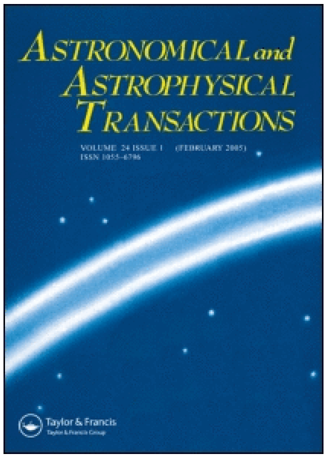

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ON THE CENTRAL SURFACE BRIGHTNESS OF GALACTIC DISKS

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In the framework of the Lin–Pringle model of the galactic disk evolution, the observed approximate constancy of the central surface brightness I_o of an exponential disk is discussed. The disk is assumed to be embedded in a massive halo of the cold dark matter. We suppose that a protogalaxy acquired its angular momentum from tidal torques by interaction with surroundings at early stages of its formation. A self-similar solution for the stellar component evolution and the condition of total disk angular momentum conservation allow to estimate the disk central surface density (and luminosity) as a function of halo parameters. These parameters are determined by the amplitude and spectral index (n) of the density fluctuations from which the galaxy has been formed. It is shown that if $n \approx -2$ at the galactic scale (CDM-model), I_o is constant and independent of other galactic parameters. The predicted values of I_o are in a good agreement with the observed ones if galaxies have been formed at rather high peaks of cosmological density fluctuations.

KEY WORDS Galactic disks, galactic evolution, structure of galaxies

1. INTRODUCTION

Photometric investigations of the galactic disk structure have revealed an exponential surface brightness distribution along the disk radius (Freeman, 1970):

$$I = I_o \exp(-\alpha R). \tag{1}$$

The magnitudes of the face-on central brightness of different galaxies turned out to be very close to each other. Moreover, I_o does not correlate with any galactic parameter. There have been considerable debates on the reality of the I_o constancy (for discussion, see Van der Kruit, 1987), but now it seems well justified that galactic disks really have very close central surface brightness $\sim$21.65 m/arcsec^2 within a small range of magnitude.

Van der Kruit (1987) was perhaps the first who made a successful attempt to explain this phenomenon. His model is based on Gunn's (1982) scenario of the disk formation. According to this scenario, the galactic disk is formed as a result of the homological contraction of barionic matter under the conservation of specific angular momentum h. The disk formation proceeds relatively slowly, at the time scale comparable to the lifetime of the flat subsystem. The distribution $h(r)$ has been proposed to be the same as that of a uniformly rotating, uniform sphere (Mestel, 1963). In this case the exponential distribution of the disk density is maintained permanently. A simple model of the spherically symmetric collapse of the halo yields the relation between halo and disk parameters. A specific link between the cosmological density fluctuation amplitude (from which the galaxy originated) and the halo mass leads to the independence of I_o of other galactic parameters. An appropriate parameterization of the problem has allowed Van der Kruit to obtain the values of the face-on brightness close to the observed ones.

Unfortunately, Gunn's model has some essential shortcomings. For example, it is hard to conceive that each element of gaseous protogalaxy "knows" the place in the disk where it must fall during the collapse. For the exponential distribution to be maintained, this element must accrete to the distance from the center where its specific angular momentum is equal to that of the disk. Otherwise, a momentum exchange occurs between the disk and the infalling matter and the resulting strong radial flow distorts the exponential distribution. It is difficult also to understand why the protogalaxy could rotate uniformly (Gunn, 1987).

Another approach to the galactic disk evolution considers a relatively fast formation of the flat subsystem instead of a slow accumulation of matter in the disk. We believe that the flat galactic subsystem has been formed due to the contraction of the gas in the gravitational field of the massive halo. In this case the typical time of the disk formation is the free-fall time determined by the mass M_h and the dimension R_h of the halo:

$$t_d \approx (R_h^3/2GM_h)^{1/2} \tag{2}$$

This time is known to be quite short in comparison with the subsequent evolution. For typical galaxies it is of the order of billion years. t_d can be considered as the initial moment of the disk evolution.

A young gaseous disk of the protogalaxy evolves due to the angular momentum redistribution and star formation. Both produce the final density profile similar to the observed one. Lin and Pringle (1987a) have confirmed this scenario by direct numerical simulations. Note that the distribution (1) has been shown to appear when the typical star formation time τ_s and the matter flow time $\tau_v = R^2/v$ are comparable to each other (here R is the radius and v denotes the kinematic viscosity):

$$\tau_v = \beta\tau_s. \tag{3}$$

(It can be shown that β has to be close to unity to reproduce both the density profile of the disk and the chemical abundance of the gas—see below). It is natural to expect that these scales are close to each other when they are determined by the same physical process. A good candidate for such a process can be the large-scale gravitational instability. As proposed by Lin and Pringle (1987b), this instability promotes the angular momentum redistribution. These authors have assumed that the disk did not split into distinct selfgravitating bodies but rather instabilities give rise to density waves which transfer angular momentum. One can say that this complicated motion of gaseous masses stimulated by instability reminds turbulence. So, the gas of the disk behaves like a viscous medium. Following Lin and Pringle (1987b), we define the corresponding effective viscosity as

$$v = G^2\Sigma^2/\Omega^3, \tag{4}$$

where we omit a numerical factor of order unity. Here G and Ω are the gravitational constant and the angular velocity, respectively and Σ is the gas surface density. Eq. (4) can be easily obtained if we choose the typical length scale of the "turbulence" to be equal to the largest wavelength of the gravitationally unstable mode: $L \approx G\Sigma/\Omega^2$ (Toomre, 1964). Since the instability growth rate is of the order of Ω, the effective kinematic viscosity takes the above form.

On the other hand, the large-scale instability can control the gas consumption (or the star formation rate) in the disk. This conclusion is supported by the fact that the density profile and magnitude in a major part of a gaseous galactic disk are very close to ones predicted by the marginal stability condition (Zasov and Simakov, 1988; Kennikutt, 1990).

Fall and Efstathiou (1980) also confirm the idea that such instability is a good stimulator of star formation. These authors found a good correlation between the typical size of luminous galactic disks and the distance R_* (in the disk plane) where the instability condition is violated. Beyond this distance, the rate of star formation is very low (there is no stimulator) and the disk luminosity decreased sharply. Due to this, the present-day galaxies look truncated at this radius. Further we assume that the gravitational instability is the prime mover of the disk evolution.

Besides the exponential luminosity distribution, the L–P model predicts a negative gradient of the gas abundance distribution $Z(R)$ is late-type galaxies. According to this model, the behavior of $Z(R)$ is accounted for by the following processes: the increase of the gas abundance owing to metal production in stars and dilution by a weakly enriched matter flowing in from outer parts of the galactic disk. This scheme is supported by both numeric (Yoshii and Sommer-Larsen, 1990; Clarke, 1989) and analytic calculations (Simakov, 1990, hereafter—SGS).

So, two important properties of galactic disks (the distribution (1) and a nagative Z gradient) have been explained by the model. Below we shall show that the proposed scenario naturally explains an approximate constancy of I_o.

2. BASIC EQUATIONS AND RESULTS

Let us consider a protogalactic gaseous disk surrounded by a massive dark halo. These halos are known to manifest themselves in long, flat rotation curves of late-type galaxies (Einasto *et al.*, 1974; Bosma, 1981) and in the hot coronae of the ellipticals (Forman *et al.*, 1985). We suppose below that the angular velocity $\Omega(R) \propto \bar{R}^{-k}$ does not depend on time and is determined by the matter distribution in the spherical component (the halo and the bulge). Note that the rotation curve in the inner parts of a galaxy is strongly affected by the mass distribution in the flat subsystem. Therefore, one should solve a self-consistent problem taking into account the evolution of the gravitational field due to the disk matter redistribution. It was made by Saio and Yoshii (1990) who repeated Lin and Pringle's numerical calculations under the assumption that the disk is selfgravitating. They also found that the surface density of such a disk embedded in a massive halo evolves to an exponential distribution.

The viscous redistribution of the gas in the disk plane makes the conditions in the disk to be very similar to those in the well-known ordinary accretion disks around stars. This similarity allows us to use the accretion disk theory. Taking into account the star formation process (the function $\psi(R, t) = \Sigma/\tau_s$ below) we obtain the following set of basic equations describing the evolution of the galactic disk (Lin and Pringle 1987a; Yoshii and Sommer-Larsen; SGS):

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{1}{(\Omega R^2)'}\frac{\partial}{\partial R}(\nu \Sigma R^3 \Omega')\right) - (1 - R_s)\psi(R, t), \tag{5}$$

$$\frac{\mathrm{d}\Sigma_s}{\mathrm{d}t} = (1 - R_s)\psi(R, t). \tag{6}$$

Here the prime denotes the derivative $\mathrm{d}/\mathrm{d}R$ and Σ_s is the surface density of the disk of stars. These equations involve the instantaneous recycling approximation. by introducing the fact $1 - R_s$, we take into account the mass of the gas coming back to the interstellar medium. According to the well-known definition of R_s (e.g. Marochnik and Suchkov, 1984), we can write

$$1 - R_s = 1 - \int_{m_l}^{m_{ms}} (m - r_m)\phi(m)\,\mathrm{d}m. \tag{7}$$

Here r_m is the mass of a stellar remnant, m and m_{ms} denote the current star mass and the mass of a star leaving the main sequence, respectively, and $\phi(m)$ is the stellar mass distribution.

It is naturally to choose the following initial condition: $\Sigma_s(t = t_{\text{initial}} = t_d) = 0$. Indeed, the gravitational instability, provided playing the key role in the disk evolution, cannot develop before the initial chaotic motions (resulting from the collapse) have decayed to a considerable extent. Therefore, the disk had enough time to remain purely gaseous.

To solve the problem (5–6), it is convenient to introduce new variables similar to those proposed by Lyubarskii and Shakura (1987): $h = \Omega R^2$ and $F = h\Sigma^3/\Omega^3$. Then Eq. (5) takes the form

$$\frac{\partial F}{\partial t} = D\frac{F^m}{h^n}\frac{\partial^2 F}{\partial h^2} - \beta\frac{D}{d}\frac{F^{m+1}}{h^{n+2}}, \tag{8}$$

where $d = k(2-k)/(1-R_s)$, $D = 3k(2-k)G^2$, $m = 2/3$ and $n = -1/3$. By virtue of dimensional arguments, solution of Eq. (8) can be found in the form

$$F(h, t) = \frac{h^{(n+2)/m}}{(Dt)^{1/m}}\,Y(\xi), \tag{9}$$

where Y is a dimensionless function of the dimensionless variable $\xi = h/At^{\alpha}$, $0 \leqslant \xi \leqslant 1$. One should determine the power-law index α and the constant A. Let us choose $Y(1) = 0$ as the outer boundary condition and suppose that the viscous force vanishes at the inner edge of the disk: $F(h \to 0) \to 0$.

Using standard similarity methods (Sedov, 1972; Barenblatt, 1978) one can easily obtain the solution of Eq. (8) for F and subsequently, for Σ_s. Following Lin and Pringle (1987), Yoshii and Sommer-Larsen (1989) and SGS, we present the solution at $t \gg t_v$, which is formally written as for $t \to \infty$, in the form

$$\Sigma_s = \Sigma_{s0}\frac{h^{\gamma}}{R^2}g(h/At^{\alpha}). \tag{10}$$

Here g is given by

$$g(x) = \begin{cases} B_x(p, (m+1)/m) & \text{if} \quad x < 1, \\ B(p, (m+1)/m) & \text{if} \quad x \geqslant 1, \end{cases} \tag{11}$$

where B and B_x are the complete and incomplete β-functions, respectively. The argument p reads

$$p = \frac{(1 - m(\alpha + |a|))}{\alpha\,|a|\,m}. \tag{12}$$

Furthermore,

$$\Sigma_{s0} = \frac{\beta(1-R_s)G^2 b^{1/m} A^{(1-m)/m\alpha}}{\alpha D^{1/m} a^{1/m}}, \tag{13}$$

$$\gamma = \frac{1}{m} + \frac{m-1}{\alpha m}, \tag{14}$$

$$a = 0.5(-4 + (1 + 4\beta/d)^{1/2}), \tag{15}$$

$$\alpha = (4 + (1 + 4\beta/d)^{1/2})^{-1}, \tag{16}$$

$$b = -3\alpha.$$

The only parameter which remains to be found is A. Its value can be easily derived basing on the total angular momentum conservation in the disk:

$$\mathcal{T}_d = 2\pi \int_0^\infty \Sigma_s(R) h(R) R \, \mathrm{d}R. \tag{17}$$

The upper limit of the integral is infinite because the outer boundary of the disk is defined as $\xi = 1$. Since $\xi \propto h/t^\alpha$, for $t \to \infty$ we have $h \to \infty$ and $R \to \infty$. At the latest stage of the evolution, the disk consists mainly of stars. The mass of the gaseous component is small in comparison with the total mass of the disk. Therefore, the stellar component contains all the angular momentum of such a disk. This is why we write Σ_s in Eq. (17). With the help of Eq. (10), Eq. (17) can be rewritten as

$$\mathcal{T}_d = \frac{2\pi\beta(1-R_s)G^2 b^{1/m} t_d^{\alpha(\gamma+1)} (A)^{(m+1)/m}}{\alpha(2-k) D^{1/m} a^{1/m}} \int_0^\infty x^\gamma g(x) \, \mathrm{d}x. \tag{18}$$

Let us assume now that the protogalaxy has acquired its rotation due to tidal interaction with the environment at an early stage of evolution (Peebles, 1969). It is natural to expect in this case that the specific angular momentum in the disk and the halo are approximately equal (Fall and Efstathiou, 1980):

$$\mathcal{T}_d / M_d = \mathcal{T}_h / M_h. \tag{19}$$

Let

$$\mathcal{T}_d = f M_h h_h, \tag{20}$$

where $f = M_d / M_h$, and h_h is the specific angular momentum at the halo boundary. Below we put $f = 0.1$ in accordance with the numerical simulations of galactic halo formation (Blumental *et al.* 1986) and $A = \Delta / t_d^{\alpha}$. The latter relation and Eq. (18) yield

$$\Delta = \left[\frac{\mathcal{T}_d \alpha D^{1/m} (2-k) t_d^{1/2} a^{1/m}}{2\pi\beta(1-R_s) G^2 \zeta b^{1/m}} \right]^{m/(m+1)}, \tag{21}$$

where

$$\zeta = \int_0^{\infty} X^{\gamma} g(X)\, \mathrm{d}X. \tag{22}$$

Using the expressions for t_d (2) and $\mathcal{T}_d$ (20) given above we rewrite (21) as

$$\Delta = \left[\frac{\alpha (3k)^{1/m} (2-k)^{1+1/m} f \pi^{1/2} a^{1/m}}{2\pi\beta(1-R_s)\zeta 2^{1/4} b^{1/m}} \right]^{2/5} h_h = \chi h_h. \tag{23}$$

The term in a square bracket is denoted by χ. This parameter being multiplied by h_h gives the length scale of the stellar disk.

Finally, using Eqs. (10) and (23) we obtain

$$\Sigma_s = \Sigma_{s0} (h/\chi h_h)^{\gamma - 2/(2-k)} g(h/\chi h_h). \tag{24}$$

The behavior of $\Sigma_s(h/\chi h_h)$ for some certain values of β and k is illustrated by SGS (similar results can be found in Yoshii and Sommer–Larsen, 1989). An approximately exponential surface density profile appears almost everywhere except the central part. (We should stress that such distortion of the exponential density profile near the center is reproduced in numerical calculations which take into account the self-gravity of the disk (Saio and Yoshii 1990).) Therefore, one can assume that $\Sigma_s(x) = \Sigma_{s0} \exp(-x)$ for the whole range of integration. This assumption can be used to estimate ζ. For $k = 1$ we have, for example, $\zeta = 2$ (this is the case used below).

Since $\Sigma_s(R)$ has an almost exponential behavior, it is natural to identify Σ_{s0} with the central density of an exponential disk. After some algebra the following expression can be easily obtained:

$$\Sigma_{s0} = \frac{\beta(1-R_s) 2^{1/4} b^{3/2} \chi^{-2/(2-k)+3/2}}{\alpha [3k(2-k)]^{3/2} \pi^{1/2} a^{3/2}} \cdot \frac{M_h}{R_h^2}. \tag{25}$$

Here we use the definition $h_h = \sqrt{G M_h R_h}$.

The last step is to be done. We need a relationship between the halo dimension R_h and its mass M_h. This relation can be deduced from a simple spherically-symmetric galaxy collapse model (Peebles 1980; Faber 1982):

$$R_h = 0.45 M_{12}^{1/3} h_{50}^{-2/3} (\nu_t \sigma)^{-1} \quad \text{Mpc}, \tag{26}$$

where ν_t denotes the amplitude of the density fluctuation from which the galaxy has originated, σ is the mean square amplitude of the fluctuations and $M_{12} = M_h / 10^{12} M_\odot$. It should be noted that σ depends on the mass of the forming galaxy. As follows from simple similarity analysis of hierarchical clustering, $\sigma = \sigma_0 M_{12}^{-1/2-n/6}$. The value of σ_0 is rather strictly limited by the observations of the galaxy distribution in the Universe. It should be set $\sigma_0 = 5$ for the galactic correlation function to be well reproduced (Blumental *et al.*, 1986). As far as the index n is concerned, it is approximately equal to -2 at the galactic mass scale (Davies *et al.*, 1985). Such a value is predicted by the popular CDM model of

Universe structure generation with a flat initial spectrum. In this case Eq. (26) gives $R_h \propto M_h^{1/2}$ and we have

$$\Sigma_{s0} = \frac{15.9\beta(1-R_s)b^{3/2}\chi^{-2/(2-k)+3/2}}{\alpha[k(2-k)]^{3/2}a^{3/2}} \cdot \nu_t^2\sigma_5^2\left(\frac{M_\odot}{pc^2}\right), \tag{27}$$

where $\sigma_5 = \sigma_0/5$.

DISCUSSION

As follows from Eq. (27), there is no dependence of Σ_{s0} on the galactic mass or galactic size in a full accordance with observations. In Table 1 we present some numerical values of Σ_{s0}, I_o and $B_0 = 27 - 2.5lg(I_o)$. The transition from Σ_{s0} to I_o has been made with the help of the mass-luminosity ratio, M/L, which is assumed to be equal to 4. Comparing the predicted magnitudes with the observed ones ($B_0 \approx 21.65$ m/arcsec^2), we can conclude that the agreement of the theory with observations is achieved under the condition that galaxies have been generated from rather high peaks of cosmological density fluctuations, $\nu_t > 1$. A similar conclusion has been obtained on the basis of the numerical simulations of the large-scale structure of the Universe (Davies *et al.*, 1985).

Besides ν_t, there is another parameter which is important for the magnitude I_o. This parameter is $\beta(1-R_s)$ (below we take $1-R_s = 0.7$ in accordance with Salpeter's stellar mass distribution), and ν_t. An admissible range of the β values is narrow. If $\beta > 3$, the final disk density profile would not be similar to the observed one. The values $\beta < 0.3$ correspond to a very small abundance gradient which is in conflict with observations of spiral galaxies. We do not know now what is the reason for such a narrow range of β. A detailed theory of star formation would be able to answer this question.

The dependence of I_o on the power-law index of the fluctuations spectrum, *n*, is weak.

The dependence of I_o on the cosmological parameters stems from the significance of the halo in galactic dynamics. In our case, the halo mass distribution determines the magnitude of the angular velocity which, in its turn, controls the star formation rate and intensity of the matter redistribution through

Table 1

β	ν_t	Σ_{s0} ($M_\odot/pc^2$)	I_o ($L_\odot/pc^2$)	B_0 (m/arcsec^2)
3.0	1.5	1339.0	334.7	20.7
	1.75	1822.5	455.6	20.4
	2.0	2380.5	595.1	20.1
2.0	1.5	399.0	99.8	22.0
	1.75	543.1	135.8	21.7
	2.0	709.4	177.4	21.4
1.5	2.0	370.7	92.7	22.1
	2.25	469.2	117.3	21.8
	2.5	579.2	114.8	21.6

the instability. One can say that the central surface brightness is, in some sense, a bridge connecting two essentially different scales of the cosmic hierarchy, the galaxy cluster and supercluster scale and the scale at which the features of the galactic disks are exhibited.

We would like to stress that the CDM model of the Universe structure has some difficulties in explanation of the galaxy cluster distribution (Frenk, White and Davies, 1983; Collins, Joseph and Robertson, 1986). Some other theory might be more successful in reproducing the matter distribution. But the fact that fluctuations with the spectral index equal to -2 at the galactic scale provide a good description of the galaxy formation and properties implies that the spectrum of the fluctuations in this new theory should be close to the CDM one at the galactic scale.

CONCLUSION

To summarize, taking into account results of other works, one can say that the Lin–Pringle model of the galactic disk evolution successfully explains all most important large-scale features of galactic disks; the exponential luminosity distribution, the approximate constancy of I_o and the negative gradient of the chemical abundance.

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