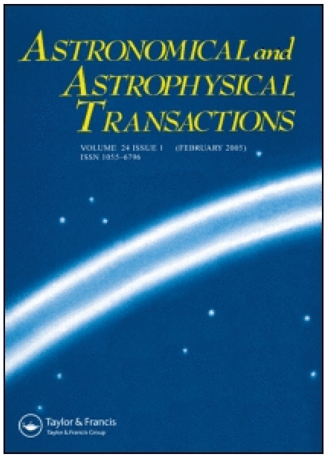

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DISC ACCRETION ONTO MAGNETIZED COMPACT OBJECTS

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In a slab jet model, the influence of a strong magnetic field and density contrasts on the development of instabilities caused by velocity contrasts is studied in application to disc accretion onto a magnetized compact object.

The perturbations propagation transverse to the magnetic field in external regions are shown not to be stable. Strong density contrast at the slab boundary ($R = \rho_{ex}/\rho_{in}$) cannot stabilize the instability of the acoustic resonance type (ARTI), the fundamental symmetric and antisymmetric modes being still unstable for any finite R. At the same time, a critical value of R exists ($R \sim 1/M^2$, M is the Mach number) at which higher reflection harmonics are stable.

We present a comparative analysis of the ARTI and the Kelvin-Helmholtz instability which develops via surface modes at the interfaces between the disc material and magnetic field (magnetosphere). The ARTI can be responsible for the penetration of the accreting material into the magnetosphere.

KEY WORDS Accretion discs, magnetohydrodynamics, hydrodynamics, instabilities.

1. INTRODUCTION

In the case of disc accretion onto magnetized compact objects (neutron stars and white dwarfs) the following situation may occur: the accretion disc (AD) material is rotating at the velocity $|\mathbf{v}| = r\Omega \gg c_{s_{in}}$ ($c_{s_{in}}$ is the sound velocity in the AD) while the disc is over-pressed by the rotating magnetic field whose field lines are parallel to the AD plane and orthogonal to $\mathbf{v}$ (see Kundt & Rubnik, 1980; Anzer *et al.*, 1987; Lipunov 1980).

Having penetrated into the magnetosphere, the matter becomes frozen into the magnetic field and rotates together with it at the velocity $r\Omega_B$. Meanwhile, the relative velocity of the material in the disc and the magnetosphere may be supersonic in a broad region of the disc ($|v_0| = r(\Omega - \Omega_B) \gg c_{s_{in}}$). The boundary between the AD and the magnetosphere is sharp ($d \ll h$, here d is the transition layer thickness and h is the half-thickness of the AD) due to magnetic pressure (Anzer *et al.*, 1988). Thus, a model of two parallel interfaces for the matter velocity, density and magnetic field contrasts seems to be appropriate.

With supersonic interfaces, the model under consideration may support at least two types of instability: the magneto-hydrodynamic instability of the acoustic resonance type (ARTI) (see Ferrari *et al.*, 1982; Hardee and Norman, 1988) if there is some matter in the magnetosphere, or unstable surface modes at the interfaces between the AD and magnetic field in vacuum if there is no matter in the magnetosphere. Following Northrop (1956), Wang & Welter (1982) and Lipunov (1978), we call the latter the Kelvin–Helmholtz instability (KHI).†

Lipunov (1978) investigated the KHI for a model of two interfaces in the limit of incompressible medium; Wang & Welter (1982) discussed a single–interface model but for a compressible fluid and for arbitrary directions of $\mathbf{v}_0$ and $\mathbf{B}$. The ARTI in the absence of magnetic field was well studied for jets in the discontinuous model (cf. Ferrari *et al.*, 1982; Hardee and Norman, 1988; Payne and Cohn, 1985; Hoperskov *et al.*, 1990) and in the model with continuous density and velocity profiles—for AD (Mustsevoj and Hoperskov, 1990). Bodo *et al.* (1989) showed that the ARTI can develop in a magnetized jet with non-magnetic environment.

The growth rate of the KHI can be estimated by the order of magnitude as $\operatorname{Im}\omega_{\mathrm{KHI}} \sim kv_A v_0/c \ll \Omega$, where $v_A^2 = B_{\mathrm{ex}}^2/(4\pi\rho_{\mathrm{in}})$. B_{ex} is the magnetic field in outer regions, ρ_{in} is the AD matter density, k is the wave number along the interfaces, and C is the speed of light (Northrop, 1956). Since the density contrast $R = \rho_{\mathrm{ex}}/\rho_{\mathrm{in}}$ can be considered small (ρ_{ex} is the density of the plasma frozen into the magnetic field above the AD), similar estimate $\operatorname{Im}\omega_{\mathrm{ARTI}} \ll \Omega$ seems to be evident. The problems which we consider here are as follows: the influence of a strong external magnetic field (and small R) on the ARTI; the relation between the growth rates of the ARTI for $R \ll 1$ and the KHI for $R = 0$ when all other relevant parameters are equal; are the mechanisms of excitation of the ARTI and KHI physically different or not?

2. THE MODEL

In the absence of magnetic field, the transverse component of the gravity force stabilizes long-wave ($kh \ll 1$) perturbations of both ARTI fundamental modes ($n = 0$, see the mode classification in Section 3) and has just no influence on higher ($n \geqslant 1$) harmonics (Mustsevoj and Hoperskov, 1990). However, this effect is significantly weaker for an AD over-pressed by magnetic field than in the case discussed by these authors because of a smaller thickness of the disc and, hence, shorter-wavelength perturbations can be important. Since the transition region between the AD and the magnetosphere is thin in comparison with the disc thickness ($d \ll h$), the gravity force does not participate in the equilibrium at the interface. For all these reasons, we should neither include the transverse component of the gravity force nor consider the perturbations of the wavelength comparable to the disc radius (but there may be $kh \ll 1$ in general).

It is known that, in non-magnetic case, the ARTI growth rate has a maximum at the frequencies such that $\Omega^2/|\omega|^2 \ll 1$ (Hoperskov *et al.*, 1990; Mustsevoj and Hoperskov, 1990). This is true in our case as well, so we neglected the effects of rotation recalling that rotation effects are significant for the perturbations with $|\omega|^2 \leqslant \Omega^2$ (of course, we have $\lambda_\parallel \ll r$, where $\lambda_\parallel$ is the perturbation wavelength along the disc plane).

† We should note that terminological inconsistency is rather traditional in this field serving to emphasize the difference between the models.

The magnitude and direction of the internal magnetic field $\mathbf{B}_{\text{in}}$ are rather difficult to estimate. A model with the magnetic field pushed out of the disc (the diamagnetic disc), that is $B_{\text{in}}=0$, seems to be the simplest one. If the magnetic field in the AD is chaotic and has a small scale being non-stationary with the time scale $\tau \gg 1/|\omega|$ (recombination of field lines, α-dynamo, etc.), then B_{in} contributes into the pressure balance only and modifies the gas equation of state, so that the adiabatic speed of sound $c_{s_{\text{in}}}$ has to be replaced by $c_{\perp_{\text{in}}}=[c_{s_{\text{in}}}^2+B_{\text{in}}^2/(4\pi\rho_{\text{in}})]^{1/2}$. In the opposite case of $\tau \lesssim 1/|\omega|$ when the internal magnetic field is strong enough ($B_{\text{in}} \lesssim B_{\text{ex}}$), our further analysis is not valid. Under certain conditions, a large-scale quasi-stationary magnetic field can be present in the AD possessing having a dominant azimuthal component (Lominadze *et al.*, 1985), i.e., parallel to the velocity. In the present paper we restrict ourselves to the simplest case of $B_{\text{in}}=0$ and small-scale $B_{\text{in}}\neq 0$ when considering the ARTI; the case of azimuthal field B_{in} is a subject of another paper.

Thus we consider homogeneous layers as a model for the analysis of ARTI adopting the following equilibrium distributions of velocity, density, sound speed and magnetic field:

$$v(z);\, \rho(z);\, c_s(z);\, B(z)=\begin{cases} v_0;\, \rho_{\text{ex}};\, c_{s_{\text{ex}}} & \text{at } |z|>h, \\ 0;\, \rho_{\text{in}};\, c_{s_{\text{in}}};\, B_{\text{in}} & \text{at } |z|<h, \end{cases} \tag{1}$$

and $\mathbf{v}_0 \perp \mathbf{B}_{\text{ex}}$.

The equilibrium pressure balance is given by

$$p_{\text{ex}}+\frac{B_{\text{ex}}^2}{8\pi}=p_{\text{in}}+\frac{B_{\text{in}}^2}{8\pi}, \tag{2}$$

where p_{ex} and p_{in} are the equilibrium hydrodynamic pressures ($\gamma p_{\text{ex;in}}=\rho_{\text{ex;in}}c_{s_{\text{ex;in}}}^2$ and γ is the specific heats ratio).

Note that the case $\rho_{\text{ex}}=0$ cannot be considered in the framework of magnetohydrodynamics; displacement current can be neglected, i.e., magnetohydrodynamic equations are applicable only when (Landau and Liftshitz, 1982)

$$B_{\text{ex}}^2 \ll \rho_{\text{ex}}c^2. \tag{3}$$

Wang and Welter (1982) gave a detailed derivation of the dispersion equation describing small oscillations of the interface between magnetic field in vacuum and moving MHD-fluid. Since the transition from their consideration to the model with two interfaces is evident enough, we shall give it in Section 4 without comments, noting that we have to assume $\rho_{\text{ex}}=0$ in (1), and to substitute the speed of light c instead of $c_{s_{\text{ex}}}$; in addition, Eq. (2) is modified by replacing p_{ex} by (Wang and Welter, 1982)

$$-\frac{E_{\text{ex}}^2}{8\pi}=-\frac{[\mathbf{v}_0\mathbf{B}_{\text{ex}}]^2}{8\pi c^2}. \tag{4}$$

Our consideration is valid outside the inner region of the AD, that is al larger distances from the central object and for $v_0^2 \ll c^2$.

3. THE ACOUSTIC RESONANCE TYPE INSTABILITY

The following dispersion equation† comes from linearized non-dissipative MHD equations for small perturbations whose Fourier harmonics have the form $f \sim \exp\{ik_x x + ik_y y + \varkappa \bar{z} - i\omega t\}$:

$$[\mathrm{th}(kh\beta_{\mathrm{in}})]^{\alpha} = -\frac{Z_{\mathrm{in}}}{Z_{\mathrm{ex}}}, \tag{5}$$

where

$$k = (k_x^2 + k_y^2)^{1/2}, \qquad Z_{\mathrm{in}} = \frac{z^2 - A_{\mathrm{in}}^2}{z\beta_{\mathrm{in}}}, \qquad Z_{ex} = \frac{R[(z-M)^2 - A_{\mathrm{ex}}^2]}{z\beta_{\mathrm{ex}}},$$

$$\beta_{\mathrm{in}} = \left(1 - \frac{z^4}{z^2(a_{\mathrm{in}}^2 + 1) - A_{\mathrm{in}}}\right)^{1/2}, \qquad \beta_{\mathrm{ex}} = \left(1 - \frac{s(z-M)^4}{(z-M)^2(sa_{\mathrm{ex}}^2 + 1) - A_{\mathrm{ex}}^2}\right)^{1/2},$$

$$z = \frac{\omega}{kc_{s_{\mathrm{in}}}}, \qquad M = \frac{(\mathbf{kv}_0)}{kc_{s_{\mathrm{in}}}}, \qquad s = \frac{c_{s_{\mathrm{in}}}^2}{c_{s_{\mathrm{ex}}}^2}, \qquad R = \frac{\rho_{\mathrm{ex}}}{\rho_{\mathrm{in}}},$$

$$a_{\mathrm{in}} = -\frac{B_{\mathrm{in}}}{(4\pi\rho_{\mathrm{in}})^{1/2} c_{s_{\mathrm{in}}}}, \qquad a_{\mathrm{ex}} = \frac{B_{\mathrm{ex}}}{(4\pi\rho_{\mathrm{ex}})^{1/2} c_{s_{\mathrm{in}}}},$$

$$A_{\mathrm{in}} = -\frac{(kB_{\mathrm{in}})}{(4\pi\rho_{\mathrm{in}})^{1/2} kc_{s_{\mathrm{in}}}}, \qquad A_{\mathrm{ex}} = \frac{(kB_{\mathrm{ex}})}{(4\pi\rho_{\mathrm{ex}})^{1/2} kc_{s_{\mathrm{in}}}}.$$

Here Z_{in} and Z_{ex} are the normal impedances k is the wave vector in the AD plane, $\beta_{\mathrm{in}} = \varkappa_{\mathrm{in}}/k$ and $\beta_{\mathrm{ex}} = \varkappa_{\mathrm{ex}}/k$ are the dimensionless wave numbers transverse with respect to the AD, $\mathrm{Re}\, z$ is the dimensionless phase velocity of the perturbation, and $\mathrm{Im}\, z$ is its growth rate. The amplitude does not grow at infinity if we select $\mathrm{Re}\, \beta_{\mathrm{ex}} > 0$.

In our notation, Eq. (2) reduces to the following:

$$s = \frac{R}{1 + \frac{\gamma}{2}(a_{\mathrm{in}}^2 - Ra_{\mathrm{ex}}^2)}. \tag{6}$$

When $\alpha = 1$, Eq. (5) describes even perturbed pressure functions in the middle layer, i.e., the symmetric (S) mode; $\alpha = -1$ corresponds to odd functions, i.e. the antisymmetric (AS) mode. The spectrum of eigenfrequencies defined by Eq. (5) is discrete. The perturbation harmonics corresponding to these frequencies are usually treated as fundamental modes ($\eta = 0$ for the S-modes and $\eta = 1$ for the AS-modes) and reflection ones ($\eta \geqslant 2$) depending on the number of the nodes of the perturbed pressure eigenfunction, η, between the interfaces. For the S-modes, $\eta = 2n$, for the AS-modes, $\eta = 2n + 1$, with n being the harmonics order.

The number of unstable modes described by Eq. (5) is, strictly speaking, infinite. The transition region has a finite (though small) thickness thus leading to

† Here we suppose the disc to be at rest and the outer regions to move. In the opposite case, it is sufficient to introduce usual Doppler shift of the frequency.

a significant (but not total) stabilization of the short-wavelength ($kh \gg 1$), higher harmonics ($n \gg 1$) (Mustsevoj and Hoperskov, 1990), so that we may consider the harmonics with $n = 1, 2, 3$, only.

3.1. *Incompressible Fluid*

Before we specify the magnitude and direction of the magnetic field according to Section 2, we shall write out solutions of Eq. (5) under the only assumption of incompressibility ($c_{s_{\rm in}}$ large enough). Although this case is rather unrealistic since $M \gg 1$ for a real AD, we shall need these results to compare them with those of Section 3.2 and 4.

Putting in Eq. (5) $|z|^2 \ll 1$, $M \ll 1$ and $s|z - M|^2 \ll 1$, we find:

$$z \simeq \frac{1}{M+1}[NM \pm ((N+1)(NA_{\rm ex}^2 + A_{\rm in}^2) - NM^2)^{1/2}], \tag{7}$$

where $N = R\,\mathrm{th}^{\alpha}(kh)$. An obvious stability condition follows from Eq. (7): $(N+1)(NA_{\rm ex}^2 + A_{\rm in}^2) > NM^2$. In the limit $kh \to \infty$ ($N \to R$) this meets the one obtained by Syrovatskij (Landau and Lifshitz, 1982).

Unstable roots of Eq. (7) (those having "+" at the square root) correspond to the fundamental ($n = 0$) harmonics of S- and AS-modes, into which the single-interface Kelvin–Helmholtz mode splits in the case of two interfaces. The bending (AS-mode) perturbations are, evidently more unstable.

3.2. *The case of* $a_{\rm in} = 0$

In real accretion systems, the density contrast between the disc and the magnetosphere probably exceeds significantly the temperature constract: $R \ll s$.† In this case Eq. (6) implies that $a_{\rm ex}^2 \simeq 2/(\gamma R)$. $A_{\rm ex} = 0$ for the perturbations propagating across the magnetic field $\mathbf{B}_{\rm ex}$ which are, consequently, the most unstable ones. It is easy to see that in this case Eq. (5) is equivalent (within the factor $\gamma/2 \sim 1$ at the second term under the square root in $\beta_{\rm ex}$) to the non-magnetic case (cf. Hopersokv *et al.*, 1990).

Numerical solution of Eq. (5) demonstrates a good qualitative agreement of the solution considered and the hydrodynamic one for arbitrary M, R and kh. But the dependence of the eigen-frequencies on these parameters at $R \ll 1$ is rather complicated and needs some comment.‡

First of all, we note that the development of reflection modes ($n \geq 1$) is energetically allowed, with a negative wave energy flux outgoing into the magnetosphere. Miles (1957) and Ribner (1957) showed that the energy flux in the neutral waves going from the interface can be negative (that is, directed to the interface) whenever (in our notation) $1 < z < M - (a_{\rm ex}^2 + 1/s)^{1/2}$, that is in the dispersive range of frequencies. This range is slightly wider for unstable preturbations, the upper limit going higher—see Figure 1 (see Mustsevoj, 1991,

† Note that Eq. (5) is valid only if (3) is satisfied.

‡ The case of very small R seems to be hardly realistic for a hydrodynamic model, for such a density contrast corresponds to a very strong temperature constrast (since $s = R$), so that such a model is thermally unstable.

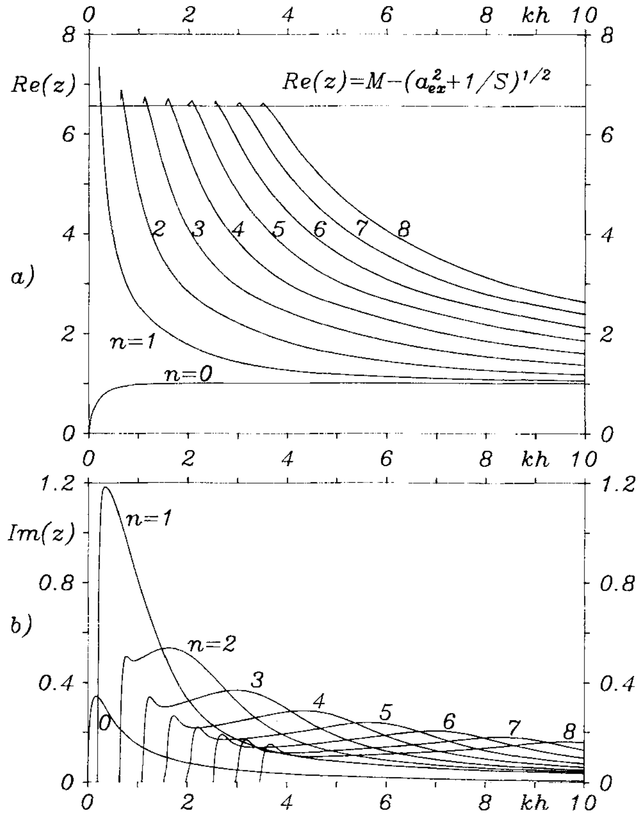

Figure 1 Dimensionless phase velocities (a) and growth rates (b) against kh for the symmetric mode. Fundamental ($n=0$) and first 8 reflection harmonics are shown. $M=10$, $R=0.1$, $S=1$, $\mathbf{k} \parallel \mathbf{v}_0$, $\mathbf{k} \perp \mathbf{B}_{\mathrm{ex}}$.

for more details). In the reactive range of frequencies $-M-(a_{\mathrm{ex}}^2+1/s)^{1/2} \leqslant z \leqslant M$ and $0 \leqslant z \leqslant 1$—the perturbations appear to be the surface ones (i.e., exponentially decaying along the transverse coordinate without oscillations) whose fundamental modes have another development mechanism; the reflection harmonics can never exist in this region. These arguments allow to interpret easily the form of the growth rate, $\mathrm{Im}\, z$, and phase velocity, $\mathrm{Re}\, z$, contour plots in the $R-kh$ plane (Figures 2–5).

The cut in the contour plots of the reflection harmonics (Figures 2–3) corresponds to the value of Rat which the reactive regions overlap, that is $1+(a_{\mathrm{ex}}^2+1/s)^{1/2} \equiv M$ (Eq. (5) is satisfied identically at $z \equiv 1$ for S, as well as AS-modes) and no reflection modes can exist at lower R. Note that the $\mathrm{Im}\, z$ and $\mathrm{Re}\, z$ surfaces are polyvalent: the fundamental S-mode ($n=0$) splits into two neutral branches while crossing the cut from higher to lower R's, and while crossing the cut from lower to higher R's we come from the S-mode $n=0$ harmonics onto the surface of the reflection S-harmonics with n the greater the higher kh is at which we cross the cut. Below the cut (at lower kh), only the fundamental mode exists. The AS-mode behaves in the same way.

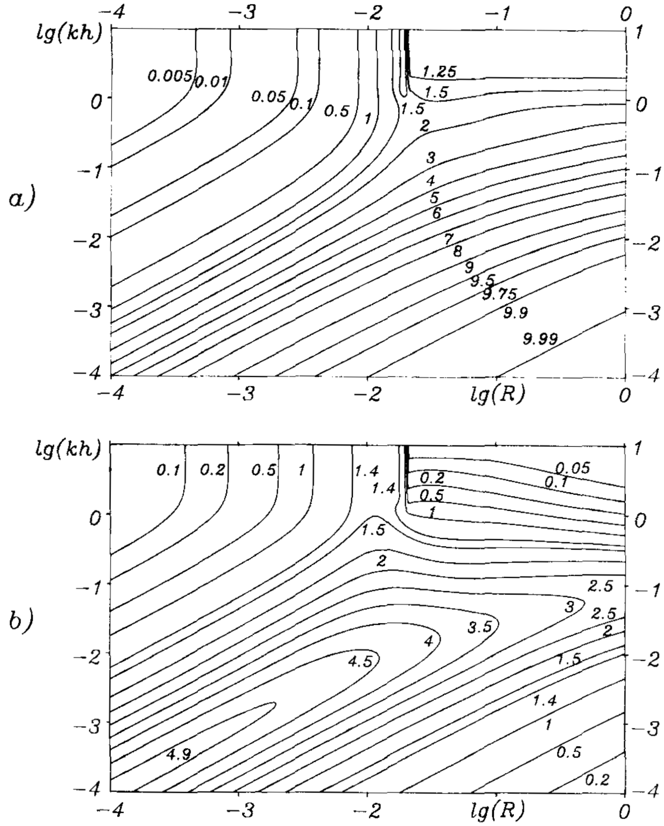

Figure 2 Contour plots of dimensionless phase velocities (a) and growth rates (b) in the $R - kh$ plane velocities (a) and growth rates (b) in the $R - kh$ plane for the fundamental ($n = 0$) AS-mode. $M = 10$, $S = 1$, $\mathbf{k} \parallel \mathbf{v}_0$, $\mathbf{k} \perp \mathbf{B}_{ex}$.

Figure 4 shows $\operatorname{Im} z$ contour plots for the reflection S-mode harmonics $n = 1, 2$ (AS-mode harmonics behave in the same way); the ranges of kh are shown for which a given harmonics can be obtained from the $n = 0$ one. These ranges lie between the first order branching points (see Figures 5a, b). Going clockwise around the mth branching point ($m = 1, 2, 3, \ldots$) at some small distance ε from it we should subsequently get (for a continuous, closed path) from the $\operatorname{Im} z$ (or $\operatorname{Re} z$, respectively) surface of the S-mode (or AS as well) $n = 0$ harmonics to the $n = m - 1$ harmonics, then to the $n = m$ harmonics of same mode, and return to the starting point of the $n = 0$ harmonics completing two full circles† (that is, a combination of Figures 2–5 appears to be, in fact, a Riemannian surface of the multiciphered function $z(R, M, kh)$).

A sharp distinction in the fundamental mode behavior to the left and to the right from the cut (Figures 2–3) is noticeable. It can be explained by the fact that

† Strictly speaking, this occurs for $m = 2$ and higher while for the first ($m = 1$) branching point we come to the neutral mode instead of the $n = m - 1$ harmonics.

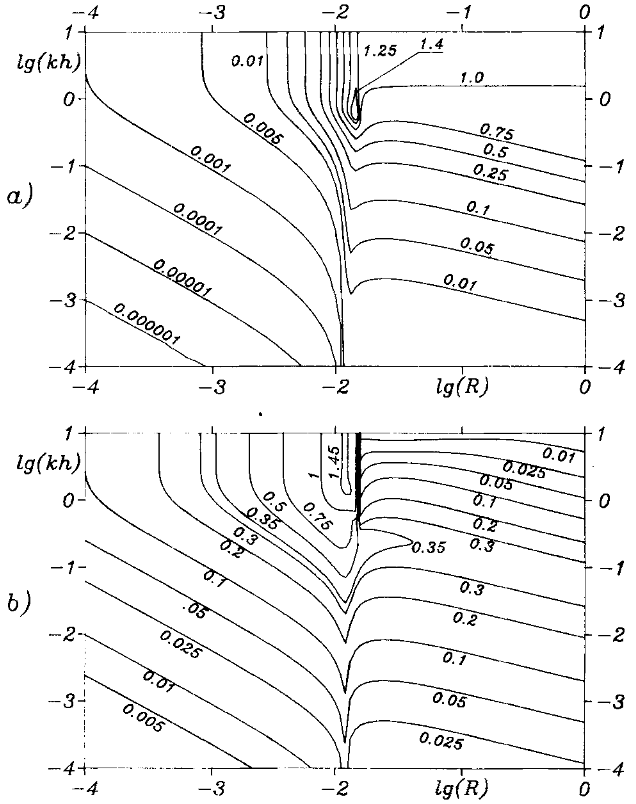

Figure 3 Contour plots of dimensionless phase velocities (a) and growth rates (b) in the R–kh plane for the fundamental ($n = 0$) S-mode. $M = 10$, $S = 1$, $\mathbf{k} \parallel \mathbf{v}_0$, $\mathbf{k} \perp \mathbf{B}_{ex}$.

for R low enough to ensure that $M < (a_{ex}^2 + 1/s)^{1/2}$, the perturbations in the magnetosphere become subsonic and for $M \ll (a_{ex}^2 + 1/s)^{1/2}$ the matter frozen into the magnetosphere behaves as an incompressible fluid. It is reasonable to expect that in this range of parameters solutions of Eq. (5) are similar to those of Eq. (7). In fact, Eq. (7) is an asymptotic solution of Eq. (5) that agrees well with the numerical solution in this ($M \ll (a_{ex}^2 + 1/s)^{1/2}$) range. Thus, we can state that for the values of R corresponding to the above range, the instability arises due to the Kelvin–Helmholtz mechanism (the Bernoulli effect) despite a strong compressibility in the disc ($M \gg 1$).

It is possible to obtain the analytical asymptotics of the eigen frequencies in the opposite limit of $M \gg (a_{ex}^2 + 1/s)^{1/2}$ when the compressibility of matter is considerable and the instability is of the acoustic resonance type.

For the range of short and medium wavelength ($kh \gtrsim 1$), supposing $RM^2 \gg 1$ and $M^2 \gg A_{ex}^2$ we can obtain from Eq. (5) for the S-mode $n = 0$ harmonics:

$$z \simeq 1 - \frac{1}{2(b^2+1)} + \frac{ib}{2(b^2+1)}, \tag{8}$$

where $b = C^{1/2} Mkh(1 - A_{ex}^2/M^2)$ and $C = R^2(a_{ex}^2 + 1/s)$.

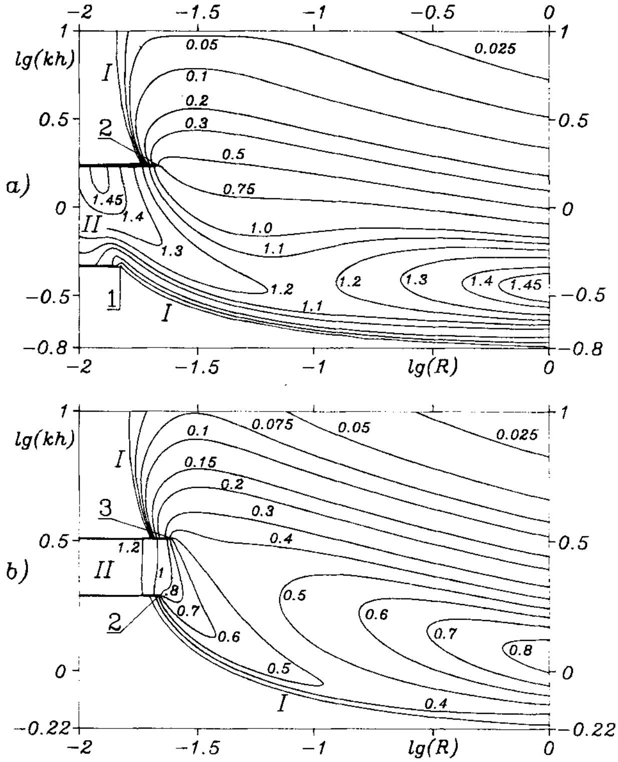

Figure 4 Contour plots of dimensionless growth rates for the $n = 1$ (a) and $n = 2$ (b) harmonics of the S-mode in the $R - kh$ plane. I denotes the marginal stability curves, II, the region where the reflection mode turns into the fundamental one. 1, 2, 3 are the branching points. $M = 10$, $S = 1$, $\mathbf{k} \parallel \mathbf{v}_0$, $\mathbf{k} \perp \mathbf{B}_{ex}$.

For the long-wavelength ($kh \ll 1$) perturbations of that mode we get under the same assumption:

$$z \simeq \left(\frac{b}{b^2+1}\right)^{1/2}\left[\left\{\frac{(b^2+1)^{1/2}+b}{2}\right\}^{1/2} + i\left\{\frac{(b^2+1)^{1/2}-b}{2}\right\}^{1/2}\right]. \tag{9}$$

In the same range ($M \gg 1$, $Mkh \ll 1$), for the AS-mode $n = 0$ harmonics, an expression follows from Eq. (5) that coincides in form with Eq. (7), but now we have $N = R/kh$.

For the reflection ($n \geqslant 1$) harmonics of both modes and the fundamental ($n = 0$) of the AS-mode at $kh \geqslant 1$, $|z|^2 \gg 1$, $R|z - M|^2 \gg 1$ and $|z - M|^2 \gg A_{ex}^2$, it follows from Eq. (5):

$$z \simeq z_0\left\{1 + \frac{z_0 - M}{iC^{1/2}kh[(z_0 - M)^2 - A_{ex}^2] - 2z_0 + M}\right\}, \tag{10}$$

where

$$z_0 = \left(\frac{n^2\pi^2}{k^2h^2} + 1\right)^{1/2} \qquad \text{for the S-mode}$$

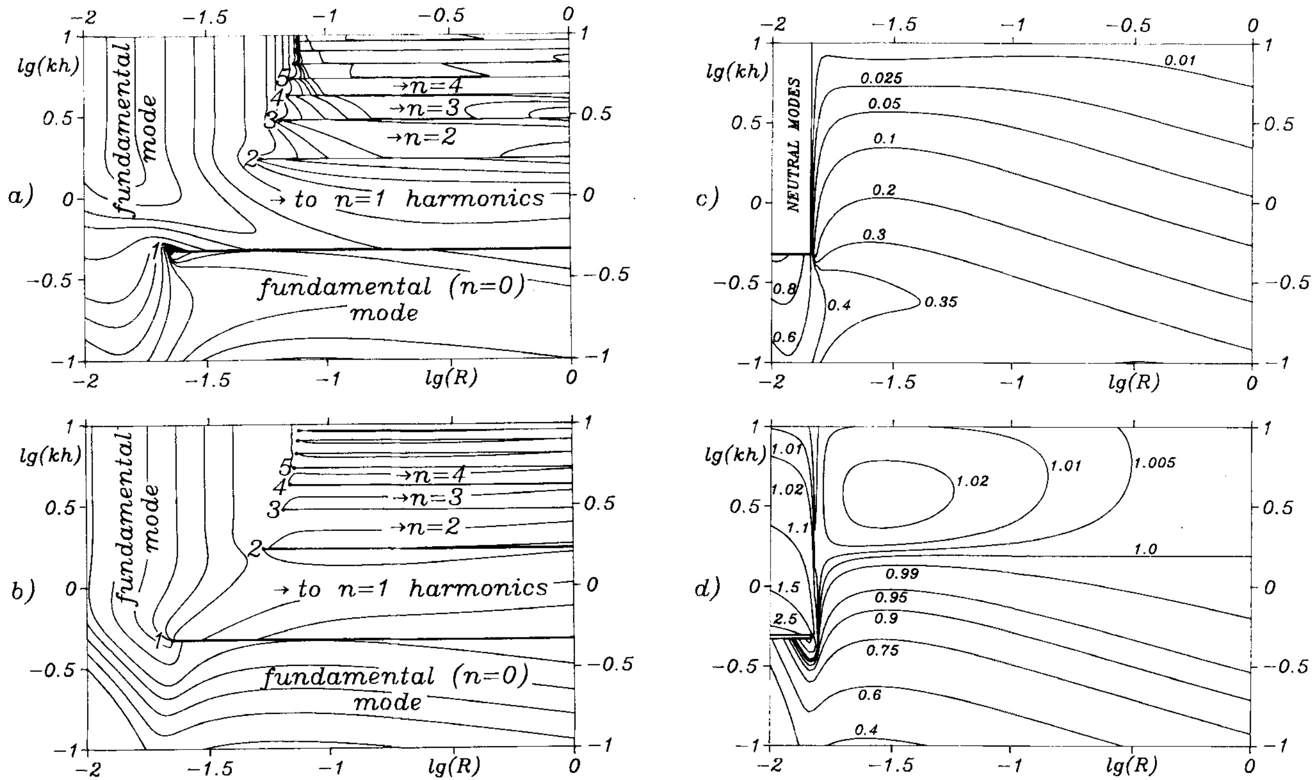

Figure 5 Contour plots of dimensionless growth rates (a) and phase velocities (b) in the $R-kh$ plane for the S-mode harmonics in the vicinity of branching points. The results of calculation from lower to higher R's are shown. $M=10$, $S=1$, $\mathbf{k} \parallel \mathbf{v}_0$, $\mathbf{k} \perp \mathbf{B}_{ex}$. (c) and phase velocities (d) in the $R-kh$ plane for the fundamental ($n=0$) S-mode in the vicinity of the branching point. The results of calculation from higher to lower R's are shown. $M=10$, $S=1$, $\mathbf{k} \parallel \mathbf{v}_0$, $\mathbf{k} \perp \mathbf{B}_{ex}$.

and

$$z_0 = \left(\frac{(n+1/2)^2\pi^2}{k^2h^2} + 1\right)^{1/2} \quad \text{for the AS-mode}$$

Note that, in the particular case of $A_{ex} = 0$ at $a_{ex} \neq 0$, Eq. (10) coincides with a similar asymptotics obtained by Morozov *et al.* (1991) (where a similar model but without magnetic field was discussed) with C being replaced by R. The requirement of $s > 0$ (with account of (6)) necessarily yields the following inequality:

$$\frac{\gamma}{2} < \frac{R}{C} \leq 1, \tag{11}$$

thus we can state that for $\mathbf{k} \parallel \mathbf{v}_0$, in the parameter range considered, magnetic field diminishes the ARTI growth rate not more than by the factor $(2/\gamma)^{1/2} \sim 1$.

Eqs. (8) and (10) are valid for the values of M and kh exceeding those at which the growth rate is maximal. The estimate of the maximal growth rate can be obtained by analyzing reflection coefficients at the interfaces. The complex reflection coefficient (in pressure) at the interface in the model discussed is the following:

$$\mathcal{R} = \frac{Z_{ex} - Z_{in}}{Z_{ex} + Z_{in}}. \tag{12}$$

Substituting the dispersion Eq. (5) into (12) one can show that

$$\mathcal{R} = \pm\exp\{2kh(1 - z^2)^{1/2}\}, \tag{13}$$

with the "+" sign corresponding to S, "−" to AS-modes.

Since the reflection harmonics ($n \geq 1$) have a high frequency ($|z|^2 \gg 1$), it follows from Eq. (13) for them:

$$\operatorname{Im} z \simeq \frac{1}{2kh} \ln|\mathcal{R}|. \tag{14}$$

Equation (14) shows that the growth rate is maximal at the value of $|\mathcal{R}|$ which is the largest for a given harmonics.

Miles (1957) and Ribner (1957) showed that vanishing of the denominator of (12) is a necessary condition for over-reflection ($|\mathcal{R}| \to \infty$). Naturally, this condition can be satisfied only for neutral oscillations ($\operatorname{Im} z \equiv 0$)† and implies that the frequency at which this occurs is the eigen-frequency for a single ($kh \to \infty$) interface. Since we are interested in solutions with $\operatorname{Im} z > 0$, $|\mathcal{R}|$ may be large though always finite.

The main resonance angle at which the maximum of $|\mathcal{R}|$ (and of the growth rate) occurs corresponds to that root of equation $Z_{in} + Z_{ex} = 0$ at which $\varkappa_{in} \sim \varkappa_{ex}$ (that root is $\operatorname{Re} z \simeq M/(1 + 1/R^{1/2})$ at $a_{ex} \equiv 0$). Supposing $|z|^2 \gg 1$, $R\,|z - M|^2 \gg 1$

† We consider an essentially supersonic case ($M \gg 1$) when the reflection harmonics exist. At $M \leq 1$, to the contrary, the denominator of (12) turns into zero for a single complex-conjugate pair of z only.

and $|z - M|^2 \gg A_{ex}^2$, we find the corresponding value of Re z:

$$\operatorname{Re} z = z_R \simeq \frac{M}{2}\frac{2C^{1/2}+1}{C^{1/2}+1}[1-(1-4D)^{1/2}], \tag{15}$$

where $D = (1 - A_{ex}^2/M^2)C^{1/2}(C^{1/2}+1)/(2C^{1/2}+1)^2$.

Substituting (15) into (12) represented in the form $z \simeq z_R + \operatorname{Im} z$, where $\operatorname{Im} z \ll z_R$, we obtain an approximate expression for the "envelope" curve of the growth rate maxima for different harmonics of both modes:

$$\max(\operatorname{Im}\omega) \simeq \frac{c_{s_{in}}}{2h}\ln\left|\frac{2C^{1/2}Mkh}{(C^{1/2}+1)^2}\left(1 - \frac{1-2D(2C^{1/2}+1)/C^{1/2}}{(1-4D)^{1/2}}\right)\right|. \tag{16}$$

Finally, we can get an asymptotic formula describing the marginal stability boundary for different reflection harmonics. The nth harmonics is stabilized at the following combination of parameters:

$$\nu \simeq M_* - \frac{1}{2^{1/2}}\left\{a_{ex*}^2 + \frac{1}{s_*} + \left[\left(a_{ex*}^2 + \frac{1}{s_*}\right)^2 - 4\frac{A_{ex*}^2}{s_*}\right]^{1/2}\right\}^{1/2}. \tag{17}$$

Here $\nu = (\eta^2\pi^2/k_*^2h_*^2 + 1)^{1/2}$, $\eta = 2n-1$ for S-modes, $\eta = 2n$ for AS-modes, $n \geqslant 1$. The accuracy of Eq. (17) grows with n. In Figure 4, the dashed line shows the marginal stability boundary according to (17).

Analytical and numeric results show that for the most unstable harmonics (AS-mode $n = 0$ and $n = 1 \div 3$ of both modes) the maximum of In z occurs at such z that $|z|^2 \gg 1$. Besides, we started supposing $kr \gg 1$ (it is more correct to say that the assumption $\lambda_{\|} \ll r$ made in Section 2 leads to stronger restriction: $kr \gg 2\pi$). Meanwhile, the Mach number maximum (at r fixed and $\mathbf{k} \parallel \mathbf{v}_0$) never exceeds $M_{max}(r) = r\Omega/c_{s_{in}}$ and $\Omega(r) \sim r^{-3/2}$ for the regions far from the central object, so the inequality $kr \gg r\Omega/c_{s_{in}}$ is valid. Therefore, $\Omega^2/|\omega|^2 = M_{max}(r)/(|z|\,kr)^2 \ll 1$ and so the effects of rotation can be neglected indeed, as assumed in Section 2.

3.3. *The Case of a Small-Scale Field* a_{in}

If $a_{in} \neq 0$ but the field has a small scale as compared to the perturbation wavelength, we can assume $A_{in} = 0$ in (5). Obviously, this case can be reduced to the one discussed above through transformation of variables:

$$z \to z' = \frac{\omega}{kc_{\perp in}}; \qquad M \to M' = \frac{(\mathbf{k}\mathbf{v}_0)}{kc_{\perp in}}; \qquad s \to s' = \frac{c_{\perp in}^2}{c_{s_{ex}}^2};$$

$$a_{ex} \to a'_{ex} = \frac{B_{ex}}{(4\pi\rho_{ex})^{1/2}c_{\perp in}}; \qquad A_{ex} \to A'_{ex} = \frac{(\mathbf{k}\mathbf{b}_{ex})}{(4\pi\rho_{ex})^{1/2}kc_{\perp in}};$$

where $c_{\perp in} = c_{s_{in}}(1 + a_{in}^2)^{1/2}$.

4. THE KELVIN–HELMHOLTZ INSTABILITY ($\rho_{ex} = 0$)

The dispersion equation describing the KHI in the double-interface model coincides in form with Eq. (5) if we introduce the following notation:

$$Z_{ex} = \frac{a_{ex}^2\delta^2(z^2 - 2Mz\sin\theta_{Bk} + M^2) - A_{ex}^2}{z\beta_{ex}},$$

$$Z_{in} = \frac{z^2(1+\delta^2 a_{in}^2) - A_{in}^2}{z\beta_{in}}, \qquad \beta_{in} = \left(1 - z^2\frac{z^2(1+\delta^2 a_{in}^2) - \delta^2 A_{in}^2}{z^2(1+a_{in}^2) - A_{in}^2}\right)^{1/2},$$

$$\beta_{ex} = (1 - z^2\delta^2)^{1/2}, \qquad a_{ex} = \frac{B_{ex}}{(4\pi\rho_{in})^{1/2}c_{s_{in}}}, \qquad A_{ex} = \frac{(\mathbf{kb}_{ex})}{(4\pi\rho_{in})^{1/2}kc_{s_{in}}},$$

$$M = \frac{v_0}{c}s_{in}\sin\theta_{vB}, \qquad \delta = \frac{c_{s_{in}}}{c}, \qquad \sin\theta_{Bk} = \frac{[\mathbf{kB}_{ex}]}{kB_{ex}}, \qquad \sin\theta_{vB} = \frac{[\mathbf{v}_0\mathbf{B}_{ex}]}{v_0 B_{ex}}.$$

The variables z, a_{in} and A_{in} are defined as above.

The condition (2) modified as discussed in Section 2 is

$$2/\gamma + a_{in}^2 = a_{ex}^2(1 - \delta^2 M^2). \tag{18}$$

The dispersion equation being considered (just like in Sections 3.1 and 3.2) can have two roots corresponding to the unstable $n = 0$ harmonics of both modes in incompressible case ($c_{s_{in}} \to \infty$), as well as for arbitrary compressibility in the disc. Supposing $|z| \ll 1$ and $M\delta \ll 1$ we obtain:

$$z \simeq \frac{LM\sin\theta_{Bk} \pm F^{1/2}}{L + 1 + \delta^2 a_{in}^2}, \qquad L = \mathrm{th}^{\alpha}(kh)\delta^2 a_{ex}^2, \tag{19}$$

$$F = \left(\frac{LA_{ex}^2}{\delta^2 a_{ex}^2} + A_{in}^2\right)(L + 1 + \delta^2 a_{in}^2) - LM^2\left(L\frac{A_{ex}^2}{a_{ex}^2} + 1 + \delta^2 a_{in}^2\right).$$

The values of z given by Eq. (19) agree well with the exact solutions of the dispersion equation for any reasonable ranges of parameters. It is evident from (19) that instability occurs if $F < 0$. For $kh \to \infty$ (single interface) this conditions coincides with the one obtained by Wang and Welter (1982).

The comparison of (19) and (7) exhibits a similar character of the instabilities arising in these models despite distinctions in the latter. This similarity occurs when the matter in the magnetosphere (in the ARTI case) behaves like an incompressible fluid (the range of $R < [M^2\gamma/2 - (\gamma/2 - 1)/s]^{-1}$ for both ARTI fundamental modes and, besides, the range of $kh \ll 1$ at any R for AS fundamental mode) and arises because magnetic field in vacuum is equivalent to an incompressible fluid with the density $\rho_m = B_{ex}^2/(4\pi c^2)$.

In the model considered, there can exist higher ($n \geq 1$) harmonics, but they appear to be neutral. Thus, supposing $|z|^2 \gg 1 + a_{in}^2$ (there exist no higher harmonics in the opposite limit), we obtain the solution of the form, $z = z_0(1 + \Delta)$, where:

$$z_0 \simeq (1 + a_{in}^2)\left[\frac{\eta^2\pi^2}{k^2h^2} + 1\right],$$
$$\Delta \simeq -\frac{a_{ex}^2\delta^2(z_0^2 - 2Mz_0\sin(\theta_{Bk}) + M^2) - A_{ex}^2}{kh(z_0^2 - A_{in}^2) - A_{ex}^2}, \tag{20}$$

$\eta = n + 1/2$ for S-modes, $\eta = n$ for AS-modes, $n \geqslant 1$. This result is quite clear. In the case of the ARTI over-reflection of waves at the interfaces (accompanied by the development of unstable reflection harmonics) is possible, since energy flux in outgoing (into magnetosphere) transmitted wave appears to be directed to the interface. In the case of the KHI, this is impossible. In fact, in the reference frame co-moving with the disc material, electric field in vacuum is $\mathbf{E}_{ex} = [\mathbf{v}_0\mathbf{B}_{ex}]/c$, and hence the Pointing vector is:

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{E}_{ex}\mathbf{B}_{ex}] = \frac{1}{4\pi}[[\mathbf{v}_0\mathbf{B}_{ex}]\mathbf{B}_{ex}] \equiv 0. \tag{21}$$

5. DISCUSSION

Thus, the local short-wavelength analysis shows that in the case of disc accretion onto a magnetized compact object in the AD, besides the KHI arising at the interfaces between the AD material and magnetic field in vacuum (e.g. Northrop, 1956; Wang and Welter, 1982; Lipunov, 1978), there can exist also a magnetohydrodynamic ARTI, for whose development the presence of matter in the magnetosphere is essential.

Then an interesting effect occurs: the unstable modes of the KHI and ARTI demonstrate a strong similarlity in behavior (with no dependence on the disc material compressibility) when the medium in the magnetosphere (magnetic field in a vacuum for the KHI and matter for the ARTI) acts like an incompressible fluid, which is true for any range of parameters in the case of the KHI. In the case of the ARTI, this situation appears whenever the fast magnetosonic velocity in the magnetosphere material exceeds the relative velocity of the material in the disc and magnetosphere and, besides, when the bending oscillation wavelength is long enough ($kh \ll 1$). Therefore, only the fundamental modes exist for both instability types (the higher harmonics are neutral), the bending mode (AS-modes with $n = 0$) being most unstable. It is remarkable that the frequencies of these modes are described by analytical expressions obtained in the incompressible limit (for disc material $-c_{s_{in}} \to 0$) even if the disc material compressibility is large ($M \gg 1$). In other words, this allows us to suggest that the same mechanism is responsible for the KHI and ARTI development at $M \ll 1$ as well as at $M \gg 1$.

This effect is due to a subsonic character of the perturbations in the magnetosphere in all cases considered above (inhomogeneous plane waves, that is surface modes) and subsequently, there is no energy flux along the $\tilde{z}$ coordinate orthogonal to the disc plane. Inside the AD the energy flux in this direction is also zero in that case, since the waves are stationary in the $\tilde{z}$-coordinate (unstable modes of a wave guide). Consequently, the only energy source remaining for the instability is the pressure forces at the interfaces (the Kelvin–Helmholtz mechanism by itself—the Bernoulli effect).

The ARTI mechanism appears to be fundamentally different only when the compressibility of material in the magnetosphere is important and the perturbations are supersonic. Then higher unstable harmonics can be supported, their growth rates being of the same order as that of the fundamental bending mode (and, therefore, exceeding the fundamental S-mode growth rate), but for shorter wavelengths (or higher n).

We should to emphasize some points we believe to be important in the results obtained as applied to AD stability.

1. First of all, the ARTI growth rate exceeds the KHI one even at $\rho_{cx} \ll \rho_{in}$ ($\mathrm{Im}\, \omega_{KHI} < \mathrm{Im}\, \omega_{ARTI}$ cf. (7) and (19)).

2. In the model being discussed ($\mathbf{v}_0 \perp \mathbf{B}_{ex}$), external magnetic field of arbitrary magnitude does not stabilize the most "dangerous" perturbations (with $\mathbf{k} \parallel \mathbf{v}_0$) both for KHI and the ARTI. It seems to be significant that the perturbations with $\mathbf{k} \parallel \mathbf{v}_0$ have the shortest (at fixed k) azimuthal wavelength and, subsequently, suffer less from gyroscopic effects.

3. A good qualitative agreement in the results considered above and those for the hydrodynamic cases allows to state, according to Mustsevoj and Hoperskov (1991), that the presence of matter in the magnetosphere is essential only in the layer of the thickness of the order of $1/|\varkappa_{ex}|$ close to the disc, the ARTI growth rate being poorly sensitive to the presence of material elsewhere in magnetosphere.

4. In the case of AD, non-linear growth of the fundamental bending mode cannot lead to initial flow disruption (unlike the jets—see Norman and Hardee, 1988) since the disc material is in a potential well and, hence, the fluid particle transverse motion is finite.

One of the problems of accretion onto magnetized compact objects is the way for plasma to penetrate into the magnetic field surrounding the accreting object. The following scenario of this process can be proposed. At the initial stage, when there is no material in the magnetosphere, the KHI (or another instability—see Lipunov, 1987) can act as a starting mechanism. As it develops, the density in the magnetosphere regions close to the disc becomes finite, and the fundamental bending mode of the ARTI having the maximum growth rate arises. If the density in the above regions grows, as a result, to the values of order $\rho_{ex} \sim 2p_{in}/v_0^2$, then, in addition, the short-wavelength reflection harmonics of S- and AS-modes becomes arise which, in turn, can lead to turbulence in the disc due to the appearance of a hierarchy of spatial scales and perturbation frequencies.

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