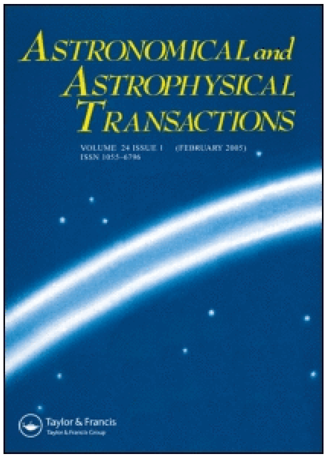

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EQUATIONS OF PERTURBED KEPLERIAN MOTION IN A QUASI-LINEAR FORM

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A method of transformation of perturbed Keplerian motion equations using integrals of motion together with transformation of coordinates $\bar{x} = A\bar{q}$ and time transformation $dt = br^n\,ds$ is proposed. Here $\bar{x}$ is the physical relative position vector, $r = |\bar{x}|$, A is the square matrix having $m(m \geqslant 3)$ *lines*, $\bar{q}$ is the parametric vector having m components, t is the physical time, s is the new independent variable, $b(>0)$ and $n(>0)$ are constants.

The matrix A is chosen in the form $A = r^\alpha E$, where α is the arbitrary constant and E is the unit matrix. Let $m = 3$. In this case we denote the new three-dimensional vector of parameters by $\bar{\rho} = \bar{q}$. Then the coordinate transformation reduces to $\bar{x} = r^\alpha \bar{\rho}$. The required differential equation of motion takes the form

$$\begin{aligned}\bar{\rho}'' = {} & 2(\alpha-1)(\alpha-n)b^2hr^{2n-2}\bar{\rho} - \alpha(\alpha-n+2)b^2c^2r^{2n-4}\bar{\rho} \\ & - (n-2\alpha)b^2r^{2n-\alpha-2}\bar{a} + (2\alpha-1)(\alpha-n+1)\mu b^2r^{2n-3}\bar{\rho} \\ & + b^2r^{2n-\alpha}\bar{\mathscr{F}},\end{aligned}$$

where h is the Keplerian energy, c^2 is the squared angular momentum, $\bar{a}$ is the Laplace vector, μ is the gravitation parameter, $\bar{\mathscr{F}}$ is the perturbing force vector, prime denotes differentiation with respect to s.

A special choices of α and n lead to linear and regular equations of Keplerian motion. For instance, in the case $\alpha = 0$ and $n = 1$, equations in the Sperling–Burdet form (Silver, 1975) are obtained. If $\alpha = 1$ and $n = 2$, we have equations of a perturbed harmonic oscillator with the frequency c.

The matrix $A = r^\alpha L(\bar{q})$, where $L(\bar{q})$ is the square four-line KS-matrix, is considered as the second example. Let us denote the new 4-dimensional parametric vector by $\bar{Q} = \bar{q}(m = 4)$. Then the transformation of coordinates takes the form $\bar{x} = r^\alpha L(\bar{Q})\bar{Q}$. This transformation is a generalization of the familiar KS-transformation (Stiefel and Scheifele, 1971). In this case we come to the following equation of the perturbed Keplerian motion:

$$\begin{aligned}\bar{Q}'' = {} & \tfrac{1}{2}(\alpha-1)(\alpha-2n+1)b^2hr^{2n-2}\bar{Q} - \alpha(\tfrac{1}{4}\alpha - \tfrac{1}{2}n + 1)b^2c^2r^{2n-4}\bar{Q} \\ & - \tfrac{1}{2}(n-\alpha-1)b^2r^{2n-3}L^T\bar{a} + \tfrac{1}{2}(\alpha^2 - 2\alpha n + n + 2\alpha + 1)b^2\mu \\ & \times r^{2n-3}\bar{Q} + \tfrac{1}{2}b^2r^{2n-1}L^T\bar{\mathscr{F}}.\end{aligned}$$

For $\alpha = 0$ and $n = 1$, this reduces to equations of motion in KS-variables. With $\alpha = 1$ and $n = 2$, we have equations of perturbed motion in the Rodrigues-Hamilton parameters.

Equations obtained from the above ones provide certain practical advantages if we take $\alpha = 0$, $n = 3/2$ and $\alpha = -1$, $n = 3/2$,

To obtain a closed systems of differential equations, the above equations should be supplemented by equations for the distance r, time t and by equations governing variations of the Keplerian parameters h, c^2 and $\bar{a}$.

Equations of motion in the form proposed here are especially convenient for numerical integration because of a higher accuracy of the result and shorter computation time.

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