the field strength after $10^{10}$ yrs reaches about 0.001 of its original value at $Q = 0.01$ and about 0.01 at $Q = 0.001$. Such a character of evolution is consistent with the current understanding of the pulsar statistics. In particular, it can provide a natural explanation for the millisecond pulsar magnetic field. Therefore, it is possible that the main currents maintaining magnetic configurations in neutron stars are concentrated within surface layers having $\rho \sim 10^{11} - 10^{12}$ g/cm$^3$ while the central regions with a high density are current-free.

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