

This article was downloaded by:[Bochkarev, N.]
On: 19 December 2007
Access Details: [subscription number 788631019]
Publisher: Taylor & Francis
Informa Ltd Registered in England and Wales Registered Number: 1072954
Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



Astronomical & Astrophysical Transactions

The Journal of the Eurasian Astronomical Society

Publication details, including instructions for authors and subscription information:
<http://www.informaworld.com/smpp/title~content=t713453505>

Equations of perturbed keplerian motion in a quasi-linear form

V. A. Shefer ^a

^a Research Institute of Applied Mathematics and Mechanics, Tomsk

Online Publication Date: 01 June 1993

To cite this Article: Shefer, V. A. (1993) 'Equations of perturbed keplerian motion in a quasi-linear form', *Astronomical & Astrophysical Transactions*, 4:1, 39 - 40

To link to this article: DOI: 10.1080/10556799308205357

URL: <http://dx.doi.org/10.1080/10556799308205357>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article maybe used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

EQUATIONS OF PERTURBED KEPLERIAN MOTION IN A QUASI-LINEAR FORM

V. A. SHEFER

Research Institute of Applied Mathematics and Mechanics, Tomsk

(Received November 4, 1991; in final form January 15, 1992)

A method of transformation of perturbed Keplerian motion equations using integrals of motion together with transformation of coordinates $\bar{x} = A\bar{q}$ and time transformation $dt = br^n ds$ is proposed. Here \bar{x} is the physical relative position vector, $r = |\bar{x}|$, A is the square matrix having $m(m \geq 3)$ lines, \bar{q} is the parametric vector having m components, t is the physical time, s is the new independent variable, $b(>0)$ and $n(>0)$ are constants.

The matrix A is chosen in the form $A = r^\alpha E$, where α is the arbitrary constant and E is the unit matrix. Let $m = 3$. In this case we denote the new three-dimensional vector of parameters by $\bar{\rho} = \bar{q}$. Then the coordinate transformation reduces to $\bar{x} = r^\alpha \bar{\rho}$. The required differential equation of motion takes the form

$$\begin{aligned} \bar{\rho}'' = & 2(\alpha - 1)(\alpha - n)b^2hr^{2n-2}\bar{\rho} - \alpha(\alpha - n + 2)b^2c^2r^{2n-4}\bar{\rho} \\ & - (n - 2\alpha)b^2r^{2n-\alpha-2}\bar{a} + (2\alpha - 1)(\alpha - n + 1)\mu b^2r^{2n-3}\bar{\rho} \\ & + b^2r^{2n-\alpha}\bar{\mathcal{F}}, \end{aligned}$$

where h is the Keplerian energy, c^2 is the squared angular momentum, \bar{a} is the Laplace vector, μ is the gravitation parameter, $\bar{\mathcal{F}}$ is the perturbing force vector, prime denotes differentiation with respect to s .

A special choices of α and n lead to linear and regular equations of Keplerian motion. For instance, in the case $\alpha = 0$ and $n = 1$, equations in the Sperling–Burdet form (Silver, 1975) are obtained. If $\alpha = 1$ and $n = 2$, we have equations of a perturbed harmonic oscillator with the frequency c .

The matrix $A = r^\alpha L(\bar{q})$, where $L(\bar{q})$ is the square four-line KS-matrix, is considered as the second example. Let us denote the new 4-dimensional parametric vector by $\bar{Q} = \bar{q}$ ($m = 4$). Then the transformation of coordinates takes the form $\bar{x} = r^\alpha L(\bar{Q})\bar{Q}$. This transformation is a generalization of the familiar KS-transformation (Stiefel and Scheifele, 1971). In this case we come to the following equation of the perturbed Keplerian motion:

$$\begin{aligned} \bar{Q}'' = & \frac{1}{2}(\alpha - 1)(\alpha - 2n + 1)b^2hr^{2n-2}\bar{Q} - \alpha(\frac{1}{4}\alpha - \frac{1}{2}n + 1)b^2c^2r^{2n-4}\bar{Q} \\ & - \frac{1}{2}(n - \alpha - 1)b^2r^{2n-3}L^T\bar{a} + \frac{1}{2}(\alpha^2 - 2\alpha n + n + 2\alpha + 1)b^2\mu \\ & \times r^{2n-3}\bar{Q} + \frac{1}{2}b^2r^{2n-1}L^T\bar{\mathcal{F}}. \end{aligned}$$

For $\alpha = 0$ and $n = 1$, this reduces to equations of motion in KS-variables. With $\alpha = 1$ and $n = 2$, we have equations of perturbed motion in the Rodrigues–Hamilton parameters.

Equations obtained from the above ones provide certain practical advantages if we take $\alpha = 0, n = 3/2$ and $\alpha = -1, n = 3/2$,

To obtain a closed systems of differential equations, the above equations should be supplemented by equations for the distance r , time t and by equations governing variations of the Keplerian parameters h, c^2 and \bar{a} .

Equations of motion in the form proposed here are especially convenient for numerical integration because of a higher accuracy of the result and shorter computation time.

References

- Silver, M. (1975). *Celest. Mech.* **11**, 39–41.
Stiefel, E. and Scheifele, G. (1971). *Linear and Regular Celestial Mechanics*, Springer, New York.