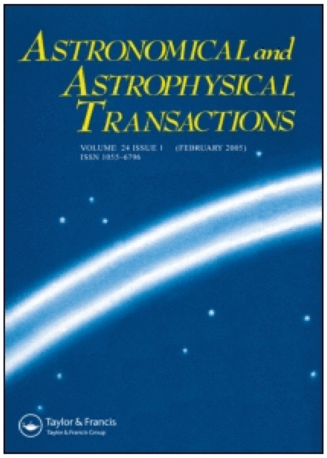

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COMPTON SCATTERING IN A STRONG MAGNETIC FIELD: RESONANCES IN THE CROSS SECTIONS

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The resonances in the cross section of Compton scattering in strong magnetic field are investigated. The distributions of the resonances over the Landau level number, on the energy of the initial photon and on the direction of the scattered photon are studied. Two methods of regularization of the resonances are proposed.

KEY WORDS Compton scattering, neutron stars, radiation, magnetic fields.

1. INTRODUCTION

Among various interactions processes between radiation and matter in the presence of magnetic field, the important one is the scattering of photons by electrons usually called the Compton scattering as in the case when magnetic field is negligible. The Compton scattering in a strong magnetic field can transform the spectra of primary sources of radiation in the magnetospheres of neutron stars where the strength of the field is comparable to the critical value, $B_c = 4.412 \cdot 10^{13}$ G. The X-ray lines in the spectra of pulsars and gamma-ray lines in gamma-burst spectra are believed to be formed by the Compton scattering in magnetic field.

General expressions for the cross sections of processes in external fields and of the Compton scattering in the magnetic field in particular have been obtained in quantum electrodynamics long ago (see, for example, Akhiezer *et al.* (1969)). However, numerical calculations without strong constraints on the field strength and the energies of photons and electrons appeared only after 1970.

In the work of Herold (1979), the differential and total cross sections have been calculated for the case when the initial and final electrons are on the ground Landau level. Daugherty and Harding (1986) have removed this restriction for the final electron. In both these articles, the momentum of the initial electron along the magnetic field has been set equal to zero. Note that in the latter work the expressions for the cross sections have been obtained for an arbitrary longitudinal momentum but in those expressions there are inaccuracies which may produce errors.

The most complete results for the Compton scattering in a strong magnetic field have been given by Bussard *et al.* (1986) where the polarization of radiation is taken into account. No constraints on the values of the parameters of interacting particles have been imposed. Simultaneously, the divergencies in cross sections have been regularized and the total cross sections have been averaged over some distributions of scattering electrons by Bussard *et al.* (1986).

It is worthwhile to note that the results of Daugherty *et al.* (1986) have been used by Harding *et al.* (1986) to interpret the observations of spectral gamma-ray bursts. The Compton scattering of synchrotron radiation has been considered by Alexander *et al.* (1989, 1991) as a mechanism of formation of resonance features in gamma-burst spectra. The Comptonization of soft photons by relativistic electrons results in the continuous spectrum of gamma-ray bursts as well (Dermer, 1990), Vitello *et al.*, 1991).

We have written a computer code for calculating complete 4×4 matrices of scattering cross sections for arbitrary polarization states of interacting electrons and photons. As in Bussard *et al.* (1986), the energies and moments of electrons and photons are considered to be arbitrary.

We have reduced the quantum electrodynamics formulae to the form convenient for calculations. The regularization of singularities (resonances) have been also made. Using the calculated values of the matrices, the cross sections for unpolarized electrons and the total cross sections are evaluated.

The formulae for elements of the matrices mentioned above and the scheme of the calculation will be given elsewhere. In the present note we concentrate our attention on the following problem: how to obtain the positions of the resonances as functions of the parameters of interacting photons and electrons. Note that these resonances are identified with the linelike features observed in gamma-ray bursts. We propose also the way to regularize the resonances.

To simplify our formulae we will use the relativistic quantum system of units where the Plank constant, the speed of light and the mass of electron are the basic units: $\hbar = c = m = 1$. In this system, the length is measured in the units of the Compton wavelength $\hbar/mc$, the energy, in mc^2, the frequency, in mc^2/h and the momentum, in mc. The electron charge is then equal $e = e/(\hbar c)^{1/2} = 1/(137.036)^{1/2}$.

2. THE ELECTRON AND THE PHOTON IN MAGNETIC FIELD

Let the magnetic field be constant, uniform and its direction to be coincident with the z-axis, i.e. $\vec{B} = B(0, 0, 1)$, where $B > 0$ is arbitrary. The product of the electron charge and the field strength B will be denoted as b. In usual CGS units, $b = B/B_c = eB\hbar/m^2 c^3$.

The states of an electron in the magnetic field are described by the solutions of the Dirac equation. These solutions are the functions of time and space coordinates. They depend also on the number of the Landau level, $n = 0, 1, 2, \ldots$, and on the longitudinal momentum of the electron Z. The transverse components of the electron momentum enter intermediate formulae only: all physically meaningful values do not depend on them. For example, for the electron energy we have

$$E = R_n(Z) = (1 + Z^2 + 2bn)^{1/2} > 0. \tag{1}$$

We need some more variables connected with the electron energy:

$$b_n = (2bn)^{1/2}, \qquad s_n^2 = 1 + b_n^2 = 1 + 2bn, \qquad R_n^2 = s_n^2 + Z^2. \tag{2}$$

Let

$$\vec{k} = k(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \tag{3}$$

be the usual three-dimensional momentum of photon.

All values in the initial state (before scattering) will be labelled with the subscript i and in the final state (after scattering), with the subscript f, for example, n_i and n_f are the corresponding numbers of the Landau levels, k_i and k_f are the frequencies of photons. Some notations will be abbreviated, for example, we shall write R_i and R_f for the energies of the electron in initial and final states. We shall write also the subscripts i and f instead of n_i, and n_f in s_i and s_f.

Now we formulate the conservations laws.

3. CONSERVATION LAWS

We need the following notations:

$$R_Z = R_i - Z_i\cos\theta_i, \; Z_R = R_i\cos\theta_i - Z_i, \tag{4}$$

$$R_k = R_i + k_i, \; Z_k = Z_i + k_i\cos\theta_i, \tag{5}$$

$$k_Z = k_i\sin^2\theta_i + R_Z, \qquad k_R = k_i(k_Z + R_Z), \; k_b = k_R + 2b(n_i - n_f), \tag{6}$$

$$\Delta^2 = (R_k\cos\theta_f - Z_k)^2 + s_f^2\sin^2\theta_f, \qquad \Delta > 0. \tag{7}$$

For the Compton scattering of photon by electron, two conservation laws are essential, namely those of energy and longitudinal momentum:

$$R_i + k_i = R_f + k_f, \qquad Z_i + k_i\cos\theta_i = Z_f + k_f\cos\theta_f. \tag{8}$$

As we have already mentioned, the transverse components have no importance.

In order to compute the differential cross section and the elements of the polarization matrices, it is necessary to know Z_i, n_i, k_i, θ_i, n_f, θ_f, $\varphi_f - \varphi_i$ and the polarization states of electrons and photons. Then we find succesively R_i from (1), Z_f from the second equation in (8) and R_f from (1). The momentum (or the frequency, which is the same in our system of units) of the scattered photon k_f may be obtained using both Eqs (8). If we substitute Z_f from the second Eq. (8) into the first one we obtain the equation

$$\sin^2\theta_f k_f^2 - 2k_f(R_k - Z_k\cos\theta_f) + k_b = 0. \tag{9}$$

Then k_f may be expressed as

$$k_f = \frac{k_b}{R_k - Z_k\cos\theta_f + \Delta}. \tag{10}$$

The momentum k_f must be positive. Consequently, we have the constraint on the number of the final Landau level:

$$n_f \leqslant n_f^0 = n_i + \mathrm{Entier}(k_R/2b), \tag{11}$$

where Entier(x) is the integer part of x.

4. THE RESONANCES IN THE CROSS SECTIONS

Two Feinman diagrams correspond to the Compton scattering. Both initial and final states contain one electron and one photon. According to the diagram denoted as a, the electron first interacts with the initial photon being converted into a virtual electron (or positron). The latter then emits the final photon. In the diagram b the interaction points are transposed: at the beginning of the process the electron emits the final photon and then the virtual particle absorbs the initial photon.

Denote the number of the Landau level of the virtual electron by n and its longitudinal momentum, by Z. Then its energy is determined by Eq. (1). The difference between the virtual electron and positron is that the energy of the latter is equal to $-R_n(Z)<0$, where $R_n>0$ is the energy of the former with the same $|Z|$ and n (the signs of their Z may be, but not necessarily, opposite).

The derivation of the expressions for the amplitudes of the Compton scattering involves several integrations which lead to rather complicated formulae. These formulae include the summation over all Landau levels of the virtual particles. Each term of the sums contains one of the four denominators: $(R_i+k_i-R_n)$ and $(R_i+k_i+R_n)$ for the diagram a, $(R_i-k_f-R_n)$ and $(R_i-k_f+R_n)$ for the diagram b. The braces with the + at R_n correspond to virtual positrons and are always positive whereas the others, electron ones may be equal to zero, so that the cross sections become infinite. These cases are called resonances.

The fact that the above braces are equal to zero means that the energy is conserved during the interaction. The longitudinal momentum is also conserved. So, for the diagram a we have

$$R_n=R_i+k_i=R_k,\; Z=Z_i+k_i\cos\theta_i=Z_k,\; R_n=R_f+k_f, \quad Z=Z_f+k_f\cos\theta_f. \tag{12}$$

For the resonance of diagram b, the following equations hold:

$$R_n=R_i-k_f,\; Z=Z_i+k_f\cos\theta_f,\; R_n=R_f-k_i,\; Z=Z_f-k_i\cos\theta_i. \tag{13}$$

Of course, if we eliminate from (12) and (13) all the quantities corresponding to the virtual particles, we shall obtain the conservation laws (8).

As in the case of the scattering in a spectral line, the resonances are to be regularized. In this work we study the distributions of resonances over the Landau levels, the energy of the incident photon and the direction of the scattered photon.

It is rather easy to find the resonances of diagram a. When the particle parameters before scattering are given, such a resonance can occur only at a single Landau level. Indeed, the first two equations in (12) imply

$$n=n_i+k_R/2b\geq n_f^0. \tag{14}$$

So, the resonance is possible if the expression for n in (14) is a non-negative integer. We see that the resonance n is determined only by the parameters of the initial electron and photon and coincides with the upper limit of n_f, which is equal to n_f^0.

We can easily find the frequency of the initial photon at which the resonances exist. Substituting the expression for k_R from (6) into (14), we obtain that the resonance occurs for any integer $n\geq 0$ at

$$k_i=\frac{[R_Z^2+\sin^2\theta_i(s_n^2-s_i^2)]^{1/2}-R_Z}{\sin^2\theta_i}=\frac{2b(n-n_i)}{[R_Z^2+2b\sin^2\theta_i(n-n_i)]^{1/2}+R_Z}. \tag{15}$$

It is much more difficult to study the resonances of the diagram b. First we shall investigate the n values of the resonance Landau level and then the directions of scattered photon which produce the resonances at certain n from this range.

5. THE RESONANCE LEVELS

The resonance condition (13) includes, together with the initial parameters of electron and photon, the frequency k_f and the angle θ_f of the scattered photon. Formally, this condition leads to the same expression for the number of the resonance level as (14) with the replacements $\theta_i \to \theta_f$ and $k_i \to -k_f$.

$$n = n_i + [k_f^2 \sin^2\theta_f - 2k_f(R_i - Z_i)]/2b \tag{16}$$

We should remember that now the frequency k_f is not arbitrary but has to be found from (10).

With the aid of (9), Eq. (16) can be rewritten as linear in k_f:

$$n = n_i + [2k_i k_f(1 - \cos\theta_i \cos\theta_f) - k_b]/2b \equiv n(\cos\theta_f). \tag{17}$$

Keeping in mind Eq. (10), we see that in (17) n is a function of $\cos\theta_f$. We denote this function as $n(\cos\theta_f)$. Consider this function in more detail.

We are interested in non-negative integer numbers between the largest and the smallest values of $n(\cos\theta_f)$, which form the range of resonance levels. The smallest value is either equal to 0 or reached at the limiting values of $\cos\theta_f = \pm 1$. The largest value also can be one of them. Therefore, we first study $n(+1)$ and $n(-1)$.

Using notations (6), we have

$$n(\pm 1) = \frac{k_i(1 \mp \cos\theta_i)n_i + (R_i \mp Z_i)(n_f - k_R/2b)}{R_k \mp Z_k}. \tag{18}$$

The difference between these two values is proportional to Z_R from (4):

$$n(1) - n(-1) = -2k_i k_b Z_R/(k_R + s_i^2), \tag{19}$$

that is $n(1) > n(-1)$ when $Z_R < 0$ and vice versa.

For the determination of the largest value of the function $n(\cos\theta_f)$, we should take into account the possibility that this function has a maximum inside the interval $(-1, 1)$.

It is easy to show that this maximum coincides in the position with the minimum of the function

$$h(\cos\theta_f) = \frac{R_i - Z_i\cos\theta_f + \Delta}{1 - \cos\theta_i\cos\theta_f}. \tag{20}$$

Indeed,

$$n(\cos\theta_f) = n_i + \frac{k_i}{b}\frac{k_b}{k_i + h(\cos\theta_f)}. \tag{21}$$

To find the extremum point let us take the derivative of (20) with respect to

$\cos\theta_f$. Note that Eq. (7) implies

$$\frac{\partial\Delta}{\partial\cos\theta_f}=\frac{1}{\Delta}\left[(R_k\cos\theta_f-Z_k)R_k-s_f^2\cos^2\theta_f\right],\tag{22}$$

where $s_f^2=1+2bn_f$. Then

$$h'(\cos\theta_f)(1-\cos\theta_i\cos\theta_f)^2$$
$$=\left(-Z_i+\frac{\partial\Delta}{\partial\cos\theta_f}\right)(1-\cos\theta_i\cos\theta_f)+(R_i-Z_i\cos\theta_f+\Delta)\cos\theta_i.\tag{23}$$

After the substitution of (22) into (23), we obtain the condition of the extremum:

$$Z_R\Delta+(R_k\cos\theta_f-Z_k)k_Z+s_f^2(\cos\theta_i-\cos\theta_f)=0.\tag{24}$$

If we eliminate Δ from (24) taking square of it, we obtain the following equation for $\cos\theta_f$ at the extremum:

$$(R_k\cos\theta_f-Z_k)^2\sin^2\theta_i=s_f^2(\cos\theta_i-\cos\theta_f)^2.\tag{25}$$

Consequently, the extremum is possible if

$$\cos\theta_f=\frac{Z_k\sin\theta_i\pm s_f\cos\theta_i}{R_k\sin\theta_i\pm s_f}\tag{26}$$

When we obtain (25) from (24), additional solutions may arise. So we substitute (26) into the initial Eq. (24) and take into account that

$$\Delta=s_f\frac{|k_Z\pm s_f\sin\theta_i|}{|R_k\sin\theta_i\pm s_f|}.\tag{27}$$

Then we find out that (26) is a solution of (24) if we choose the lower signs in both the numerator and the denominator and the following inequality holds:

$$\Delta=s_f\frac{k_Z-s_f\sin\theta_i}{R_k\sin\theta_i-s_f}>0.\tag{28}$$

This inequality with the obvious requirement $-1\le\cos\theta_f\le1$ leads to the conclusion that the maximum value of $n(\cos\theta_f)$ exists inside the interval $(-1,1)$ if

$$s_f=(1+2bn_f)^{1/2}<s_f^m=\begin{cases}(R_k+Z_k)\tan(\theta_i/2), & Z_R>0,\\ (R_k-Z_k)\cot(\theta_i/2), & Z_R<0.\end{cases}\tag{29}$$

This maximum is reached at the point

$$\cos\theta_f^0=\frac{Z_k\sin\theta_i-s_f\cos\theta_i}{R_k\sin\theta_i-s_f}\tag{30}$$

and its magnitude is

$$n_{\max}=n(\cos\theta_f^0)=[(k_i\sin\theta_i-s_f)^2-1]/2b.\tag{31}$$

We can check that $h''(\cos\theta_f^0)>0$ and, consequently, $h(\cos\theta_f^0)$ is the minimum value whereas $n(\cos\theta_f^0)$ is the maximum one.

So we need to compare the three (or two, if (29) is not fulfilled) quantities, $n(\pm 1)$ and $n_{\max}$, and to choose the smallest and the largest of them. When a non-negative integer number occurs between the smallest and the largest values, the resonances of diagram b exist there. If the function $n(\cos\theta_f)$ has a maximum and $Z_R > 0$ then there is one resonance value of $\cos\theta_f$ for each integer $n \geq 0$ such that $n(1) \leq n < n(-1)$ and, otherwise, two values for $n(-1) \leq n \leq n_{\max}$. If the maximum does not exist then there is one value of $\cos\theta_f$ for each $n \geq 0$ in the interval $n(1) \leq n \leq n(-1)$. If $Z_R < 0$ then two values $n(1)$ and $n(-1)$ exchange their positions. At last, if $Z_R = 0$ then $n(-1) = n(1)$ and the maximum does always exist at $\cos\theta_f = \cos\theta_i = Z_i/R_i$.

For possible resonance values of k_i in the case of diagram b we can obtain only inequalities. These inequalities are the same which determine the region of resonance Landau levels. It is easy to see that the level with the number $n = 0, 1, 2, \ldots$ will be resonant in the case $Z_R > 0$ if either

$$s_f \geq s_f^m, \qquad n(-1) \geq n, \qquad n(1) < n, \tag{32}$$

or

$$s_f < s_f^m, \qquad n_{\max} \geq n, \qquad n(1) < n. \tag{33}$$

In more detail, these inequalities can be written as

$$\begin{gathered} 2\cos^2(\theta_i/2)k_i \geq s_f \cot(\theta_i/2) - R_i - Z_i, \\ -Z_R\left(1 - \frac{s_n^2}{s_i^2}\right) < k_i \sin^2\theta_i + \frac{s_n^2 - s_f^2}{k_i} + R_Z\left(1 + \frac{s_n^2}{s_i^2}\right) \leq Z_R\left(1 - \frac{s_n^2}{s_i^2}\right). \end{gathered} \tag{34}$$

$$\begin{gathered} 2\cos^2(\theta_i/2)k_i < s_f \cot(\theta_i/2) - R_i - Z_i, \qquad s_n \leq |k_i \sin\theta_i - s_f|, \\ -Z_R\left(1 - \frac{s_n^2}{s_i^2}\right) < k_i \sin^2\theta_i + \frac{s_n^2 - s_f^2}{k_i} + R_Z\left(1 + \frac{s_n^2}{s_i^2}\right). \end{gathered} \tag{35}$$

In (34) and (35), $s_n^2 = 1 + 2bn$ as before.

For $Z_R < 0$ the inequalities can be obtained if we repalce $Z_i \to -Z_i$ and $\theta_i \to \pi - \theta_i$. All other quantities do not change their values, among them there is R_Z.

If $Z_R = 0$ the resonances exist at level n for all k_i that satisfy

$$s_f^2 \frac{R_i}{R_i + k_i} - s_i^2 \frac{k_i}{R_i} \leq s_n^2 \leq \left(k_i \frac{s_i}{R_i} - s_f\right)^2. \tag{36}$$

The resonance values of k_i can be obtained if we solve the corresponding inequalities.

Thus, there exists a region of non-negative integer numbers n at which the resonances of diagram b occur. One or two resonance values of $\cos\theta_f$ occur when these resonances really arise. Now we find these values.

6. THE DISTRIBUTION OF RESONANCES OVER DIRECTIONS

If the number n of a Landau level is in the resonance region found above then from (16) we can obtain the following equation for $\cos\theta_f$:

$$A_n \cos^2\theta_f - 2B_n \cos\theta_f - C_n = 0, \tag{37}$$

where

$$A_n = k_n^2 + 4k_iZ_i \cos\theta_i k_n + 4k_i^2 \cos^2\theta_i (s_n^2 - s_i^2), \tag{38}$$

$$B_n = 2k_i[(Z_i + R_i \cos\theta_i)k_n + 2(s_n^2 - s_i^2)k_i \cos\theta_i], \tag{39}$$

$$C_n = k_n^2 - 4k_iR_ik_n - 4(s_n^2 - s_i^2)k_i^2. \tag{40}$$

In the last three formulae, the following notation is used

$$k_n = k_R + 2b(n - n_f) = k_R + s_n^2 - s_f^2. \tag{41}$$

From Eq. (37) we find

$$\cos\theta_f = (B_n \pm |k_n|\, D_n)/A_n \tag{42}$$

where

$$D_n^2 = B_n^2 + A_nC_n = [s_n^2 - (k_i \sin\theta_i - s_f)^2][s_n^2 - (k_i \sin\theta_i + s_f)^2]. \tag{43}$$

Double sign in (42) reflects the possibility of two resonant directions at one resonance n. The dependence of $\cos\theta_f$ on n is suppressed in these expressions.

If we introduce auxiliary variables

$$Y_n = [s_n^2 - s_f^2 + k_i^2(1 + \cos^2\theta_i) + 2k_iR_i]/2b, \qquad y = (R_k^2 - s_f^2)^{1/2}, \tag{44}$$

then (38)–(41) and (43) will be expressed in term of (44) as follows:

$$k_n = 2k_i(Y_n - Z_k \cos\theta_i), \qquad A_n = 4k_i^2(Y_n^2 - y^2\cos^2\theta_i), \tag{45}$$

$$B_n = 4k_i^2[(Z_k + R_k \cos\theta_i)Y_n - \cos\theta_i(y^2 + Z_kR_k \cos\theta_i)], \tag{46}$$

$$C_n = 4k_i^2[Y_n^2 - 2(R_k + Z_k \cos\theta_i)Y_n + y^2 + Z_k(2R_k \cos\theta_i - Z_k \sin^2\theta_i)], \tag{47}$$

$$D_n^2 = 4k_i^2[Y_n^2 - 2R_kY_n + R_k^2\cos^2\theta_i + y^2\sin^2\theta_i]. \tag{48}$$

If we substitute (42) into expression (10) for k_f we see that at the resonance,

$$k_f = \frac{k_n}{2k_i(1 - \cos\theta_i \cos\theta_f)} \tag{49}$$

and, consequently, $k_n > 0$. At the same time A_n can be both positive and negative. If $A_n = 0$, only one resonant value of $\cos\theta_f$ is possible, namely

$$\cos\theta_f = -C_n/2B_n. \tag{50}$$

If $A_n \to 0$, the second solution goes to infinity (unphysical case).

The formulae obtained show that the procedure of calculation of resonances must be the following. Using given values n_i, Z_i, k_i, θ_i, n_f, θ_f we find all the quantities which enter Eqs (4)–(6). We check if n_f satisfies the inequality (11) and there is a resonance of the diagram a. Then we calculate the values $n(-1)$ and $n(1)$, find whether the maximum of $n(\cos\theta_f)$ exists and, if it does, obtain $n_{\max}$. Then we search for the region of resonance levels n. At last we calculate the quantities (38)–(42) and use them to obtain the resonant directions (36) taking care that all the $\cos\theta_f$ do not exceed unity by modulus.

7. REGULARIZATION OF THE RESONANCES

The regularization of the resonance levels is necessary first of all in order to calculate the differential cross sections and polarization matrices for all admissible values of the parameters of colliding particles without care that some of the calculated values may be infinite. Of course we might avoid such situations. The behavior of the cross sections can be understood from their values out of resonance. But if we want to calculate the total cross sections or some average values, for example the mean frequencies of scattered photons then the presence of resonances does not allow these calculations.

Bussard *et al.* (1986) inform that they regularize the resonances using the mean lifetimes of Landau levels. But there are no details in their work. The lifetimes were calculated by Herold *et al.* (1982).

We may propose two methods of regularization. The first one is less accurate but it is sufficient for many purposes, for example for the calculation of total cross sections. According to this method, the resonance denominators $R_i + k_i - R_n$ and $R_i - k_f - R_n$ are replaced by $R_i + k_i - R_n - i\Gamma$ and $R_i - k_f - R_n - i\Gamma$, where the width of the transition region Γ can be taken as in the theory of spectral lines in the form of a sum of the widths of the levels involved:

$$\Gamma = \Gamma_n + (\Gamma_i + \Gamma_f)/2. \tag{51}$$

For the width of the level, it is sufficient to take the following empirical fitting formula proposed by Pavlov (1986):

$$\Gamma_n = bn^2/137.036(1 + bn)[1 + (bn^3)^{1/2}]^{1/3}. \tag{52}$$

It is possible to take into account the dependence of the level width on the longitudinal momentum of the electron, i.e., to set

$$\Gamma_n(Z) = \Gamma_n(0)R_n/s_n, \qquad \Gamma_n(0) = \Gamma_n. \tag{53}$$

At the present time the shifts of Landau levels and the widths of shifted levels have been calculated. These results are given by Pavlov *et al.* (1991). In the same work, satisfactorily accurate parametrizations have been obtained for the widths and the shifts. The shifts are not equal for the levels with different projections of spin, so degenerate levels become non-degenerate. Therefore the possibility arises to take into account the widths of shifted levels which can be strongly different from each other. This is particularly important if we investigate the polarization of scattered photons. This is the second method of regularization.

To calculate the total cross sections we need to integrate the differential ones over the directions of scattered photons. The integral over φ_f can be calculated analytically (cf. Bussard *et al.* (1986)). As for the integral over θ_f, the terms of the diagram *a* do not depend on θ_f. At the same time, the terms of the diagram *b* may contain the resonances. Near the resonance, the integrand varies very quickly with θ_f even being regularized. So we have to be careful. Bussard *et al.* (1986) have increased the order of the quadrature formula. We use a method based on a special quadrature of order two which is adapted to the regularized integrand. In fact, these quadratures were deduced for each resonance separately by the method described by Chandrasekhar (1950). Note that we use similar quadratures in the case of the so-called quasi-resonances when the denominators do not vanish but become very small. This method gives accurate results and saves computer time.

CONCLUSION

We have described the procedure to search for the possible resonances in the cross sections of Compton scattering and to regularize them. The results of the calculations of the cross sections and the polarization matrices and the corresponding formulae will be given in the following publication. We suppose to use them for the investigation of various characteristics of Compton scattering, for averaging these values over the distributions of electrons and photons and for the calculation of the spectra formed by Compton scattering.

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