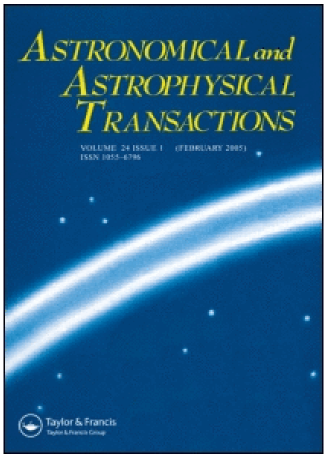

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Neutrino emissivity from **$e^- e$** annihilation in a strong magnetic field: Non-degenerate plasma

A. D. Kaminker [a]; O. Yu. Gnedin [a]; D. G. Yakovlev [a]; P. Amsterdamski [b]; P. Haensel [b]
[a] A. F. Ioffe Institute of Physics and Technology, St. Petersburg, Russia
[b] N. Copernicus Astronomical Center, Polish Academy of Sciences, Warszawa, Poland

Online Publication Date: 01 March 1994
To cite this Article: Kaminker, A. D., Gnedin, O. Yu., Yakovlev, D. G., Amsterdamski, P. and Haensel, P. (1994) 'Neutrino emissivity from **$e^- e$** annihilation in a strong magnetic field: Non-degenerate plasma', Astronomical & Astrophysical Transactions, 4:4, 283 - 289
To link to this article: DOI: 10.1080/10556799408205385
URL: http://dx.doi.org/10.1080/10556799408205385

Astronomical and Astrophysical Transactions, 1994,
Vol. 4, pp. 283–289
Reprints available directly from the publisher.
Photocopying permitted by license only

NEUTRINO EMISSIVITY FROM e^-e^+ ANNIHILATION IN A STRONG MAGNETIC FIELD: NON-DEGENERATE PLASMA

A. D. KAMINKER, O. Yu. GNEDIN and D. G. YAKOVLEV

A. F. Ioffe Institute of Physics and Technology, 194021 St. Petersburg, Russia

P. AMSTERDAMSKI and P. HAENSEL

N. Copernicus Astronomical Center, Polish Academy of Sciences, Bartycka 18, PL-00-716 Warszawa, Poland

(17 July 1992)

The neutrino emissivity from the e^-e^+ pair annihilation is calculated and fitted by a simple formula for a hot, nondegenerate plasma in a magnetic field B of arbitrary strength. A modest magnetic field, $b = B/B_c \ll 1$ ($B_c = 4.41 \times 10^{13}$ G), enhances the emissivity of a moderately hot plasma, at $t = T/T_c \lesssim b$ ($T_c = 6 \times 10^9$ K), and does not affect the emissivity at larger T. Stronger fields, $b \gg 1$, influence the pair annihilation if $t \lesssim \sqrt{b}$. At $t \gtrsim b^{1/4}$ they suppress the process, and at $t \ll b^{1/4}$ they enhance it. As a rule, the pair annihilation dominates over other neutrino production mechanisms in the hot plasma of neutron star envelopes.

KEY WORDS Neutron stars, neutrino, magnetic fields.

1. INTRODUCTION

The aim of this paper is to study the neutrino energy loss rate Q resulting from the e^-e^+ pair annihilation,

$$e^- + e^+ \rightarrow \nu + \bar{\nu}, \tag{1}$$

is a nondegenerate plasma (nonrelativistic and relativistic) with a strong magnetic field $B \sim 10^{12}$–10^{14} G. The pair annihilation is known to be one of the main neutrino energy loss mechanisms in hot neutron star envelopes. At $B = 0$, this process was examined by many authors (Beaudet, Petrosian and Salpeter, 1967; Discus, 1972; Munakata *et al.*, 1985; Itoh *et al.*, 1989). Huge magnetic fields modify the neutrino production.

Some (not fully successful) attempts to study the effects of magnetic field onto the process (1) have been analysed in a recent article of Kaminker *et al.* (1992, hereafter Paper I). These authors obtained a general equation for Q in a magnetic field, and calculated Q for a nonrelativistic plasma with a moderate field $B \ll 4 \times 10^{13}$ G. In the present paper we extend these studies to a relativistic plasma and to larger B.

2. GENERAL EQUATIONS

An equation for the neutrino energy loss rate Q [erg cm^{-3} s^{-1}] in the pair-annihilation process (1) has been derived (Paper I) with exact wave functions of relativistic electrons (e^-) and positrons (e^+) in a quantizing magnetic field B in the framework of the Weinberg–Salam theory. For brevity, we use the Compton units expressing energies in units of mc^2 and momenta in units of mc, where m is the electron mass. The energy of an e^- or e^+ is then written as

$$\varepsilon = \sqrt{1 + p_z^2 + 2bn}\,, \qquad b = \frac{B}{B_c}, \qquad B_c = \frac{m^2c^3}{\hbar e} = 4.414 \times 10^{13}\,\mathrm{G}, \tag{2}$$

where p_z is the particle momentum along B, $n = 0, 1, \ldots$ enumerates the Landau levels.

According to Paper I,

$$Q = Q_c \frac{b}{3(2\pi)^5} \sum_{n,n'=0}^{\infty} \int_{-\infty}^{+\infty} dp_z \int_{-\infty}^{+\infty} dp_z' \int_0^{\sqrt{\omega^2 - q_z^2}} q_\perp\, dq_\perp A \omega f f', \tag{3}$$

where

$$Q_c = \frac{G^2}{\hbar}\left(\frac{mc}{\hbar}\right)^9 = 1.023 \times 10^{23}\,\mathrm{erg\,cm^{-3}\,s^{-1}}, \tag{4}$$

may be called the Compton neutrino emissivity unit and G is the Fermi weak-coupling constant. Other quantities on the right-hand side of (3) are dimensionless (see above). Nonprimed quantities refer to e^-, while primed ones refer to e^+; $\omega = \varepsilon + \varepsilon'$, $q_z = p_z + p_z'$, and $q_\perp$ denote, respectively, the energy, the longitudinal momentum, and the transverse momentum carried away by a neutrino pair; f and f' are the Fermi-Dirac distributions of e^- and e^+, respectively. The integrand in (3) contains the factor A that is proportional to the spin trace of the squared matrix element of the process. A is a complicated function of n, n', p_z, p_z' and $q_\perp$. The upper limit of integration over $q_\perp$ follows from the energy-momentum conservation. Equation (3) is similar to the equation for the neutrino energy loss rate in the electron synchrotron process ($e \to e + \nu + \bar{\nu}$) (Paper I).

3. PHYSICAL CONDITIONS AND LIMITING CASES

We shall study the pair-annihilation neutrino radiation in a hot *nondegenerate* plasma at $T > T_F$, where $T_F = T_c(\sqrt{x^2+1} - 1)$ is the e^- degeneracy temperature, $T_c = mc^2/k_B \approx 5.930 \times 10^9$ K, $x = p_F/(mc)$, and p_F is the e^- Fermi momentum in the case of a strong degeneracy. When magnetic field is not very strong one has $p_F = \hbar(3\pi^2 n_-)^{1/3}$ and $x = x_0 \approx 0.0101(\rho/\mu_e)^{1/3}$, where n_- is the e^- number density, ρ is mass density (expressed in g cm^{-3}), μ_e being the number of baryons per proton in matter. When the magnetic field is strong enough and/or plasma density small, $b \gtrsim x_0^2$, the degenerate e^- are forced to occupy the ground Landau level, and p_F is essentially reduced as compared to the field-free case (e.g., Yakolev, 1984). Then $x = x_B = 2x_0^3/(3b)$. When $x \ll 1$ (low density and/or strong magnetic field), one has $T_F \ll T_c$, i.e. the nondegenerate electron gas may be either *nonrelativistic* ($T_F \ll T \ll T_c$) or *relativistic* ($T \gg T_c$). In the opposite case ($x \gg 1$) we have $T_F \gg T_c$, and the nondegenerate gas is essentially relativistic. In the nondegenerate plasma, it is useful to consider the cases when $e^{\mp}$ occupy many Landau levels ($b \ll t(1+t)$, $t \equiv T/T_c$) and the magnetic field is *nonquantizing*, and when $e^{\mp}$ occupy mostly the ground level ($b \gg t(1+t)$) and the field is *quantizing*. We will also distinguish *nonrelativistic* ($b \ll 1$) and *relativistic* ($b \gg 1$) magnetic fields.

A careful analysis of Eq. (3) supplemented by the equations of thermodynamic equilibrium to determine the number densities of e^- and e^+ shows that the neutrino emissivity of the pair-annihilation process in the nondegenerate plasma is actually independent of density. This is because the product of $e^{\mp}$ number densities, n_-n_+, is almost density independent at $T \gg T_F$. As a result, Q is determined by two parameters, B and T, (Figure 1) and has different behaviour in the six domains denoted by I–VI in the (t, b)-plane.

Let us present asymptotic forms for Q in each domain.

Domain I corresponds to a nonrelativistic plasma ($t \ll 1$) with a nonquantizing magnetic field ($b \ll t$), while *Domain II* corresponds to a nonrelativistic plasma with a quantizing nonrelativistic field ($t \ll b \ll 1$). In Paper I, an asymptotics of Q has been obtained which is equally valid in Domains I and II (at $t \ll 1$, $b \ll 1$):

$$Q = \frac{Q_c}{8\pi^4}(C_+^2 + C_-^2)b^2 t \exp\left(-\frac{2}{t}\right)\left[\coth^2\left(\frac{b}{2t}\right) - \frac{1}{3}\right], \tag{5}$$

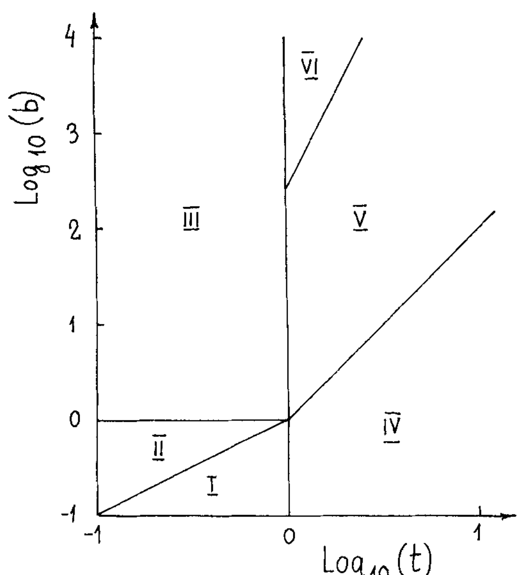

Figure 1 Domains I–VI of temperature ($t = T/T_c$) and magnetic field ($b = B/B_c$), where the pair annihilation energy losses in a nondegenerate plasma have different characters.

where

$$C_+^2 = C_V^2 + C_A^2 + N(C_V'^2 + C_A'^2) \approx 1.675$$
$$C_-^2 = C_V^2 - C_A^2 + N(C_V'^2 - C_A'^2) \approx 0.175, \tag{6}$$

$C_V = 2\sin^2\theta_W + 0.5$ and $C_A = 0.5$ are, respectively, the vector and axial-vector coupling constants appropriate to the emission of the electron neutrinos. $C'_{V,A} = C_{V,A} - 1$ correspond to emitting the neutrino of other types whose number is denoted by N; θ_W is the Weinberg angle. Below we shall set $\sin^2\theta_W = 0.23$, and $N = 2$ (including muonic and taonic neutrinos).

In Domain I, (5) reproduces the field-free expression $Q = Q_0 = Q_c(2\pi^4)^{-1}(C_+^2 + C_-^2)t^3\exp(-2/t)$.

In Domain II, one has $Q = Q_c(12\pi^4)^{-1}(C_+^2 + C_-^2)tb^2\exp(-2/t)$. The quantizing field in Domain II enhances the neutrino emissivity, $Q/Q_0 \approx (b/t)^2/6$, by increasing the number densities of $e^\mp$ (Paper I).

Domain III corresponds to the nonrelativistic plasma ($t \ll 1$) with a relativistic quantizing field ($b \gg 1$). In this case we get from (3) $Q = Q_c(6\pi^4)^{-1}C_+^2 tb \times \exp(-2/t)$. We see that the relativistic field also enhances the emissivity, although the enhancement is modified, $Q/Q_0 \sim b/t^2$.

Now consider the case when the plasma is relativistic, $t \gg 1$. If $b \ll t^2$ (*Domain IV*), the field is nonquantizing and the field-free expression (see, e.g., Dicus, 1972) holds: $Q = Q_0 = 7Q_c(12\pi)^{-1}\zeta(5)C_+^2 t^9$, where $\zeta(s)$ is the zeta function.

For much stronger fields, $b \gg t^2$, the particles populate mostly the ground Landau level and one has $Q = Q_c(48\pi^3)^{-1}\zeta(3)C_+^2 t^5 b$. This equation deserves further comments because it does not match Q_0, and gives $Q/Q_0 \sim 1/(28\pi^2 b) \ll 1$ at $b \sim t^2$. This stiking circumstance comes from the fact that the contribution of the $n = n' = 0$ term into Q at $t \gg 1$ can be suppressed in comparison with the contributions of other terms due to the fixed spin orientations of e^- and e^+ on the ground Landau level. Actually, when b increases and approaches $b \sim t^2$, the neutrino emissivity suffers a strong decrease: the contribution from the excited Landau levels decreases (the levels are depopulated), whereas the contribution from the ground-level particles is suppressed. That is why the region $t \gg 1$, $b \gg t^2$ can be separated into two domains, V and VI.

Domain V corresponds to $t^2 \ll b \ll 28\pi^2 t^4$. In this domain the neutrino emissivity is suppressed as compared to the field-free one.

Domain VI corresponds to stronger fields, $b \gg 28\pi^2 t^4$. In Domain VI, $Q/Q_0 \sim b/(28\pi^2 t^4) \gg 1$.

4. CALCULATIONS AND FITTING

In addition to an analytical study of Q in various domains of B and T, we have calculated Q directly from Eq. (3). First we have calculated the field-free neutrino emissivity Q_0. In all our calculations, we have taken $\rho/\mu_e = 10^5\,\mathrm{g\,cm^{-3}}$ although the results are actually independent of ρ. We have determined Q_0 for eleven values of temperature T ranging from 10^9 K to 10^{11} K. With an average error of ~1.5%, these results are fitted by

$$Q_0 = \frac{Q_c}{\pi^4}\left[\tfrac{1}{2}t^3(C_+^2 + C_-^2)(1 + \tfrac{15}{4}t) + t^4 C_+^2 P(t)\right]\exp\left(-\frac{2}{t}\right), \tag{7}$$

where $P(t) = 1 + 3.581t + 39.64t^2 + 24.43t^3 + 36.49t^4 + 18.75t^5$.

Then we have calculated Q for the same eleven values of temperature at several B chosen in such a way as to recover all the asymptotic regimes outlined in Section 3.

For all 309 values of B and T included in the computation, with an average error of ~3.8%, the results are fitted by a single formula

$$Q = \frac{Q_c}{\pi^4}\left\{[\tfrac{1}{2}t^3(C_+^2 + C_-^2)(1 + \tfrac{15}{4}t) + t^4C_+^2P(t)]F(t, b) + \frac{tb^2}{12(1+b)}\left[(C_+^2 + C_-^2) + (C_+^2 - C_-^2)\frac{b}{1+b}\right]S(t)\right\}\exp\left(-\frac{2}{t}\right). \tag{8}$$

where $S(t) = 1 + 1.058t + 0.6701t^2 + 0.9143t^3 + 0.472t^4$,

$$F(t, b) = \frac{1}{R_1R_2R_3}, \qquad R_i = 1 + c_i\frac{b}{t_2}\exp\left(\frac{\sqrt{2b}}{3t}\right), \tag{9}$$

$c_1 = 3.106 \times 10^{-6}$, $c_2 = 1.491 \times 10^{-3}$, and $c_3 = 4.839 \times 10^{-6}$.

One can easily verify that our fitting expressions reproduce all the asymptotic forms of Q presented in Section 3.

5. DISCUSSION AND CONCLUSIONS

The effect of magnetic field on the annihilation energy loss rate $Q(T, B)$ in a hot nondegenerate plasma is demonstrated in Figure 2. The figure presents the level lines of $q = Q(T, B)/Q(T, 0) = Q(B, T)/Q_0(T)$. The field effects are different in

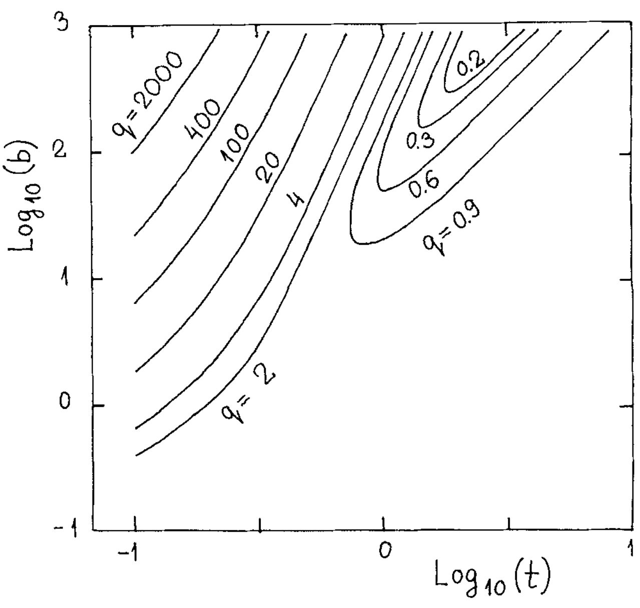

Figure 2 Level lines of $q = Q(T, B)/Q(T, 0)$ in the (t, b)-plane.

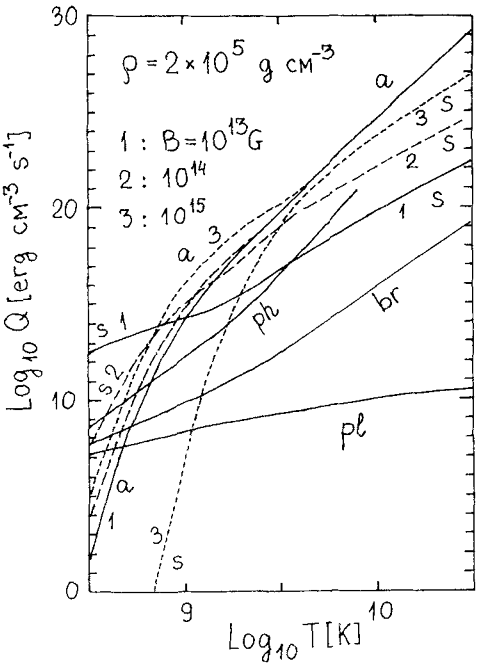

Figure 3 The temperature dependence of energy loss rates from some neutrino production mechanisms at $\rho/\mu_e = 10^5\,\mathrm{g\,cm^{-3}}$. Curves ($a$) correspond to the pair annihilation and (s), to the synchrotron radiation, respectively, for $B = 10^{13}$ G (solid lines, labelled 1), 10^{14} G (long dashes, 2), and 10^{15} G (short dashes, 3). Other solid curves show the loss rates from the photon decay (ph), plasmon decay (pl) and electron bremsstrahlung (br) on ^6C nuclei at $B = 0$.

Domains I–VI as explained in Section 3. The quantizing field in Domains II, III and VI enhances the emissivity, $q > 1$, and the enhancement can be very large. On the other hand, a moderately quantizing field in the relativistic plasma suppresses the emissivity ($q < 1$). This creates a "suppression valley' in Domain V. The curves in Figure 2 do not exactly follow the map of Domains in Figure 1 because the map is rather approximate.

Figure 3 demonstrates the temperature dependence of the annihilation losses Q for $B = 10^{13}$, 10^{14}, and 10^{15} G. The parameters displayed correspond to Domains II and III. In Figure 3 we compare the annihilation losses with the energy losses from other neutrino production mechanisms: the $e^{\mp}$ synchrotron radiation ($e \rightarrow e + \nu + \bar{\nu}$) at the same B (Kaminker and Yakovlev, 1992), the photon decay ($\gamma + e \rightarrow e + \nu + \bar{\nu}$), the plasmon decay ($\hbar\omega_p \rightarrow \nu + \bar{\nu}$) (Itoh *et al.*, 1989), and the bremsstrahlung on nuclei ($e + Z \rightarrow e + Z + \nu + \bar{\nu}$) (Munakata *et al.*, 1987). The three latter mechanisms have not yet been studied in a magnetic field, and we present the $B = 0$ curves for illustration.

As can be seen from Figure 3, synchrotron radiation can be the main source of neutrino energy losses in a magnetized nonrelativistic nondegenerate plasma while the annihilation radiation dominates at higher temperatures. These conditions are appropriate to hot neutron star envelopes. The results obtained are required, particularly, for studying the cooling of young (10–100 years old) magnetized neutron stars.

References

Beaudet, G., Petrosian, V., and Salpeter, E. E. (1967). *Astrophys. J.* **150,** 979.
Discus, D. A. (1972). *Phys. Rev.* **D6,** 941.

Itoh, N., Adachi, T., Nakagawa, M., Kohyama, Y., and Munakata, H. (1989). *Astrophys. J.*, **339**, 354.
Kaminker, A. D. and Yakovlev, D. G. (1992). *Zh. Eksperim. Teor. Fiz.*, in press.
Kaminker, A. D., Levenfish, K. P., and Yakovlev, D. G. (1991). *Pis'ma Astron. Zh.* **17,** 1090.
Kaminker, A. D., Levenfish, K. P., Yakovlev, D. G., Amsterdamski, P., and Haensel, P. (1992). *Phys. Rev.* in print (Paper I).
Munakata, H., Kohyama, Y., and Itoh, N. (1985). *Astrophys. J.* **296,** 197.
Munakata, H., Kohyama, Y., and Itoh, N. (1987). *Astrophys. J.* **316,** 708.
Yakovlev, D. G. (1984). *Astrophys. Space Sci.* **98,** 37.