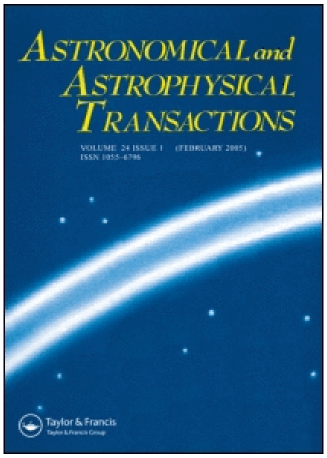

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THE POLOIDAL FIELD OF A SLOWLY ROTATING AXISYMMETRIC ROTATOR (AN AXISYMMETRIC ROTATOR MAGNETOSPHERE)

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The equation governing the poloidal field of an axisymmetric rotator is derived. The equation is obtained in the MHD approximation with allowance for relativistic effects, plasma pressure and gravitation of the central object. The problem of cold plasma ejection from the magnetosphere of an axisymmetric rotator at small angular velocities is solved with the help of this equation. Rotation is considered as a small perturbation of a magnetosphere. An analytical expression for the corrections to the poloidal magnetic field is obtained. The solution shows a poleward deflection of the flow due to the magnetic force of spiraling field lines.

KEY WORDS Pulsars, magnetosphere, plasma ejection.

Rapidly rotating neutron star with strong magnetic field represents a generally accepted model of radio pulsars. It is well known that the magnetic dipole rotating in a vacuum emits the magnetodipole radiation if the angle between the axis of rotation and the direction of the magnetic moment differs from zero. The magnetodipole emission is absent for the axisymmetric rotation in vacuum. The rotating object does not lose energy of rotation in the this case.

Real pulsars do not rotate in vacuum. Even for an axisymmetric rotation, a magnetized neutron star produces dense e^+e^- plasma in the magnetosphere (Ruderman and Sutherland, 1975; Arons, 1983). The plasma is ejected to infinity along open field lines. The presence of the plasma strongly changes the energy losses of an axisymmetrically rotating neutron star. The so-called current losses appear. They are connected with the generation of an azimuthal magnetic field in the magnetosphere. The energy loss rate for an axisymmetric rotator is (Beskin, Gurevich & Istomin 1983)

$$L = \frac{1}{4}\frac{H^2R^6\Omega^4}{c^3}i, \tag{1}$$

where H is the magnetic field on the surface of the pulsar, R is the radius of the neutron star, Ω is the angular velocity of the star rotation, c is the speed of light and i is the electric current density flowing from the polar cap normalized by the Goldreich—Julian electric current density $L_{\mathrm{GT}} = \Omega H/2\pi$ (Goldreich and Julian, 1969). For pulsars, $i \simeq 1$ (Bogovalov, 1991). Therefore, an axisymmetric rotating, magnetized neutron star losses the energy of rotation at the rate comparable to the rotational energy loss rate of an obliquely rotating star in vacuum.

The energy losses (1) are equal to the total flux of electromagnetic field energy from the star. This energy is transformed into the kinetic energy of particles at the Alfvén surface of the axisymmetric rotator magnetosphere. In this region, a supersonic wind of relativistic particles with the total power equal to the total energy losses of the neutron star is formed (Bogovalov, 1990). For example, in such a pulsar as Crab, the particles have the mean energy $10^4\,\mathrm{mc}^2$ inside the Alfvén surface. Beyond the Alfvén surface, the particles are accelerated to the energy of the order of $10^7\,\mathrm{mc}^2$.

So, we see that all basic processes observed in real pulsars such as the plasma generation, the effective deceleration of the star, the acceleration of the particles to the energy as high as $10^7\,\mathrm{mc}^2$, and the formation of a supersonic wind of the power comparable to the total spindown losses of the star take place in the magnetosphere of an axisymmetric rotator. In this connection we suppose that the model of an axisymmetric rotator is adequate for real pulsars.

Below we discuss an analytical solution of the problem of the structure of the axisymmetric rotator magnetosphere. Due to the stationarity and azimuthal symmetry of the magnetosphere, there are certain integrals of the motion of the plasma (Ardavan, 1976; Bogovalov, 1991). The conservation of the energy flux along the field lines is described by

$$\Gamma + \varphi(\vec{r}) - F(\Psi)q(\Psi)xH_\varphi = W(\Psi). \tag{2}$$

Here $\Gamma = (\varepsilon + p)\gamma$, ε and p are the internal energy and plasma pressure per particle in mc^2 units, γ is the Lorentz factor, $\varphi(r)$ is Newton's potential of gravitational field divided by c^2, $F = H_p/4\pi mchv_p$, where H_p is the poloidal field strength, n is the plasma density, m is the mass of the particles, v_p is the plasma velocity along the force lines of the poloidal field, x is the distance to the axis of rotation expressed in the units of the light cylinder radius, $q(\Psi)$ is the function which appears in the relationship $E = q(\Psi)xH_p$ between the electric field E and the poloidal magnetic field H_p, H_φ is the azimuthal component of the magnetic field, Ψ is the potential defining the poloidal field as

$$\vec{H}_p = \frac{1}{x}[\nabla\Psi, \vec{e}_\varphi]. \tag{3}$$

Ψ is constant along the force lines of the poloidal magnetic field. All the functions depending on Ψ are also constant along the poloidal field lines.

The conservation of the angular momentum flux is expressed by

$$xU_\varphi - F(\Psi)xH_\varphi = M(\Psi). \tag{4}$$

Here $U_\varphi = \dfrac{v_\varphi}{c}\Gamma$, where v_φ is the azimuthal component of the velocity. The frozen in condition has the form

$$xq(\Psi)\Gamma + U_p h = U_\varphi, \tag{5}$$

where $U_p = (v_p/c)\Gamma$ and $h = H_\varphi/H_p$. The relativistic relationship between Γ, U_p and U_φ is

$$\Gamma^2 = (\varepsilon + p)^2 + U_p^2 + U_\varphi^2. \tag{6}$$

Equations (2), (5), (6) and (7) govern the dynamics of adiabatic plasma in a given poloidal field.

The equation for the potential Ψ can be obtained from the flux conservation, across the field lines, for the energy-momentum tensor (Ardavan, 1979; Bogovalov, 1991, 1992a). This equation has the form (Bogovalov, 1992b)

$$\frac{(U_p - U_A)}{xFH_p^2}\Big[(U_pH_x^2 - H_p^2U_m)\frac{\partial^2\Psi}{\partial x^2} + 2H_xH_zU_p\frac{\partial^2\Psi}{\partial x\,\partial z} + (U_pH_z^2 - H_p^2U_m)$$
$$\times\frac{\partial^2\Psi}{\partial z^2} + H_z\Big\{FH_p(H^2+E^2) + \frac{U_\varphi^2H_p^2}{U_p} + \varepsilon H_p^2(U_p - (1+x^2q^2)FH_p)\Big\}$$
$$+ x^2\varepsilon\frac{H_p^4}{U_p}\left(U_p^2(\ln F)'_\Psi + \frac{(V)'_\Psi U_\varphi}{xq}\right) + \frac{H_p^2}{qU_p}\Big[\left(E_i\frac{\partial\varphi}{\partial r_i}\right) + \frac{x^2H_p^3}{U_p}(\ln q)'_\Psi$$
$$\times(\varepsilon U_\varphi^2H_p + FU_pE^2(1-\varepsilon))\Big]\Big] = -H_\varphi\Big\{2H_z\omega - \frac{1}{q}\Big(\frac{TH_pU_m}{F}$$
$$-\frac{(V)'_\Psi H_p(\vec{U}\vec{H})}{U_p} + \left(E_i\frac{\partial\varphi}{\partial r_i}\right) + (\ln q)'_\Psi\left(\frac{xH_p}{U_p}\right)$$
$$\times(U_pH_pxq\omega + U_\varphi(\vec{U}\vec{H}))\Big)\Big\}. \tag{7}$$

Here $\omega = W - \varphi(\vec{r})$, $V = W - Mq$, $U_m = \varepsilon(U_p - U_A) + F(H^2 - E^2)/H_p$, $U_A = (1 - x^2q^2)FH_p$, U_A is the Alfvén velocity, $\varepsilon = (U_s/U_p)^2$, $U_s = (\varepsilon + p)C_s/\sqrt{1-(C_s/C)^2}$, C_s is the sound velocity and $T = (W)'_\Psi + xH_\varphi(Fq)'_\Psi$. The symbol $(\;)'_\Psi$ denotes the derivative with respect to Ψ and H is the total magnetic field.

Later we consider the simplest case of a cold plasma ejection from the magnetosphere which has a monopole-like spherically symmetric configuration in the absence of rotation (Sakurai, 1985). We suppose that, when $\Omega = 0$, the flow is spherically symmetric and has the velocity U_0. The gravitation is neglected. The function Ψ has the form

$$\Psi = \Psi_{\max}\frac{(1-\cos\theta)}{2}, \tag{8}$$

where θ is the polar angle and $\Psi_{\max} = 2H_0R^2$ is the magnetic flux through the surface of the star. The magnetic field is given by

$$H_p = H_0\left(\frac{R}{r}\right)^2, \tag{9}$$

where H_0 is the field at the star surface. The velocity is uniform throughout the flow. The Alfvén surface is a sphere of the radius r_A defined by

$$U_A = U_0 = FH_p(r_A). \tag{10}$$

At the same time, the Alfvén surface is the fast magnetosound surface. On this surface, the plasma flow becomes supersonic. It is convenient to write out Eq. (7) in the spherical system of coordinates. For a cold plasma without gravitation and

$q = 1$, it has the form

$$\frac{(U_p - U_A)}{FH_p^2}\left\{(U_pH_r^2 - (H^2 - E^2)FH_p)\frac{\partial^2\Psi}{\partial r^2} + 2H_\theta H_r U_p \frac{1}{r}\frac{\partial^2\Psi}{\partial\theta\,\partial r}\right.$$
$$+ (U_pH_\theta^2 - (H^2 - E^2)FH_p)\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2} + \cos\theta\, H_r\left[(H^2 + E^2)FH_p\right.$$
$$\left.\left. + \frac{H_p^2U_\varphi}{U_p}\right] - \sin\theta\, H_\theta\left[U_p(H_p^2 + H_r^2) + 2E^2FH_p + \frac{H_p^2U_\varphi^2}{U_p}\right]\right\}$$
$$= -xH_\varphi\left[2WH_z - (H^2 - E^2)(W'_\Psi + xH_\varphi F'_\Psi) - \frac{H_pV'_\Psi}{U_p}(\vec{U}\vec{H})\right]. \tag{11}$$

Here H_θ and H_r are the spherical components of the magnetic field.

Let us consider a slowly rotating star, so that $r_A \ll 1$ (r_A is normalized by the light cylinder radius). In this case rotation can be considered as a small perturbation. The problem can be solved by means of perturbation theory.

From the system of Eqs. (2–6), it can be shown that the first-order term in the expansion of the azimuthal magnetic field in Ω has the form (Bogovalov, 1992b)

$$h = -\frac{xW}{U_0}, \tag{12}$$

where $W = \gamma_0$. It can be shown that the corresponding correction to U_p is of the fourth order in Ω. Therefore, below we assume that $U_p = U_0$. The expansion of U_φ begins with the term proportional to Ω^3. Therefore, we assume below that $U_\varphi = 0$.

Ψ can be presented in the form

$$\Psi = \Psi_{\max}\left(\frac{1-\cos\theta}{2} + f\right), \tag{13}$$

where f is the lowest nonvanishing perturbation of Ψ. After the linearization of Eq. (11), the following equation for f can be obtained:

$$(\xi^2 - 1)\frac{\partial^2 f}{\partial\xi^2} + 2\xi\frac{\partial f}{\partial\xi} - \frac{1}{\xi^2}\left(\frac{\partial^2 f}{\partial\theta^2} - \frac{\cos\theta}{\sin\theta}\frac{\partial f}{\partial\theta}\right) = \frac{r_A^2}{U_0^2}\cos\theta\sin^2\theta, \tag{14}$$

where $\xi = r/r_A$. The solution of this equation can be presented as

$$f = \left(\frac{r_A}{U_0}\right)^2 y\left(\frac{1}{\xi}\right)\cos\theta\sin^2\theta. \tag{15}$$

Here $y(t)$ is given by

$$y(t) = Q_2(t)\frac{(t-1)(3t-1)}{2t} - \frac{P_2(t)}{P_2(t_0)}Q_2(t_0)\frac{(t_0-1)(3t_0-1)}{2t_0} - P_2(t)(I(t) - I(t_0)), \tag{16}$$

where

$$I(t) = \frac{3t^2+1}{4t}\ln\left|\frac{t+1}{t-1}\right| + \ln\left|\frac{t^2-1}{t^2}\right|, \qquad t_0 = \frac{r_A}{R}.$$

P_2 and Q_2 are the second-order Legendre functions of the first and second kind, respectively (Abramowitz and Stegun, 1964).

Bogovalov (1992b) shows that the solution (15) is valid for

$$\xi < \frac{1}{\gamma_0} \frac{U_0}{r_A}. \tag{18}$$

This inequality implies that the present solution exists if $r_A < v_0/c$. This means that rotation can be considered slow when the velocity of the rigid rotation of the magnetosphere at the Alfvén surface is smaller than the starting velocity of plasma, v_0.

The structure of the magnetosphere, as given by Eq. (15), is presented in Figure 1 for $(r_A/U_0)^2 = 0.3$. The deflection of the flow toward the axis of rotation due to the magnetic force of the spiraling field lines is seen clearly. Such effect has been discussed earlier by Sakurai (1985) for solar wind and Sulkanen and Lovelace (1990) for pulsars in the massless approximation. It is obvious that the compression of the force lines of the poloidal field toward the axis of rotation is a general phenomenon for rotating magnetized ortators ejecting plasma. It must exist at any angular velocity of rotation. We believe that such an effect of nonuniformity of the plasma flow ejected by a pulsar can play an important role in the interaction of the pulsar wind with the nebula.

In conclusion, I would like to stress one important methodical result of the

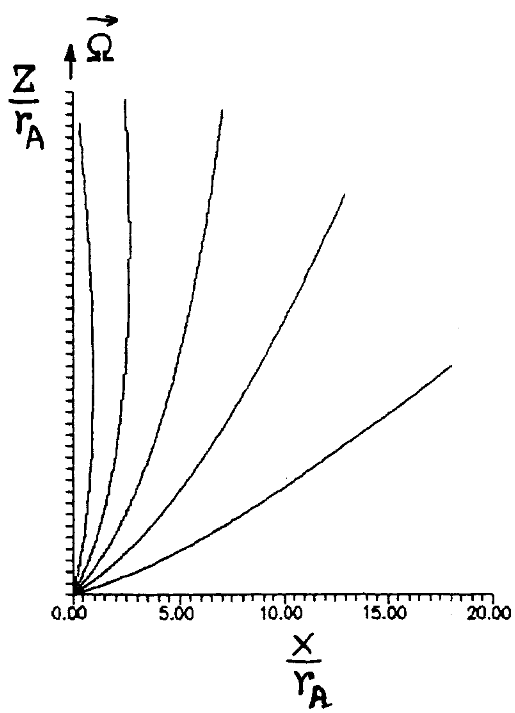

Figure 1 The structure of the poloidal field lines of slowly rotating axisymmetric rotator given by perturbation theory for $(r_A/U_0)^2 = 0.3$.

present solution. When solving the problem of the structure of the magnetosphere of an axisymmetric rotator, it is necessary to know the azimuthal magnetic field generated due to rotation. More than twenty years ago Weber and Davis (1969), investigating magnetohydrodynamics of the solar wind, proposed the hypothesis that the magnitude of the azimuthal magnetic field is determined by the critical condition at the fast magnetosound point. They did not investigate Eq. (7). But this equation has a singularity on the Alfvén surface. On this surface, the left-hand side of Eq. (7) vanishes. The present solution shows that rotation generates such azimuthal magnetic field that the right-hand side of Eq. (7) also vanishes. In other words, the magnitude of the azimuthal magnetic field is not determined by the critical condition as Weber and Davis (1969) supposed. It is defined by the condition of regularity of the equation on the Alfvén surface. For a general case, this result was discussed by Bogovalov (1992a).

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