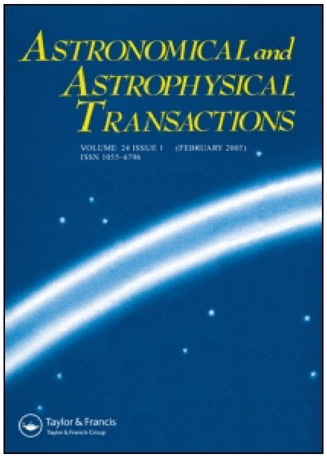

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

The distribution of old neutron stars in the galaxy

Mikhail E. Prokhorov [a]; Konstantin A. Postnov [a]
[a] Sternberg Astronomical Institute, Moscow, Russia

Online Publication Date: 01 August 1993
To cite this Article: Prokhorov, Mikhail E. and Postnov, Konstantin A. (1993) 'The distribution of old neutron stars in the galaxy', Astronomical & Astrophysical Transactions, 4:2, 81 - 89
To link to this article: DOI: 10.1080/10556799308205367
URL: http://dx.doi.org/10.1080/10556799308205367

Astronomical and Astrophysical Transactions, 1993, Vol. 4, pp. 81–89
Reprints available directly from the publisher.
Photocopying permitted by license only

THE DISTRIBUTION OF OLD NEUTRON STARS IN THE GALAXY

MIKHAIL E. PROKHOROV and KONSTANTIN A. POSTNOV
Sternberg Astronomical Institute, 119899 Moscow, Russia

(21 July 1992)

The distribution of old galactic neutron stars is computed for model galactic potential and initial neutron star velocity distribution following from evolution scenario of binary systems. Neutron stars are supposed to get their initial velocities during (asymmetric) supernova explosion of a single massive ($M_0 > 10\,M_\odot$) star or a massive component of a binary system. Old neutron star population is shown to be capable of filling a torus-like area extending to a few tens kiloparsecs above the galactic plane. Statistical tests related to the spatial distribution of gamma-ray bursts are carried out for the distribution obtained.

KEY WORDS Pulsars, old neutron stars, space distribution, gamma-ray bursts.

1. INTRODUCTION

Recent results from the BATSE device onboard the GRO observatory have shown a full isotropy of gamma-ray burst locations the sky and their inhomogeneous spatial density ($\langle V/V_{\max}\rangle = 0.33 \pm 0.02$, $\langle \sin^2 b\rangle = 0.31 \pm 0.02$, $\langle \cos\theta\rangle = 0.008 \pm 0.035$) (Meegan *et al.*, 1992), where θ is the angle between the source and the galactic center, b is the galactic latitude and $\langle\ \rangle$ denotes averaging over the whole sample. In the case of a purely homogeneous, isotropic distribution these quantities would be, respectively 1/2, 1/3 and 0. These observed statistical features of gamma-bursts greatly reinforced interest to the nature of their possible progenitors. An attractive possibility of old galactic neutron stars looses its credibility, especially in view of negative results of searching for any spectral feature in the gamma-burst spectra obtained by BATSE. Some indications appear that gamma-ray bursts can be divided into at least two types: the strong bursts having hard spectra, with $\langle V/V_{\max}\rangle > 1/2$ and the weaker ones with softer spectra and $\langle V/V_{\max}\rangle < 1/2$ (APEX experiment results, Mitrofanov *et al.*, 1992; see also Lingenfelter & Higdon, 1992). These results might indicate that two populations of gamma-ray burst progenitors having different spatial distributions actually exist, viz. that producing weak bursts and lying outside the galactic disc, and that producing stronger bursts and residing in the galactic disc. Brainerd (1992) has shown that the BATSE results can be reconciliated with the old neutron star paradigm if one considers an extended ($\gtrsim$100 kpc) halo consisting of old neutron stars. The question arises, whether there is any mechanism of natural populating such a halo by old neturon stars?

The spatial distribution of old galactic neutron stars as putative gamma-ray burst progenitors has been studied earlier (e.g. Paczyński, 1990; Hartmann *et al.*, 1990). By direct integration of the equations of motion they found the distribution of old neutron stars taking as an initial velocity that deduced from observations of radiopulsars. Their results show that it is difficult for a neutron star to run out too far away from the galactic plane (several kiloparsecs) unless a very fast tail exists in their initial velocity distribution.

In this paper we consider another method for obtaining such distributions basing essentially on stationary solutions of the Boltzmann kinetic equation for the distribution function $f(\vec{x}, \vec{v})$ in 6-dimensional phase space (here $\vec{x}$ and $\vec{v}$ are the 3-dimensional coordinate velocity, respectively). As an initial position in space, we take an ordinary exponential distribution in r- and z-coordinates, whereas a distribution following from modern binary star evolutionary scenario is taken as the initial condition for neutron star velocities.

We show that if one can use the ergodic hypothesis for the motion in the galactic potential, then the population of old neutron stars can extend very far from the galactic plane under reasonable initial conditions.

2. FORMULATION OF THE PROBLEM

The problem to be solved is to find a stationary spatial distribution of stars which have had certain initial spatial and velocity distributions and move in a given potential. The potential is supposed to be axially symmetric, constant in time and not affected by this stellar population. In principle, the most straightforward way to obtain the solution is to integrate directly the equations of motion for a sufficiently large number of stars with different initial conditions. This method was used by Paczyńsky (1990) and Hartmann *et al.* (1990). Here we propse to use another method based on the solution of collisionless Boltzmann equation. We introduce $f(\vec{x}, \vec{v})$ as the distribution function of stars in the 6-dimensional phase space. Then the spatial density is given by $\rho(\vec{x}) = \int f(\vec{x}, \vec{v})\, d\vec{v}$. Time evolution of the distribution function is generally described by the kinetic equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla\Phi(r, z) \cdot \frac{\partial f}{\partial \vec{v}} = 0,$$

where Φ is the potential. Axial symmetry implies that only two cylindrical coordinates, r and z are required. In the case considered the problem can be significantly simplified because at least two integrals of motion exist—the total energy, E, and the angular momentum along the z-axis, L_z (here and below we consider the stars of unit mass):

$$E = \tfrac{1}{2}v^2(r_0, z_0) + \Phi(r_0, z_0); \qquad L_z = r_0 \cdot v_\varphi(r_0, z_0) \tag{1}$$

with $v(r_0, z_0)$ being the absolute value of the initial velocity, and $v_\varphi(r, z) = 1/r(\partial\Phi/\partial r)$ is the circular velocity. So the problem can be reformulated as follows: starting from a given distribution function $Q(E, L_z)$ [that is, the number of stars per energy interval $(E, E + dE)$ and angular momentum interval $(L_z, L_z + dL_z)$] in 2-dimensional space to find the distribution function $f(\vec{x}, \vec{v})$ in 6-dimensional phase space. As we are interested in the investigation of old neutron star distribution, we can safely neglect the time dependence and search for an equilibrium solution of the collisionless equation.

The conservation of the total number of the stars implies that

$$\int Q(E, L_z)\, dE\, dL_z = \int f(\vec{x}, \vec{v})\, d\vec{x}\, d\vec{v}. \tag{2}$$

Introduce cylindrical coordinates r, z, ϕ in physical space and spherical coordinates $\vec{w}$, θ, α in the velocity space. The direction of $\vec{w}$ can be chosen arbitrary, say along the orbital velocity $\vec{v}_{orb}$, θ being the angle between $\vec{v}$ and $\vec{w}$ and α, the azimuthal angle. Then we perform the transformation in the velocity space from the spherical coordinates to the coordinates E, L_z, α. Using (1), we calculate the Jacobian $\vec{J} = \det(\partial\tilde{x}_i/\partial x_j)$ of the transformation $\tilde{x}_i \rightarrow x_j$: $\vec{J} = rv^2 \sin\theta$. Finally, we obtain for Q:

$$Q(E, L_z) = 4\pi^2 \int_{\mathscr{D}(E,L_z)} f\, dr\, dz. \tag{3}$$

Here $\mathscr{D}(E, L_z)$ denotes the region in the r, z-space where the motion is allowed with the given angular momentum and energy, E and L_z. Now asume that the motion, in the chosen potential, is ergodic, or equivalently, that a test particle moving in this potential long enough will sweep uniformly the hypersurface of constant E, L_z in the phase space r, z, E, L_z. Then the distribution function f in Eq. (3) can be treated as constant and follows directly from (3) provided $Q(E, L_z)$ is known. The density Q can be found straightforwardly from the initial distributions $p(r, z)$ and $p(\vec{v})$ using the same Eq. (3) with $f = f_0 = p(r, z)p(\vec{v})$ (note that $p(\vec{v})$ is expressed through $p(|\vec{v}|)$, which follows from statistical calculations, and $p_\varphi(v_\varphi)$; see below). Finally, we obtain as a solution

$$f(t \rightarrow \infty; \vec{x}, \vec{v}) \equiv f(\vec{x}, \vec{v}) = \frac{\int_{\mathscr{D}(E,L_z)} f_0\, dr\, dz}{\int_{\mathscr{D}(E,L_z)} dr\, dz}. \tag{4}$$

Basically, this result is an equivalent formulation of the ergodic hypothesis. The latter should be proved for each potential separately because no general theorem exists for axisymmetric potentials. We briefly discuss this problem below.

3. INITIAL CONDITIONS

3.1. Space and Velocity Distributions

3.1.1. Space position. Following Paczyńsky (1990), we assume the initial space distribution of young neutron stars to vary exponentially with distance from the Galactic center and from the Galactic plane, that is the probability distribution for young neutron star positions can be written in the form

$$p_z(z)\, dz \propto \exp(-z/z_0)\, dz,$$
$$p_r(r)\, dr \propto \exp(-r/r_0) r\, dr,$$

with $r_0 = 4.5$ kpc and $z_0 = 75$ pc.

3.1.2. Initial velocity. The initial velocity of young neutron stars can be taken either from the observed pulsar velocity distributions (which is still not firmly established), or from some model-dependent assumptions based on current ideas about neutron star formation. Both ways involve complications. Here we use the second way, that is we calculate the initial velocity of a new-born pulsar as a sum of its regular velocity in the Galaxy and the velocity which the pulsar acquires at the moment of its birth. The former is defined by the assumed galactic potential and is typically of the order of 200 km/s, and the latter is caused by the chaotic motion of the progenitor star and a regular motion of the pulsar progenitor in a binary system and usually $\simeq$100 km/s. We also take into account a possible assymmetry in the supernova explosion mechanism (for example, neutrino anistropy in a strong magnetic field (Chugaj, 1984), breaking of magnetic field symmetry during magneto-rotational collapse (Bisnovatyj-Kogan and Moiseenko, 1992) or splitting of fastly rotating newborn neutron star into two neutron stars with subsequent explosion of the lighter component (Imshennik 1992) by introducing a "kick" velocity for the pulsar. The "kick" velocity is supposed to be as high as $v_k = 100$ km/s and randomly oriented. (Note that if Imshennik and Nadyozhin's mechanism is actually under work, which is supposed to occur in a few percent of supernovae, one should expect a subclass of very fast pulsars ($v \geq 300$–500 km/s) to exist which will populate ultimately a very extended halo around the Galaxy).

We have performed a Monte-Carlo calculation of the evolution of 50,000 massive binaries with an initial mass of the primary star $M_i \geq 10\,M_\odot$ to obtain the initial velocity distribution of yound pulsars. The description of the method of statistical simulation can be found elsewhere (Kornilov and Lipunov, 1983;

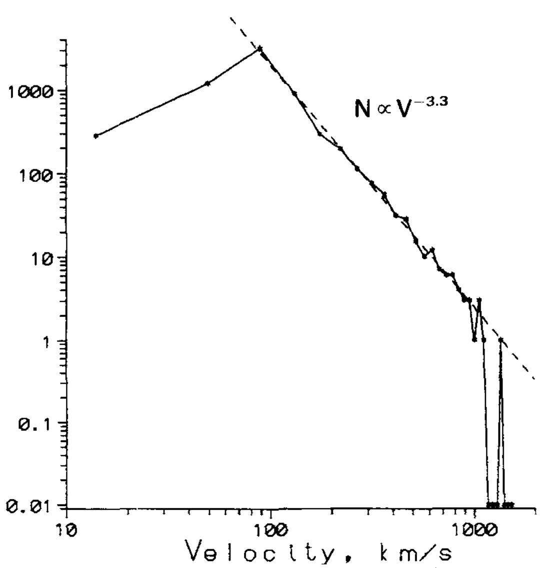

Figure 1 The Velocity distribution of young pulsars calculated from statistical modelling of the evolution of 50,000 binary systems. The "kick" velocity in supernova explosions is assumed to be 100 km/s.

Lipunov and Postnov, 1987). The resutls are presented in Figure 1. One can see that the velocity distribution $p(|\vec{v}|)$ has a maximum centered at the assumed "kick" velocity and the high-velocity tail of the distribution is a power law with the index of $\simeq -3.3$. This probably reflect the power-law form of the assumed (Salpeter) distribution of the initial stellar mass, and the power-law parametrization of binary system evolutionary tracks we used for the statistical calculations for a large number of the systems. We assume the distribution $p(\vec{v})$ to be isotropic. Furthermore, we need the distribution of projected velocities on the direction of the systematic motion in the Galaxy (i.e., on the direction of $\vec{v}_{orb}$), $p_\varphi(v_\varphi)$, in order to calculate the distribution over energy E (which depends on $p(|\vec{v}|)$) and over angular momentum L_z (which is determined by $p_\varphi(v_\varphi)$). These two distributions are connected through the well known relation

$$p_\varphi(x) = 1/2\int_x^\infty p(y)/y\, dy.$$

3.2. *Galactic Potential*

We consider motion of the stars in the model galactic axially symmetrical potential which becomes quasi-spherical at large distances from the galactic center and gives rise to the observed galactic rotation curve. As an example, we have taken the same galactic potential as Paczyńsky (1990). The potential consists of two disk components corresponding to the galactic disc and the bulge,

$$\Phi_i(r, z) = \frac{GM_i}{\{r^2 + [a_i + (z^2 + b_i^2)^{1/2}]\}^{1/2}},$$

and a quasi-spherical halo with the density distribution in the form

$$\rho = \frac{\rho_0}{1 + (d/d_0)^2}, \qquad d^2 = r^2 + z^2.$$

The parameters of the potential are the following:

Disk	$a_1 = 0$	$b_1 = 0.277$ kpc	$M_1 = 1.12 \times 10^{10} M_\odot$
Bulge	$a_1 = 3.7$ kpc	$b_1 = 0.20$ kpc	$M_1 = 8.07 \times 10^{10} M_\odot$
Halo		$d_0 = 0.277$ kpc	$M_0 = 5.0 \times 10^{10}\ M_\odot$,

here $M_0 = 4\pi\rho_0 d_0^3$.

4. RESULTS

The resulting distribution $Q(E, L_z)$ is shown in Figure 2. It can be seen that a sharp boundary exists (minimal E at a given L_s) corresponding to circular orbits on the galactic plane. If all the stars were moving strictly along circular orbits, only the positive part of the curve would be seen and all the particles would be at this boundary. The galactic z-distribution of stars and peculiar velocities slightly smear out this narrow boundary. It is clear that the shape of the isodenses in this

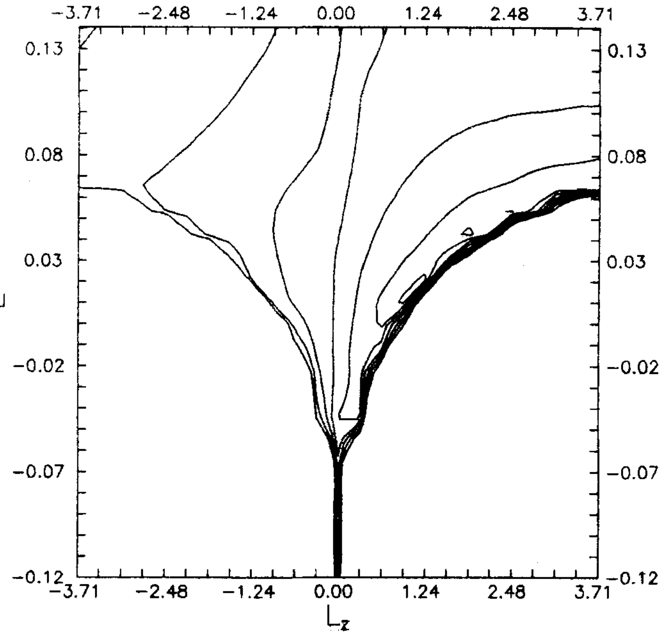

Figure 2 The contours of equal density of energy-angular momentum distribution $Q(E, L_z)$. Units: energy, in $(1000\,\mathrm{km/s})^2/2$, and angular momentum, in $1000\,\mathrm{km/s} \times \mathrm{kpc}$.

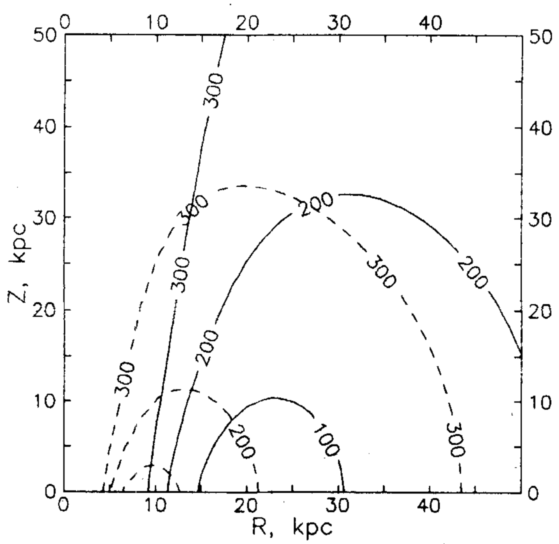

Figure 3 Equipotential lines of effective potential Φ_{eff} labelled with the component of the velocity (in km/s) which does not contribute to L_z which a newborn neutron star at the galactic plane should acquire to have this potential. Solid line is for the star at the distance $r = 20$ kpc, dashed line, for $r = 8.5$ kpc.

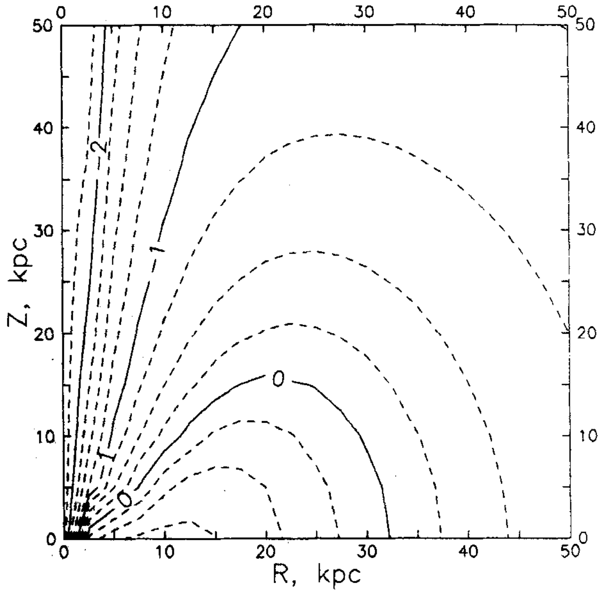

Figure 4 Equal tensity contours for the calculated distribution of old neutron stars (in arbitrary units).

plot results from peculiar velocities of the neutron stars acquired during supernova explosions. However, the majority of stars lies close to the positive boundary, as in the case of unperturbed motion. This happens because the typical velocity of 100 km/s is less than the orbital velocity in the Galaxy. The part of $Q(E, L_z)$-plane with high energies and negative angular momenta corresponding to retrograde motion are due to the high-velocity tail of the velocity distribution.

Equipotential surfaces are shown in Figure 3. If a test particle were moving with given E, L_z, the ergodic hypothesis states that ultimately it will be found at any point inside the equipotential surface (with a uniform probability density in the *phase space*). It is convenient to label the equipotential surfaces with the value of energy difference between E and the energy the star would have at the circular orbit at a certain r and, hence, with a certain L_z (recall that this corresponds to the minimum of the effective potential defined by $\Phi_{eff} = \Phi(r, z) + L_z^2/2r^2$), expressed in units of the velocity component of the star in the direction which does not contribute to L_z.

The final form Figure 4 of the old neutron star density distribution is obviously the sum of filled equipotential lobes with different L_z weighted with the probability $Q(E, L_z)$ shown in Figure 2.

5. DISCUSSION

As can be seen in Figure 4, the old galactic neutron stars fill a torus-like region extending up to tens kiloparsecs above the galactic plane with a sharp deficiency of stars near the galactic axis. As we have already mentioned above, this is a

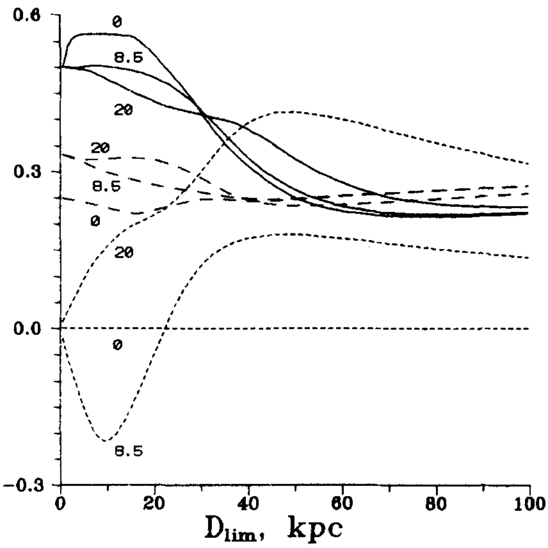

Figure 5 Plot of $\langle V/V_{max}\rangle$ (solid), $\langle \sin^2 b\rangle$ (long dashes) and $\langle \cos\theta\rangle$ (short dashes) for the calculated old neutron star space distribution versus the limiting distance of the sample, D_{lim}. Figures correspond to the distance of the observer from the galactic center (in kiloparsecs).

consequence of the shape of the chosen galactic potential and (to a smaller degree) of the calculated $Q(E, L_z)$-distribution defined by initial conditions. Unexpectedly enough, the maximum density lobe is centered close to the distance $r \approx 10$ kpc from the galactic center, that is the Sun lies inside it. Then it is intriguing to see what the well known test of space distribution give in this case. In Figure 5 we show the results of applying the $\langle V/V_{max}\rangle$ test and $\langle \cos\theta\rangle$ and $\langle \sin^2 b\rangle$ criteria to the obtained distribution for three specific positions of the observer (close to the axis, inside the torus and outside it), which describe all different situations. It is clear that a slight change of the parameters of the model will not distort dramatically the qualitative picture shown in Figure 5. If such a torus-like distribution actually exists, it will manifest itself in this manner.

Now return to the crucial question about the applicability of the *ergodicity principle* to the dynamics of stars in a real galaxy. This question is directly connected with the question of existence of the so-called third isolating integral of motion (see, e.g., Binney and Tremaine, 1987). If it exists and is a single-valued function of space coordinates, the ergodicity is not applicable. Otherwise, if the third integral is not an isolating one or does not exist at all, the situation is more complicated and in any case a strong proof of the ergodicity is required. It has been proved that the ergodicity is not applicable for a subclass of power-law potentials, but the potential or our Galaxy does not belong to this class (see Sinaj, 1982). Numerical calculations also cannot prove strongly the ergodicity due to lack of knowledge of the precise parameters of the potential. On the other hand, weak collisions (which always should be present in the real galaxy) can support the applicability of the ergodic principle if they are not strong enough to change significantly the equilibrium distribution, but are still sufficient to mix close trajectories in the phase space. After all *pro* and *contra,* considerations based on the ergodic hypothesis seem to be quit permissible and, in a sense, alternative to the methods based on the direct integration of the equations of motion.

6. CONCLUSION

Using the ergodicity hypothesis, we have shown that, under reasonable initial space and velocity distributions of young pulsars, the old neutron star population in the Galaxy can fill a torus-like region extending up to several tens kiloparsecs above the galactic plane. The general shape of this space distribution is determined mainly by the assumed galactic potential. The resulting distribution satisfies the isotropy and homogeneity tests appropriate to the gamma burst problem. At the chosen parameters, the distribution cannot simultaneously explain both the observed isotropy of gamma-burst locations on the sky, and low $\langle V/V_{max}\rangle$. However, existence of two populations of gamma-burst progenitors having different space distributions will change the picture but cannot be calculated in a proper way at present. Future improvement of the gamma-burst statistics presumably will help to resolve this problem.

Acknowledgements

The authors thank Drs. V. S. Beskin and Yu. E. Lyubarskij for valuable discussions. KAP acknowledge the support by AAS through grant 1-11-24.

References

Binney, J. and Tremaine, S. (1987). *Galactic Dynamics,* Princeton University Press, Princeton, NJ.
Bisnovatyj-Kogan, G. S. & Moiseenko, S. G. (1982). *AZh* **69,** 563.
Brainerd, J. J. (1992). *Nature,* **355,** 132.
Chugaj, N. N. (1984). *Pis'ma Astron. Zhurn.* **10,** 210.
Hartmann, D., Epstein, R. I. and Woosley, S. E. (1990). *Astrophys. J.* **348,** 525.
Imshennik, V. S. (1992). *Pis'ma Astron. Zh.* **18,** 489.
Kornilov, V. G. and Lipunov, V. M. (1983). *Atron. Zh.,* **60,** 574.
Lingenfelter, R. E. and Higdon, J. C. (1992). *Nature,* **356,** 132.
Lipunov, V. M. and Postnov, K. A. (1987). *Astrophys. Space Sci.,* **145,** 1.
Meegan, C. A. *et al.* (1992). IAU Circular No 5478.
Mitrofanov, I. G., Kozlenkov, A., Pozanenko, A. *et al.* (1992). Talk presented at the conference of Isolated Neutron Stars, February 22–28, Thaos, NM.
Paczyński, B., (1990). *Astrophys. J.* **348,** 485.
Sinaj, Ya. G. (1982). *Ergodic Theory,* Nauka, Moscow.