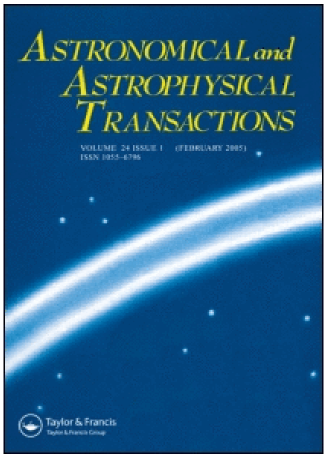

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GRAVITATIONAL LENSES (THE STATE OF THE THEORY AND OBSERVATIONS)

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(23 January, 1992)

A review of the gravitational lens theory as well as related observations and astrophysical applications is given. The geometrical optics approach is most popular but it leads to singularities at caustics where diffraction effects have to be taken into account. In this case the thin phase-screen approximation is used. Another class of difficulties is connected with irregularities in the medium. Due to enormous number of such inhomogeneities, statistical theory works very well.

The number of accepted and proposed candidates for lensed extragalactic objects is about 40. In recent years the focus has turned to using gravitational lenses as "instruments" in studying some astrophysical problems. In this way, important information about the sources and lenses themselves can be obtained.

KEY WORDS Gravitational lenses

Since the discovery of the first Gravitational Lens (GL) in 1979, a stream of journal papers mounted to the critical density at which the monographs begin to appear. There are already two books published with almost the same title (1991). One of them[1] was published in 1989, the other[2] is being prepared for publishing and will appear in the nearest future. In addition, three international conferences were devoted to GL (Cambridge, USA—1988; Toulouse, France—1989; Hamburg, Germany—1991). The proceedings of the first two Conferences also have been published.[3,4] The aforementioned publications contain numerous references, so in our review we shall confine outselves to only a few.

The diagram in Figure 1 [1] gives one an idea of the increase in the rate of publications. As a matter of fact, the number of the papers of the last decade is considerably greater than that shown in the diagram. The point is that Figure 1 corresponds to the beginning of 1989. At the Conference in Hamburg, more than about 500 published papers mentioned.

1. THE POSSIBLE MANIFESTATIONS OF A GL

The GL effect is due to the bending of rays (light, radio) in an inhomogeneous gravity field. The following manifestations can be observed even in a weak gravitational field:

(i) Displacement of a source from its true position and splitting of the image;

(ii) Modification of the spatial density of sources because of different displacements of images;

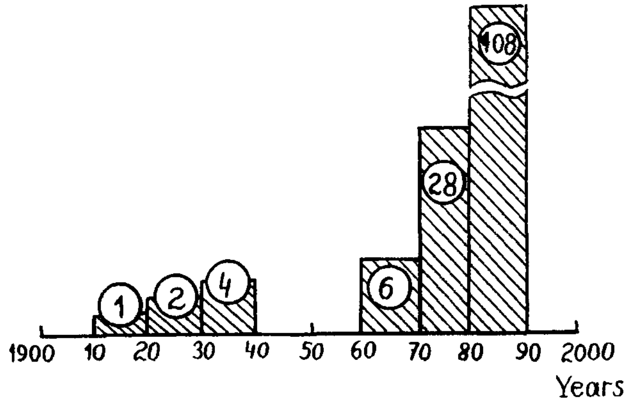

Figure 1 The rate of publications on gravitational lenses in the XX century.

(iii) Distortion of the shapes of extended sources;
(iv) Enhancement of remote source intensity which leads to the widening of the luminosity function of quasars and to increasing the quasar density near the galaxies which act as GL;
(v) Time delay in the intensity variations of different images of a single source;
(vi) Fast intensity fluctuations and apparent superluminal velocities due to transverse relative motion of the source and the GL.

2. THEORY

The physical principles of gravitational lensing have a solid foundation in the form of the General Relativity Theory. Within the framework of this theory, Maxwell's equations in gravitational field can be formulated. Analysis of these equations shows that in weak gravitational fields produced by massive bodies at rest, electromagnetic waves propagate as in an inhomogeneous medium with the refractive index:[5]

$$n_g \simeq 1 - \frac{2\Phi(\vec{r})}{c^2}, \tag{1}$$

where $\Phi(\vec{r})$ is the gravitational potential and c is the speed of light. If motions of gravitating masses are taken into account (for instance, rotation) Eq. (1) becomes more complicated. The additional term describes the effects which can be observed when electromagnetic waves propagate through a moving medium.

The problem of wave propagation in inhomogeneous media has a long history and many relevant approximate methods have been work out. These methods also are applicable to the theory of GL.

2.1. *Geometric Optics (GO). The Refracting Plane Concept*

GL scales exceed greatly the wavelengths used for astronomical observations. That is why the GO approach is widely used in making calculations. It is not difficult to write out main equation of GO, which governs ray trajectories

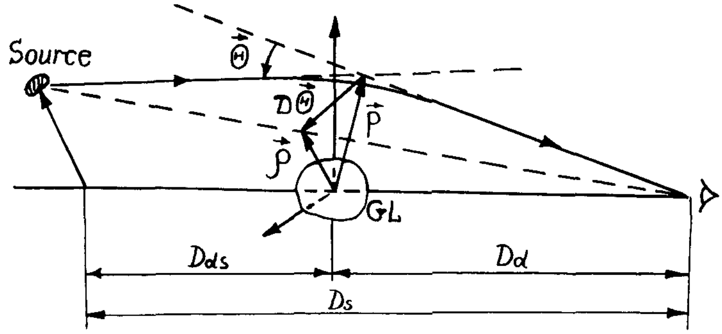

Figure 2 Scheme of the ray deflection by a gravitational lens (GL).

(eikonal equation) in arbitrary space–time metric. However, this equation can be solved only in some particular cases, but it is not necessary to solve this equation at all. The point is that at large distances from the GL the rays practically coincide with their asymptotes. This allows to introduce the refracting plane concept. Thus we replace a smooth trajectory by a two-segment broken line (Figure 2).†

In this approach, when the refractive angle $\vec{\theta}$ is small, the true position of the source $\vec{\rho}$ and its apparent position $\vec{p}$ in the GL-plane are connected by a simple equation (the lens equation) [7]:

$$\vec{\rho} = \vec{p} + D\vec{\theta}(\vec{p}), \tag{2}$$

where $D = D_d D_{ds}/D_s$ is the reduced distance (the notations D_d, D_{ds}, D_s are understandable from Figure 2).

All the effects mentioned above can be calculated with the help of Eq. (2), but it is necessary, first, to know function $\vec{\theta}(\vec{p})$ and, second, to solve the lens equation, that is to find all roots $\vec{p}_i(\vec{\rho})$. Simple analytical formulae for $\vec{\theta}(\vec{p})$ come out only in the GL models with spherical symmetry of the mass distribution. Within the framework of perturbations theory, it is possible to take into account weak deviations from spherical symmetry, which on the whole are described by multipoles.[8]

Besides, there are calculations of $\vec{\theta}(\vec{p})$ for some planar mass distributions which approximate the disk components of galaxies. It is noteworthy that oscillating trajectories can exist near a massive plane ("gravitational waveguide"[6]). On the whole, it is necessary to use numerical calculations and in this way multiple centres of attraction can be considered (separate stars in a galaxy, the so-called "microlenses"). The calculations are restricted, as a rule, to the case of all stars being located on a single plane. Allowance for a distribution of stars within some volume leads to considerable difficulties.

The analytical treatment of Eq. (2) is possible only with some specific functions $\vec{\theta}(\vec{p})$, but graphical analysis or numerical calculations are quite simple. In

† There is one exception related to our Galaxy. In this case an observer on the Earth resides inside the GL and the refractive plane concept cannot be applied.[6]

particular, it is easy to formulate conditions for multiple roots $\vec{p}_i(\vec{\rho})$, that is the conditions for a multiple image of a source.

In order to analyse the shape and intensity modification for a small source, it is necessary to consider the variation of Eq. (2) $(\delta\vec{\rho} = \delta\vec{p} + D(\delta\vec{p}\nabla)\vec{\theta}(\vec{p}))$ and rewrite it as $\delta\vec{\rho} = \hat{A}(\vec{p})\,\delta\vec{p}$.

The square symmetric matrix of the second order $\hat{A}(\vec{p})$ determines the linear transformation from the coordinates of the image to those of the source. The determinant of the inverse matrix $\hat{A}^{-1}$ is equal to the ratio of the image and source areas. This ratio is nothing but the amplification factor of the GL: $q = \det \hat{A}^{-1}(\vec{p})$. Equation $\det \hat{A}(\vec{p}) = 0 (q \rightarrow \infty)$ determines the so-called caustic curve in the GL plane. On this curve, some images correspond to the infinite intensity (for a point-like source).

2.2. *Wave Optics. Thin Phase-Screen Approximation*

The infinite intensity at the caustic is an artefact of the geometric optic approximation. In order to get rid of the divergences it is necessary to take into consideration the diffraction of waves. In this way a finite value of intensity can be obtained everywhere including caustics. But, at present, the exact solution of Maxwell's equations in a gravity field has been obtained only for a GL in the Schwarzschield metric.[9]

These results have a restricted application range for real GLs, but the exact solution serves as a starting point for approximate methods. Among them, the thin phase-screen approximation is widespread. It is possible to draw an analogy between this approach and the refracting plan concept in the GO. The essence of the thin phase-screen approximation is as follows: first, the phase shift at an infinite plane which crosses a GL is calculated. After that, the application of the Huyghens principle yields the intensity of the wave at any position. The main difficulties encountered in this procedure are connected with the evaluation of the diffraction integral. Analytical treatment can be carried out only for some symmetric mass distributions, but there are no really serious obstacles for approximate calculations.

Finally the intensity amplification factor q is represented as a ratio of two areas: $q = A_L/A_F$, where A_L is the area of the input aperture of GL and A_F is an area of the focal spot. When the observer is located on the focal semiaxis of a point-like GL, the input aperture has the shape of a thin ring, the radius of which is $l = (2r_g D)^{1/2}$(r_g is the gravitational radius) and the width is $\Delta\rho \simeq D\Psi_d$ ($\Psi_d \sim \lambda/l$ is the angle of diffraction). The focal spot is a circle with the radius $D\Psi_d$, and so $A_L \simeq 2\pi l D\Psi_d$, $A_F \simeq \pi(D\Psi_d)^2$. Thus, the amplification factor is given by

$$q \simeq l/(D\Psi_d) = \Psi_L/\Psi_d \simeq r_g/\lambda, \tag{4}$$

where $\Psi_L = l/D$. At caustics which arise in transparent lenses, the dependence of q on λ can be more complicated but always $q \rightarrow \infty$ when $\lambda \rightarrow 0$.

Equation (4) is valid for a point-like source. If a source has finite angle dimensions Ψ_0, the structure of Eq. (4) does not change but the angles Ψ_d and Ψ_0 combine as a quadratic sum:

$$q \simeq \Psi_L/(\Psi_0^2 + \Psi_d^2)^{1/2}. \tag{5}$$

If $\Psi_0 \gg \Psi_d$, diffraction effects are negligible and GO is valid everywhere including the caustics. As a rule the condition $\Psi_0 \gg \Psi_d$ is fulfilled, that is why the specific dependence of q on λ has not been observed as yet.

The finite dimension of a source also leads to the violation of the coherence between different images. The calculations show that an interference can be observed only in a very narrow frequency band.[10] The interference problem is discussed also in Ref. 11.

2.3. *Statistics in the GL Theory*

Electromagnetic waves propagate from a source to an observer through the medium which is optically inhomogeneous. The main inhomogeneity is produced by GL itself; besides, there are many irregularities of a much smaller scale (gravitational fields of individual stars, plasma clouds in the interstellar medium, etc). Due to the enormous amount of these inhomogeneities, statistical theory works very well and statistical parameters of GL can be evaluated (for instance, the average value $\langle q \rangle$ instead of q). Small irregularities in a transparent GL destroy the spatial coherence of radiation. As a consequence, the image is deformed and magnification is deteriorated.

The influence of random factors can be taken into account by considering ray scattering at the refracting plane or phase fluctuations at the phase-change screen. If the root mean square values of the angle scattering are $\langle \Psi_g^2 \rangle^{1/2}$ (for a random gravitational field) and $\langle \Psi_p^2 \rangle^{1/2}$ (for plasma inhomogeneities), eqs. (4) and (5) can be generalized in the following way:

$$\langle q \rangle \simeq \Psi_L / (\Psi_d^2 + \Psi_0^2 + \langle \Psi_g^2 \rangle + \langle \Psi_p^2 \rangle)^{1/2}. \tag{6}$$

We consider separately the effects of the random gravitational field and plasma inhomogeneities because of their different wavelength dependencies: $\langle \Psi_g^2 \rangle$ is independent of λ, but $\langle \Psi_p^2 \rangle \sim \lambda^4$. Analysis of Eq. (6) allows to establish the effective limits of the wave length range, $\lambda_{\min}$ and $\lambda_{\max}$, for gravitational magnification. The lower cutoff $\lambda_{\min}$ is not absolute. Even if $\lambda \to 0$, the value of $\langle q \rangle$ is finite. At the upper limit, $\lambda_{\max}$ $\langle q \rangle \simeq 1$. Under reasonable assumptions about medium parameters, the value of $\lambda_{\max}$ belongs to the meter range of radio waves.

There is the second aspect of the statistical theory. It allows to evaluate the probability of gravitational lensing.[12] For instance, the average value of image splitting in a galaxy-lens is of about a few arcseconds. Another example deals with a quasar-galaxy association. The following hypothesis has been put forward to explain an excess of associations between high-redshift quasars and faint galaxies: the mass associated with the galaxies gravitationally lenses the quasar. The apparent brightness of a quasar is enhanced when the line of sight passes close to a foreground galaxy, thereby increasing the probability that such a quasar is included in flux-limited samples. A consequence of this model is the steepening of the bright tail of the observed quasar luminosity function. This effect offers the possibility to determine the total mass associated with galaxies, an outstanding goal in the search for the dark matter.

The statistics of gravitational lensing can be also studied by using background galaxies projected onto selected rich, massive galaxy clusters. The background galaxies are preferentially aligned at 90° to the radial vector from the cluster centre.

3. OBSERVATIONS

The review of the accepted and proposed candidates for lensed extragalactic objects is presented in Ref. 4. It is possible to divide the list of about 40 candidates in two groups. In the first one, there are lensed compact objects, namely QSO and AGN with splitted images. The typical splitting angle is about a few arcseconds. The number of images varies from 2 to 4.

The main features of the lensing effect are the coincidence of the spectra and the time delay of brightness variations for different images as well as the existence of inverted images.

At the same time, the spectra of images are not always replicas of each other. Their identity can be slightly distorted due to the different conditions of wave propagation at the paths to different images. The time delay Δt can be accurately measured only if its value surpasses the characteristic time of the brightness variations of the source itself. For instance, this condition probably is not satisfied for the cross-like images of QSO 2237 + 0305. Besides, the measurements of Δt are complicated by interference on the wave traces. Only for the first GL, QSO 0957 + 561 A, B, regular monitoring has been carried out. The first obtained value of Δt was ~1.55 years, and the recent data favor Δt ~1.15 years.

The observation of inverted structures can be carried out only with an instrument having a very high resolution. At present, the effect has been observed only for QSO 0957 + 561 A, B.

Extended sources belong to the second group of GL. Their shapes are strongly distorted due to the lensing by a foreground cluster of galaxies. This is how the arclets and giant luminous arcs arise. The high surface density of the background galaxies (sources) and a very great number of clusters (lenses) are enough to make clear that the second group potentially contains an extremely great number of candidates as compared to the class of classical galaxy–QSO lenses. Such lenses have not been detected earlier because the probability of detecting such gravitational images of distant galaxies is high only when using extremely deep photometry.

In 1986, the first radio ring lens MG 1131 + 0456 was discovered. The second radio ring MG 1634 + 1346 was found in 1989.† The rings are found using the radio observations of double extended sources both of which are observed at better resolution and better contrast between the lens and the source than those observed at optical wavelengths. The rings are about two arcseconds in diameter.

It is necessary to mention the galaxy M82. In its vicinity, almost identical objects with a very large splitting angle (~200″) are observed. Probably, this is due to the lensing by a massive black hole.

Another exotic object is the field of twin galaxies 0249 − 186 with splitting angles ~2″. It is assumed that here the lens effect of a string is observed.

4. GL AND ASTROPHYSICAL PROBLEMS

The theory of GL is completed in its main parts and adequate methods of observations as well as of the search for new objects are well known now. In

† At the conference in Hamburg (1991) the object PKS 1830-211 was mentioned, which can possibly be added to the two former ones.

recent years, the focus has turned to using GL as an "instruments" in studying some astrophysical problems. In this way an important information about the sources and lenses themselves can be obtained.

4.1. *Mass Distribution in the GL*

Determining the mass distribution in a gravitation lens (e.g., a galaxy or galaxy cluster) from information about the source images represents the so-called inverse problem. In general, to solve the problem, it is necessary to look through the lens in all directions, i.e., to get a kind of tomography. There is no possibility to carry out such experiment and only very limited data are available: the coordinates of images and their intensities for point-like sources, and the distribution of brightness and polarization of the images for extended sources. In any case the data refer only to the single point of observation and, therefore, the inverse problem has, generally, multiple solutions. In practise, the problem even is not formulated in exact terms. Instead of the inverse problem, simpler problems are solved. They reduce to the selection of parameters in some adopted mass distribution models. It is very important that, in the inverse problem, all kinds of matter are taken into account, including the dark matter.

4.2. *The Distance to the Source. The Hubble Constant H_0*

After the lens parameters have been determined, the possiblities arise to obtain some information about the source. In particular, the distance to the source can be determined. Thus the Hubble constant H_0 can be evaluated.[13] As input data, the time delays Δt between the light curves for different images is used. The delay depends on the pathlength differences between the ray trajectories and the gravity potential along them. In spite of apparent simplicity of the method, very serious difficulties arise in practice. They are connected with the errors in evaluation of Δt. The most reliable data have been obtained, as yet, for the first GL. The value of H_0 determined in such a way is 50 ± 17 km s^{-1} Mpc^{-1}.[2]

4.3. *The Sizes of the Sources and Absorbing Clouds*

Due to the transverse motion, a ray from the source may pass near one of the stars in the galaxy-lens. In such case the apparent variations of the source brightness arise. The variation will be more visible for the smallest sources. This circumstance can be used for estimation of the source size.

In the spectra of the splitted images of some quasars, a set of absorption lines is observed. The lines are due to intervening clouds located along the ray trajectories. The similarity of the absorption spectra shows that the probability for both rays to pass through the same cloud is large. So the possibility arises to estimate the cloud size. For instance, for QSO 0957 + 561 this procedure yields the cloud size of about 10 pc.

4.4. *The nearest GLs: planets, the Sun and the Galaxy*

The action of these GLs can be observed not only in a narrow solid angle but through the whole of the celestial sphere. The maximum displacement of the source is $\sim 1''$ for the Sun, $\sim 0.1''$ for the Galaxy and $\sim 0.001''$ for Jupiter. During precise astrometric measurements, such displacements should be taken into account.

The ray bending in the gravity fields of the Sun and Jupiter can be observed by means of direct measurements, but it is impossible to perform similar measurements for the Galaxy. The point is that the direction to an extragalactic source does not vary with time. The observable effects are the variations of the number density of the sources and their brightness with galactic coordinates (the density increases and the brightness decreases near the galactic equator). The evaluation of both effects is characterized by a small parameter of $\sim 10^{-5}$–10^{-6}.

4.5. *The Stars of the Galaxy as Microlenses*

The stars of the Galaxy act as microlenses not only for distant extragalactic sources but for the stars of the bulge as well. At any moment, one bulge star out of every few million should have its brightness increased by at least 0.3 mag because of the gravitational microlensing by the Galactic disk stars. If there is a large population of brown dwarfs in the disk, it should produce microlensing events lasting a few days. There is even a possibility to discover planets around lensing disk stars through the effect they should produce on the light curves. At present about 20 observatories take part in the preparation for the microlensing effect observations (the MACHO—Massive Compact Halo Objects—project). The planned multichannel device (of about 10^5–10^6 channels) is expected to detect several microlensing events in one year.

5. CONCLUSION

We would like to conclude this short review by the words of the well-known physicist T. Rege: "We are at the beginning of a new epoch in astrophysics, when the data about the remote galaxies will be obtained using the influence of gravity on the light from more distance sources." These words were written in 1981. At present, the epoch of gravitational lenses as astrophysical instruments already starts.

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