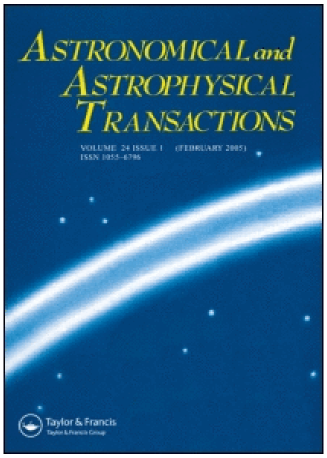

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Nonlinear effects in theory of equilibrium gravitating, rapidly rotating, magnetized barotropic configurations and the gravitational radiation from pulsars

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To cite this Article: Masjukov, V. V. and Tsvetkov, V. P. (1993) 'Nonlinear effects in theory of equilibrium gravitating, rapidly rotating, magnetized barotropic configurations and the gravitational radiation from pulsars', Astronomical & Astrophysical Transactions, 4:1, 41 - 42
To link to this article: DOI: 10.1080/10556799308205358
URL: http://dx.doi.org/10.1080/10556799308205358

Astronomical and Astrophysical Transactions, 1993,
Vol. 4, pp. 41–42
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NONLINEAR EFFECTS IN THEORY OF EQUILIBRIUM GRAVITATING, RAPIDLY ROTATING, MAGNETIZED BAROTROPIC CONFIGURATIONS AND THE GRAVITATIONAL RADIATION FROM PULSARS

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(November 4, 1991; in final form April 30, 1992)

It is shown that the exact equilibrium equation of gravitating, rapidly rotating, magnetized barotropic configurations can be reduced to the system of algebraic equations with a small parameter of order of ratio of magnetic to gravitational energy of the configuration. The essential feature of the system is an existence of the 'critical' points where even a weak internal magnetic field which is asymmetric with respect to axis of rotation leads to a considerable asymmetric strains of the configuration in question. It is estimated that the intensity of gravitational radiation from millisecond pulsars can increase at 'critical' points by a factor of 10^{12} because of the nonlinear effects taking place near these points.

KEY WORDS gravitation theory, pulsars.

1. We consider [1, 2] the barotropic configuration D which satisfies the following equation of equilibrium:

$$\Phi + \int \frac{dP}{\rho} - \frac{\omega^2}{2}((x_1')^2 + (x_2')^2) + \varkappa\Pi^{(g)} + \varkappa_1\Pi^{(m)} = \text{const}, \tag{1}$$

where Φ is the gravitational potential, ω, the angular velocity of rotation about the x_3'-axis, ρ, the density, $P = P(\rho)$, the pressure; $\varkappa_1\Pi^{(m)}$, $\varkappa\Pi^{(g)}$ are the terms describing the influence of internal magnetic field and post-Newtonian corrections. We seek for ρ and the surface δD represented in the following form:

$$\rho = \rho_0 \sum_{abc} \rho_{abc} x_1^a x_2^b x_3^c; \qquad \rho_{000} = 1. \qquad \delta D\colon x_1^2 + x_2^2 + x_3^2 + \sum_{ij} Z_{ij} x_1^i x_2^j = 1, \tag{2}$$

where $x_i = x_i'/a_i$. The relation between Z_{ij} and ρ_{abc} can be obtained by the minimization of the r.m.s. deviation of $\rho|_{\delta D}$ from zero:

$$\mathbf{F}(Z_{ij}) = \left[\frac{1}{4\pi\rho_0^2}\int_{\delta D} \rho^2\, d\Omega\right]^{1/2}, \qquad \text{where } d\Omega = \sin\theta\, d\theta\, d\varphi. \tag{3}$$

If $\mathbf{F}(Z_{ij}) = 0$, the surfaces $\rho = 0$ and δD are the same. In any case, an iterative method of minimization can give a sufficient accuracy. The ellipsoid with the semi-axes a_1, a_2 and a_3 is chosen as a zeroth-order approximation.

2. By applying the method of the Burman–Lagrange series [3], we obtain an analytical representation for all the terms in Eq. (1) in the form of absolutely convergent series in x_i. Then we substitute it in Eq. (1) and combine the terms with equal powers of x_i. So we reduce the problem of the determination of the equilibrium configuration to the exact algebraic system of equations in ρ_{abc} and Z_{ij}. The configurations of equilibrium which are asymmetric with respect to the axis of rotation are of particular interest for problems of gravitational radiation. That is why we put $\rho_{abc} = \rho_{(ab)c} + \rho_{[ab]c}$ and $Z_{ij} = Z_{(ij)} + Z_{[ij]}$, where $Z_{(ij)} = Z_{(ji)}$, $Z_{[ij]} = -Z_{[ji]}$, $\rho_{(ab)c} = \rho_{(ba)c}$ and $\rho_{[ab]c} = -\rho_{[ba]c}$. Then we solve the set of equations for the configuration of rotation to determine $\rho_{(ab)c}$ and $Z_{(ij)}$. Thus, we have an opportunity to investigate, against this background, small perturbations which are asymmetric about the axis of rotation. Such perturbations are governed by the following non-linear system of equations:

$$\sum_{l=0}^{\infty} \sum_{I_1 I_2 \cdots I_{2l+1}} \mathbf{W}_A^{I_1 I_2 \cdots I_{2l+1}} \prod_{r=1}^{2l+1} \mathbf{X}_{I_r} = \varkappa_1 \Pi_A, \tag{4}$$

where $\mathbf{X}_I$ is the column of unknown antisymmetric coefficients $\rho_{[ab]c}$ and $Z_{[ij]}$, $\mathbf{W}_A^{I_1 I_2 \cdots I_{2l+1}}$ depend on symmetric coefficient $\rho_{(ab)c}$ and $Z_{(ij)}$ which have been already defined.

3. The right-hand side of Eq. (4) is of the order of the ratio of magnetic to gravitational energy. For many pulsars, $\varkappa_1 \sim 10^{-8}$–10^{-12} [4]. From Eq. (4) it follows that in the general case $\mathbf{X}_I \sim \varkappa_1$, but at 'critical' points, where $|\det(\mathbf{W}_A^I)| \leq (\varkappa_1)^{2/3}$, we have $\mathbf{X}_I \sim (\varkappa_1)^{1/3}$ because nonlinear effects take place near these points. In the wave zone, with the choice of a transverse-traceless gauge, the time-depending parts of the metric coefficients can be estimated as $h \sim \dfrac{4G\omega^2}{R} M a_0^2 (X_I)_{\max}$, where G is the universal constant of gravity, M is the mass, R is the distance from the pulsar, and $a_0 = (a_1 a_2 a_3)^{1/3}$. If, for our pulsar, $T = 1$ ms, $R \sim 50$ kpc, $M \sim M_\oplus$, $a_0 = 10^6$ cm, and $\varkappa_1 \sim 10^{-9}$ then on the Earth we obtain $h \sim 10^{-28}$, but at the 'critical' points $h \sim 10^{-22}$. At present, the metric perturbations of the amplitude $\sim 10^{-20}$–10^{-22} can be detected. Therefore, in our opinion, millisecond pulsars are the most probable periodical sources of gravitational radiation.

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