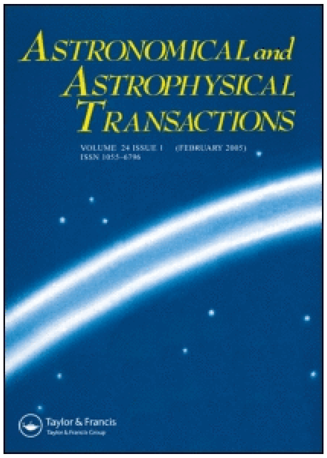

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Experimental study of turbulence in astrophysics

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EXPERIMENTAL STUDY OF TURBULENCE IN ASTROPHYSICS

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The methods of experimental study of turbulence in the ISM (interstellar medium) are discussed. The results of S. A. Kaplan and S. B. Pikelner as well as the theoretical and experimental results influenced by their works are analyzed. The conclusion of the paper is that the investigation of the ISM turbulence through observable fluctuations can be very promising and may give important information about the properties of the ISM.

KEY WORDS Interstellar matter, turbulence, fluctuations.

1. INTRODUCTION

The properties of the interstellar medium strongly suggest that it is turbulent. Turbulence is understood as the unpredictable temporal behavior of nonlinear systems, accompanied by self-organising spatial fluctuations. The spatial fluctuations cover a wide range of size scales.

Turbulence of incompressible fluids in conditions on the Earth is quite well described theoretically, and to some extent has been investigated experimentally. However, astrophysical turbulence is a much more complicated phenomenon. The importance of the turbulence for the understanding of astrophysical processes was recognized long ago (Chandrasekhar, 1949). This type of turbulence differs greatly from the laboratory one (Scalo, 1987; Elmegreen, 1991). Compressibility, shock fronts, clumpy structure, effects of magnetic field and self-gravitation are only some of the specific features of the interstellar turbulence. Injection of the energy from different sources and the dissipation processes influence different scales of the turbulent motion. These processes make the well known description of the turbulence (Kolmogorov, 1941) likely to be unacceptable[1] (see Scalo, 1984).

Statistical methods of investigating astrophysical turbulence have been discussed in a number of papers over the past few decades (Kaplan and Pikelner, 1963). In contrast to the usual use of the mean values in astronomy the authors suggested using fluctuations of the emission intensity. They recognised the

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[1] A picture of hydromagnetic incompressible turbulence (Kraichnan, 1965) is also too simplified to describe such a complex phenomena.

difficulty that particular lines of sight cross many turbulent cells inside the emitting region, so that different turbulent scales contribute to the resulting observable correlation functions. This is why only one possible situation was examined, that where the thickness of the emitting region is much smaller than the characteristic scale of the turbulence (Hoerner, 1951; Pikelner, 1954). The fact that S. A. Kaplan and S. B. Pikelner included these results in their famous books (Kaplan, 1958, 1966; Kaplan and Pikelner, 1963, 1970) contributed to attracting the attention of other scientists to these problems.

It was much later when the method of the turbulence investigation in large emitting volumes was introduced (Chibisov and Lazarian, 1987; Lazarian, 1989, 1991).

At present, different types of turbulence in different astrophysical objects can be studied by the analysis of the emission fluctuations. For example, MHD turbulence can be investigated in the disks and haloes of galaxies and supernova shells, while the random density field and the field of momentum can be studied in the HI disc of the Galaxy and molecular and atomic clouds, just to name a few objects. Turbulence is one of the intrinsic characteristics of interstellar matter. To understand the properties of the ISM one has to know the properties of the turbulence. Just one example. Turbulence can provide support against the gravitational collapse. As it was first pointed out by Chandrasekhar (1951), the stability of a region of some given size δ is determined by the turbulent energy on the scales less than δ. That is why the gravitational stability properties of the medium can depend on the turbulence spectrum, i.e., the relative distribution of power at different scales.

The observational evidence for the turbulence is, for example, the broadening of the observable line profiles due to the Doppler effect. The observable parameters of the emitting volumes of the turbulent medium are fluctuating owing to the statistical nature of the turbulence. These fluctuations can be used to construct the statistical characteristics of the emission field. These characteristics can be used to find the statistical characteristics of the turbulence.

All these different sources of information should be used not only because turbulence in astrophysical conditions should be characterised by many random fields and the ISM (interstellar medium) is a multicomponent medium (see Knapp, 1990; Brinks, 1990; Cox, 1990), but also because possible fractal structure of the ISM can greatly complicate the interpretation of data.

Different parameters of the observable emission can be used in practice. They include fluctuations of intensity, fluctuations of the characteristics constructed from line profiles, fluctuations of different Stokes parameters, for example, polarization, etc.

It is possible to state now that fluctuations in different parameters, to which S. A. Kaplan and S. B. Pikelner tried to attract the attention of the astrophysical community, are a very promising source of information about the ISM.

The structure of the paper is as follows. Statistical characteristics of the astrophysical turbulence are being discussed in Section 2, while some possible observable parameters are mentioned in Section 3. A brief discussion of the methods to obtain the information about turbulence is given in Section 4. Some observational data are discussed in Section 5. Appendix is devoted to investigation of MHD turbulence in thin layers.

2. STATISTICAL CHARACTERISTICS OF TURBULENCE

Turbulence is an incompressible fluid is usually described by the velocity correlation tensor. However, this description is not very accurate when astrophysical turbulence is concerned. The interstellar medium is far from being quiescent, but is rather dominated by the action of supernovae, stellar winds and other violent activity (Cox and Smith, 1974; McKee and Ostriker, 1977; McCray and Snow, 1979). The interstellar gas pressure varies by at least two orders of magnitude and on the time scales of 10^4 to 10^7 years, as a result of supernova explosions, large scale galactic flows and winds from HII regions and OB stars. Owing to the fact that motions in the interstellar medium are supersonic, the compressibility should be taken into account.

The full description of a compressible fluid demands the introduction of six hydrodynamic fields which are connected by the equations of momentum balance, continuity (mass balance), and the energy balance equation, as well as the thermodynamic equation (Monin and Yaglom, 1975).

In the case of the astrophysical turbulence one has to include equations for magnetic field and gravitation. This not only makes difficult the problem of theoretical description of the turbulence (Scalo, 1987) but also complicates the interpretation of the observational data. Although the procedure for study of the complete set of the interrelated random fields characterising the turbulence is still to be constructed in full, a number of statistical characteristics available from observations can be introduced in a rather simple manner.

For example, to characterize the random density field the scalar correlation function of density can be introduced:

$$B(\mathbf{r}, \mathbf{R}) = \langle \delta n_2(\mathbf{x}_1)\, \delta n_2(\mathbf{x}_2) \rangle, \tag{1}$$

where $\delta n_2(\mathbf{x}_i)$ is the variation of the "emitters" density[2] at the position $\mathbf{x}_i$, and

$$\begin{aligned} \mathbf{r} &= \mathbf{x}_1 - \mathbf{x}_2 \\ \mathbf{R} &= \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}. \end{aligned} \tag{2}$$

In the case of uniform and isotropic, at its characteristic scale, turbulence the correlation function depends on r rather than $\mathbf{r}$, (r is the distance between the points). This can considerably simplify analysis of the data. The demand for the random field to be isotropic and uniform at its characteristic scale is stronger than the condition of local uniformity (see Yaglom, 1962), but it is likely to be valid in the astrophysical conditions. In the case of locally uniform turbulence one can introduce the structure functions of the "emitters" density.[3]

$$d(r, \mathbf{R}) = \langle (n_2(\mathbf{x}_1) - n_2(\mathbf{x}_2))^2 \rangle, \tag{3}$$

where r is again the distance between the points.

[2] The term "emitter" is used here in a broad sense. In a number of cases the density of "emitters" can be proportional to the density of the atoms squared. In this sense it is possible to understand the density of "emitters" as the density influencing the signal. It makes it possible to apply the same formalism to absorption lines as well.

[3] In the case of uniform and isotropic at its characteristic scale, turbulence both descriptions are applicable, but one should remember that at small separations the structure function can be measured with greater accuracy.

To characterise the random vector field, e.g. the momentum field or magnetic field, the correlation tensor $B_{ij}(\mathbf{x}_1, \mathbf{x}_2)$ can be used. In the case of the momentum field it can be defined as

$$B_{ij}(\mathbf{x}_1, \mathbf{x}_2) = \langle \delta p_i(\mathbf{x}_1)\, \delta p_j(\mathbf{x}_2) \rangle , \tag{4}$$

where $\delta p_i(\mathbf{x})$ is the fluctuation of the i-th component of momentum at the position $\mathbf{x}$.

The structure tensor characterising a random vector field can be introduced as well:

$$d_{ij}(\mathbf{x}_1, \mathbf{x}_2) = \langle (p_i(\mathbf{x}_1) - p_j(\mathbf{x}_2))^2 \rangle . \tag{5}$$

For the momentum (velocity) field the following representations of $B_{ij}(\mathbf{x}_1, \mathbf{x}_2)$ and $d_{ij}(\mathbf{x}_1, \mathbf{x}_2)$ are valid (von Karman, 1937):[4]

$$B_{ij}(\mathbf{r}, \mathbf{R}) = [B_l(r, \mathbf{R}) - B_t(r, \mathbf{R})]\frac{r_i r_j}{r^2} + B_t(r, \mathbf{R})\, \delta_{ij} , \tag{6}$$

$$d_{ij}(\mathbf{r}, \mathbf{R}) = [d_l(r, \mathbf{R}) - d_t(r, \mathbf{R})]\frac{r_i r_j}{r^2} + d_t(r, \mathbf{R})\, \delta_{ij} , \tag{7}$$

where $B_l(r, \mathbf{R})$, $d_l(r, \mathbf{R})$ and $B_t(r, \mathbf{R})$, $d_t(r, \mathbf{R})$ are the longitudinal and transversal correlation and structure functions, respectively. These functions characterize the turbulence of a random vector field, which can vary with the vector $\mathbf{R}$ while it points to different parts of the emitting object.

If the random field is a solenoidal one, as, for example, the magnetic field, the longitudinal and transversal correlation and structure functions are not independent but the following relation is valid (e.g., Monin and Yaglom, 1970):

$$B_l(r, \mathbf{R}) - B_t(r, \mathbf{R}) = -\tfrac{1}{2} r \frac{dB_l(r, \mathbf{R})}{dr} \tag{8}$$

In many interesting astrophysical cases there exist scale separation in the sense that the correlation or structure functions vary with $\mathbf{R}$ at the scale of the object itself, which is much greater than the characteristic scale of the turbulence for which the functions vary with r.

The turbulence characteristics are of importance when only fluctuations of the observable parameters are available.

3. OBSERVABLE PARAMETERS

The fluctuations of different parameters are avilable from observations. Let us take emission in lines as an example of such a parameter.

If we are concerned with the velocity field in a layer, its component along the line of sight $v_l(\mathbf{e})$ can be measured directly (Kaplan and Pikelner, 1970) and the

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[4] Slightly more complex expressions can be introduced in the case of a turbulence having cylindrical symmetry (Batchelor, 1946).

following structure function is available:

$$D_v(\mathbf{e}_1, \mathbf{e}_2) = \tau_v \langle (v_l(\mathbf{e}_1) - v_l(\mathbf{e}_2))^2 \rangle, \tag{9}$$

where τ_v is a constant factor proportional to the number of "emitters."[5] The component of random velocity $v_l(\mathbf{e})$ which is parallel to the line of sight can be measured through the Doppler shift of the line at the direction $\mathbf{e}$.

As for other parameters, some of them have been discussed elsewhere (Chibisov and Lazarian, 1987; Lazarian, 1990, 1991a). There can be fluctuations of intensity $\delta S_0(\mathbf{e}, w_0)$, or higher moments constructed from the observable line profiles $\delta S_i(\mathbf{e}, w_0)$ (Lazarian, 1991b):

$$S_i(\mathbf{e}, w_0) = \int_{-a}^{a} J(\mathbf{e}, w_0, \delta w)(\delta w)^i \, d\delta w, \tag{10}$$

where a determines the fraction of the line profile which is used for the analysis. It can correspond to a small fraction of the profile in the case of HI disc investigation (Lazarian, 1991b) or can formally go to infinity incorporating the whole line in some other cases, The quantity $J(\mathbf{e}, w_0, \delta w)\, d\delta w$ is the energy of the photons moving in the direction $\mathbf{e}$ whose frequency lies within the element $d\delta w$ about $w = w_0 + \delta w$. The unshifted line frequency is w_0. In many cases the emission can be considered relatively constant over a frequency band equal to the intrinsic line width, which is normally several orders of magnitude smaller than the Doppler width of the line profile (see Spitzer, 1978).

There can be other types of fluctuations (Kaplan and Pikelner, 1970), for example, the polarization fluctuations. These types of fluctuations and their use for the investigation of the turbulence will be discussed in a separate paper. Returning to the fluctuations $\delta S_i(\mathbf{e})$,[6] it is possible to construct the structure function of the appropriate line emission parameters:

$$D_{ij}(\mathbf{e}_1, \mathbf{e}_2) = \langle [S_i(\mathbf{e}_1, w_0) - S_j(\mathbf{e}_2, w_0)]^2 \rangle, \tag{11}$$

where angular brackets $\langle \dots \rangle$ are used to denote the ensemble average. In reality, one has to use spatial averaging (the characteristic time for the astrophysical turbulence at the parsec scale is by far too long to enable application of the time averaging). Although there is a number of problems concerning spatial averaging (Scalo, 1984), it is the only way to get statistical information about astrophysical turbulence. That is why one should be rather suspicious about the cases when the structure functions are obtained using just one realization on the sky. It can have some justification only in the case when we consider that characteristic size of the turbulence is much smaller than the thickness of the object and the turbulence is

[5] In the two simplest cases the number of "emitters" can be proportional to the density of molecules or to the density squared.

[6] The use of the fluctuations $\delta S_i(\mathbf{e})$ requires high sensitivity of the instrument (telescope). The fluctuations are measured against a relatively strong background of the "mean emission". The smaller are the scales under consideration, the higher is the sensitivity needed to find the fluctuations. To measure the structure function $D_{ii}(\theta)$ with the same ambiguity at different θ, one has to increase the sensitivity proportionally to $1/\theta$. At the same time, the difference in the relative intensity of the fluctuations at different scales can be used to separate contributions from the small and large scale fluctuations, which can be useful in the case of turbulence showing a strong anisotropy at the largest scale.

uniform along the line of sight.[7] The latter condition is rather doubtful for the majority of astrophysical emitting objects, where characteristics of the turbulence are likely to change, for example, from the center to the edges as in the case of molecular clouds.

Different moments $S_i(\mathbf{e})$ are useful not only for investigation of various random fields like those of density and momentum as in the case of the first and the second moments of the line profile, but the difference between the correlation functions of even and odd moments of line profiles can help to investigate separately different types of the random behavior of the turbulent motion (in a general sense discussed above). If the line broadening is due to the superposition of Alfvén waves with random phases and amplitudes, then the expected line broadening should be symmetric and the correlation functions of all the odd moments of the line vanish.

In the case of ordinary turbulence accompanied by the displacements of the emitting matter or in the case of the shock fronts superposition, the correlation function of the odd moments of the line profile is not supposed to be zero. At the same time, the correlation functions of the even moments are zero. That is why using the even and odd moments of the correlation function it is possible to distinguish statistical characteristics of different nature.

It may happen that in some regions fluctuations in the observed emission are associated not with turbulence, but rather with fragmentation-agglomeration processes. These regions are not expected to give contribution to the structure functions constructed using the first moments of the line profile (characterizing the random momentum field) but to the zeroth-order moments. In this case the physical field of interest is a random density field for which structure functions and spectrum can be found.

The characteristics (11)[8] are useful for the study of the turbulence when the thickness of the object along the line of sight is greater than the characteristic scale of the turbulence.

4. DISCUSSION OF THE METHODS

1. *A Direct Method*

The simplest possible method was introduced by Kaplan (1955). He proposed finding the characteristics of the random velocity field by constructing the velocity structure functions (9). He intended to use the motions of individual molecular clouds. Unfortunately, the ambiguity in the determination of the distances to molecular clouds prevented this method from being widely used for studies of the large scale turbulence in the Galaxy. The improvements in the distance determination could lead to a revival of the method concerned.

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[7] In this case it is possible to consider different parts of the turbulent volume which are of the size of the turbulent eddy as different realizations of the stochastic process.

[8] Similar characteristics can be constructed using different fluctuations of parameters of the observable emission, like synchrotron intensity and degree of polarisation just to name a few.

2. *Investigation of Turbulence in Thin Layers*

The investigation of interstellar turbulence was pioneered by several authors in the fifties (Hoerner, 1951; Pikelner, 1954; Kaplan, 1955). The results of this work are summarized in well-known books (Kaplan, 1958, 1966; Kaplan and Pikelner, 1963, 1970) and often referred to (see Scalo, 1984; Kleiner and Dickman, 1984, 1985a, b). The case of a thin emitting layer l was discussed. This thickness was compared with the distance between the correlating points on the image of the emitting region (y).

It will be shown here that it is important to introduce three characteristic scales: the two discussed previously (l and y) and the characteristic scale of the turbulence λ.

Two cases are of interest when a thin layer is concerned:

$$y \ll l \ll \lambda,$$

and

$$l \ll y; l \ll \lambda.$$

In the first case the scalar field is considered:

$$D_0(\theta, \mathbf{E}) \approx \tau_0\, d(y/\cos\gamma, \mathbf{E}), \tag{12}$$

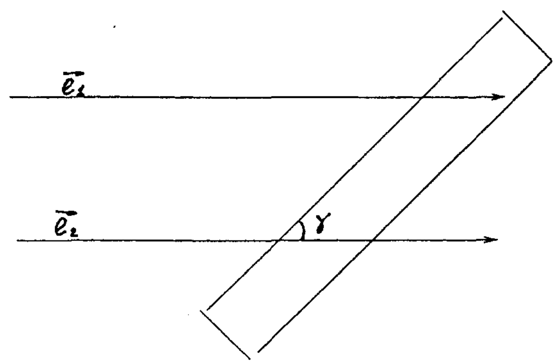
and in the second case

$$D_0(\theta, \mathbf{E}) \approx y^2/l^2 \tau_0\, d(l/\cos\gamma, \mathbf{E}), \tag{13}$$

where $\theta = y/\rho$ (ρ is the distance from the layer), τ_0 is a constant proportional to the number of the emitting particles along the line of sight, $\mathbf{E}$ is a vector in the plane of the layer (which is used instead of $\mathbf{R}$ in the case of a three dimensional object) and γ is the angle between the line of sight and the layer (see Figure 1a).

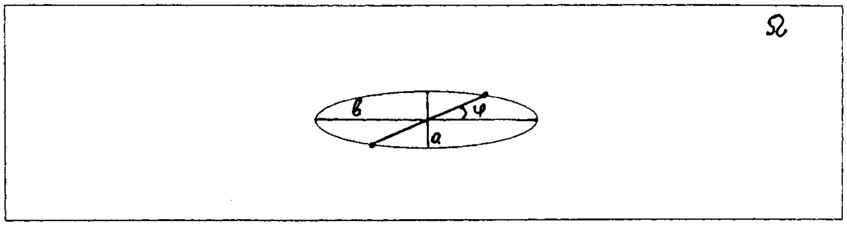
It can be easily seen that the first case is of major importance for the investigation of the turbulence in thin emitting astrophysical layers.

Similar relations are valid for the correlation functions as well. In the case of a vector field the answer depends on which component of the vector influences the observable parameters. For example, when the momentum field is concerned, the



(a)

Figure 1a The investigation of inclined layers. $\mathbf{e}_1$ and $\mathbf{e}_2$ are the vectors directed along the lines of sight. They enter the emitting layer at the angle γ.



(b)

Figure 1b Ω is the image of the layer (as seen by the observer). The correlation (structure) functions do not change with φ when the averaging is made along the ellipse with minor axis a and the major axis b. This corresponds to a circle along the actual surface of the layer. That is why the inclination of the layer is given by $\sin\gamma = a/b$.

component of the velocity parallel to the line of sight influences the resulting observable structure functions of the type (9).[9] If the layer forms an angle γ with the line of sight, the following expression is valid when $y \ll l \ll \lambda$ (see Figure 1a):[10]

$$D_1(\theta, \mathbf{E}) = \tau_1[d_l(\rho\theta/\cos\gamma, \mathbf{E}) - d_t(\rho\theta/\cos\gamma, \mathbf{E})\sin^2\gamma] + \tau_1\, d_t(\rho\theta/\cos\gamma, \mathbf{E}). \tag{14}$$

With the use of the solenoidality condition (8), we obtain in the case when the compressibility of the two dimentinal turbulence is ignored:[11]

$$d_l(r, \mathbf{E}) = \frac{2r^{-2/(1-\beta)}}{\tau_1(1-\beta)} \int_\infty^r x^{\beta+1/1-\beta} D_1\left(\frac{x}{\rho}\cos\gamma, \mathbf{E}\right) dx, \tag{15}$$

$$d_t(r, \mathbf{E}) = \frac{2\beta r^{(-2/1-\beta)}}{\tau_1(1-\beta)^2} \int_\infty^r x^{\beta+1/1-\beta} D_1\left(\frac{x}{\rho}\cos\gamma, \mathbf{E}\right) dx + \frac{1}{\tau_1(1-\beta)} D_1\left(\frac{r}{\rho}\cos\gamma, \mathbf{E}\right), \tag{16}$$

where

$$r = \rho\theta/\cos\gamma, \tag{17}$$

$$\beta = \sin^2\gamma. \tag{18}$$

The angle γ can be found by the special procedure of *ellipsometry*. The idea is very simple. Imagine an emitting layer with uniform and isotropic turbulence.[12] When the layer is perpendicualr to the line of sight, the rotation of both correlating lines through any angle φ (when the angular distance of the

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[9] The example of synchrotron emission which is proportional to the component of magnetic field perpendicular to the line of sight is discussed in the Appendix.

[10] The idea is shown on the example of $D_1(\theta, \mathbf{E})$, while similar calculations can be done for $D_i(\theta, \mathbf{E})$ and $K_i(\theta, \mathbf{E})$.

[11] In the case of a compressible turbulence one has to use a condition which takes into account the correlation between the field of momentum and the density field. This will be discussed elsewhere.

[12] It can be the only visible layer of a large turbulent volume.

correlating points does not change) does not change the resulting structure function.

If the layer is inclined by the angle γ (see Figure 1b), the correlation function will depend on the angle φ. By changing the angle θ with φ, it is possible to find the ellipse[13] moving along which the structure functions do not vary, i.e.

$$D_i(\theta, \varphi) \equiv \text{const.} \tag{19}$$

The ratio of the minor axis (a) of the ellipse to the major one (b) gives us $\sin\gamma$:

$$\sin\gamma = a/b. \tag{20}$$

Using this expression for $\sin\gamma$, it is possible to solve equations (15) and (16) and to find longitudinal and transversal correlation and structure functions, respectively. If the emitting layer varies in thickness, the thickness still being much less than the characteristic scale of the turbulence, it is possible to take this fact into account during the averaging procedure by using $S_i(\mathbf{e})/l(\mathbf{e})$ instead of $S_i(\mathbf{e})$ in definitions of the structure function (9).[14]

There are a number of objects which can be investigated by the method discussed. Observations of the CI fine structure line or CO ultraviolet bands and other density-sensitive atomic and molecular transitions indicate that many diffuse clouds have thickness much less than their transverse dimensions. Some of the 21 cm emitting regions were found to be very thin (~0.03 pc), resembling compressed layers behind shock fronts (Giovanelli *et al.*, 1978). Investigations of two dimensional turbulence in such layers is possible with the use of the method presented here. The method is also applicable to molecular clouds where only a thin layer is observable due to absorption of the radiation, or when fluctuations in a thin layer are observable against a uniform emitting background.[15]

If the velocity field pattern is determined as in the case of an expanding cloud with the velocity increasing outward, one can use the Zeeman splitting (Heiles *et al.*, 1992) to determine the magnetic field in different "velocity layers". This can result in obtaining the magnetic correlation functions with the method discussed (it is even a more simple case because the solinoidality condition (8) is exact for a magnetic field), provided the characteristic scale of the magnetic turbulence is much larger than the characteristic scale of the turbulence of the velocity field. These two characteristic scales can be determined directly from observations.

Ideas of S. A. Kaplan concerning investigation of the magnetic fields using the synchrotron intensity fluctuations in the Galaxy have been developed lately. A brief description of the results obtained is given in the next section and Appendix.

3. *Investigation of Three Dimensional Turbulence*

When the thickness of the turbulent volume along the line of sight exceeds the characteristic scale of the turbulence, many scales contribute simultaneously to

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[13] In the case of a complex curved layer the curve can be more complex.

[14] If the velocity correlation (structure) functions are examined, all the previous results (15) and (16) hold and there is no real need to take into account the variations of the thickness of the emitting layer.

[15] The later case is much more difficult for practical investigation.

the resulting correlation functions.[16] However, in the case of turbulence having spherical or cylindrical symmetry at its characteristic scale, the inverse problem can be solved for many physically interesting cases and statistical characteristics of the turbulence can be found through statistical characteristics known from observations. These properties are rather natural for the turbulence in the ISM. In the case of the energy supply at smaller scales due to stellar winds, supernova explosions etc. or when the turbulence in molecular clouds is concerned, one would expect the turbulence having spherical symmetry at its characteristic scale, while in the case when some large-scale flow is responsible for the turbulisation of the ISM the turbulence could be cylindrically symmetric at its larger scales. Estimates show that in many cases the phenomena at the smallest scales are a major source of turbulence. For example, the differential rotation is responsible for just one percent of turbulent energy in the HI disc (Ruzmaikin *et al.*, 1988). The rest of the energy comes from supernova explosions and stellar winds.

The cases of turbulence having different symmetry properties can be distinguished by analyzing the symmetry properties of the observable correlation functions. The lack of spherical symmetry of the turbulence should result in the dependence of the structure functions on the position angle φ.[17] Little is known about the symmetry properties of the turbulence. One could expect spherically symmetric turbulence (symmetry at the characteristic scale of the turbulence) occurring in the many molecular clouds, while some indications of anisotropy are discussed in the case of the HI disk (Baker, 1973). This indication cannot be considered as strictly proven, because the difference in absorption for different wings of the 21 cm line profile due to rotation of the ambient gas as well as a few other effects like possible influence of clouds outside the disc have not been discussed. The data that the characteristic scale of the turbulence is much smaller than the size of the emitting molecular cloud (see Roy and Joneas, 1985), indicate that the global properties of the molecular clouds cannot be very important for the local properties of the turbulence and it can be considered isotropic and uniform at its characteristic scale. Even in the case when, due to some reasons, turbulence is lacking either type of symmetry at its characteristic scale, the solution of the inverse problem is useful, because it yields the spectrum of isotropic or axisymmetric turbulence required to produce the observable spectrum of fluctuations.

A number of solutions have been found for different types of turbulence (Chibisov and Lazarian, 1987; Lazarian 1989, 1990, 1991). Let us discuss the random density field as an example (Lazarian, 1991b):

$$c(r, \mathbf{E}) = -\frac{1}{2\pi\tau_0^2\rho}\int_{r/\rho}^{\infty}\frac{d\theta}{\sqrt{\theta^2 - r^2/\rho^2}}\frac{d}{d\theta}[D_{00}(\theta, \mathbf{E})], \tag{21}$$

[16] This complicates analysis in comparison with the case of thin layers. Kaplan and Pikelner gave a few estimates how the thickness of the emitting volume could result in a change of the observable spectrum of fluctuations, but analytically this case has been discussed only recently (see Lazarian, 1990a).

[17] However, such a dependence can be due to other geometrical and physical properties of the emitting object.

where D_{00} is the structure function of intensity (see (11)),[18] or

$$c(r, \mathbf{E}) = \int_{\rho}^{\rho+\delta} B_0(r, x, \mathbf{E})\, dx, \tag{22}$$

with δ the thickness of the emitting object, ρ the distance to the observer and $\mathbf{E}$ a vector in the pictorial plane of the object.[19] Turbulence characteristics can be different in different parts of the object which are determined by the coordinate x and the vector $\mathbf{E}$. The solution can be rewritten in the following form:

$$c(r, \mathbf{E}) - \beta c(\beta r, \mathbf{E}) = -\frac{1}{2\pi\tau_0^2\rho} \int_{r/\rho}^{\infty} \frac{d\theta}{\sqrt{\theta^2 - r^2/\rho^2}} \times \frac{d}{d\theta}[D_{00}(\theta, \mathbf{E}) - D_{00}(\beta\theta, \mathbf{E})] \tag{23}$$

If for some β, the difference $[B_0(r, \rho, \mathbf{E}) - \beta B_0(\beta r, \rho, \mathbf{E})]$ does not depend on ρ, for these β we have[20]

$$c(r, \mathbf{E}) - \beta c(\beta r, \mathbf{E}) = \delta(B_0(r, \rho, \mathbf{E}) - \beta B_0(\beta r, \rho, \mathbf{E})). \tag{24}$$

Solution (21) corresponds to the case of $\beta \to \infty$ in the solution (23). Similar solutions are valid for the field of momentum. They will be discussed elsewhere.

Not only correlation functions can be found through the solution of the inverse problem. Analytical solutions exist for the spectrum of the turbulence (see Lazarian, 1989). In the case of the random density field the following solution is valid:

$$\int_{\rho}^{\rho+\delta} E_{00}(r, x, \mathbf{E})\, dx = 1/8\pi\tau_0^2\rho \int_0^{\infty} d\theta [D_{00}(\infty, \mathbf{E}) - D_{00}(\theta, \mathbf{E})]\theta J_0(t\rho\theta), \tag{25}$$

where $J_0(z)$ is the Bessel function of the zeroth order. The infinity can be used as the upper limit in (25) owing to the rapid decrease of $[D_{00}(\infty, \mathbf{E}) - D_{00}(\theta, \mathbf{E})]$ with θ. This is why the spectrum of the turbulence is the Fourier-Bessel (Hankel) transform of the intensity correlation function. Here only a rather simple case of the random density field is discussed. Similar solutions have been found for the fields of momentum and momentum-density. They will be discussed elsewhere.

4. *Indirect Methods*

These methods are the major source of information on astrophysical turbulence at the moment (see Armstrong *et al.*, 1981a; Ruzmaikin *et al.*, 1988). They use, for example, temporal fluctuations of the radio signal in the case of interplanetary

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[18] It can happen to be difficult to find $D_0(\infty)$ from the function $D_{00}(\infty)$ due to a relatively slow variation of $D_{00}(\infty)$ with θ. In this case the value $D_0(\infty)$ can be found by direct averaging of intensity fluctuation squared, $D_0(\infty) = 2\langle (S_0(\mathbf{e}, w_0) - \langle S_0 \rangle)^2 \rangle$.

[19] If a spatial averaging is used, the vector $\mathbf{E}$ characterises the strip over which the averaging procedure has been performed.

[20] $B_0(r, \rho, \mathbf{E})$ can be found also in a number of cases when, for example, the information about symmetry properties of the object is available or observations of different emission lines are used. In the case of the turbulence in the HI disc of the Galaxy the known rotation curve can be used to distinguish between the emitting volumes (Lazarian, 1991b).

turbulence (Coles and Kaufman, 1977), or dispersion of pulsar signals in the case of larger scales (Backer, 1974). Usually only complex integrals of the statistical characteristics of the ISM are available. To find any estimate of the characteristics of the turbulence, some assumption concerning isotropy and uniformity is adopted. It should be noted, however, that unlike the cases considered in the previous section where the isotropy at the turbulence characteristic scale is required to obtain solution of the inverse problem, here a global isotropy is required. This makes the results of such statistical analysis rather unreliable.

As it was mentioned earlier (see Kaplan and Pikelner, 1963), superposition of the contributions from different turbulent scales can make the observable fluctuations spectrum completely different from the actual spectrum of the turbulence. Similarly, much caution is needed when a result of superposition of the contributions from regions with completely different physical conditions is discussed.

Much has been done on the study of the velocity dispersion dependence on the region size for molecular clouds (Larson, 1981). Although important, such approach cannot give the local turbulence characteristics. Another reservation should be made that unambiguous interpretation of the linewidth-scale relation is quite difficult (Scalo, 1984). Different processes other than turbulence like, for example, the collapse or global oscillations may result in relations similar to those observed. That is why one should be rather cautious about the "density spectrum behaving as $(wavenumber)^{-3.6\pm0.2}$ over 12 decades of the scale size range ($\sim$100 pc to $\sim 10^7$ m)" (Armstrong *et al.*, 1981b), which is found using results of such a study.

The approach suggested by Kaplan and Pikelner and based on the use of the structure (correlation) functions is much more reliable in many cases than any indirect method. However, it involves complications due to the requirement of the sufficient spatial resolution and high sensitivity. It can be believed that it was due to the technological problems that this attitude is still widespread. To characterize the interstellar turbulence, many interdependent random fields should be introduced. That is why it is so important to use any possible channel of getting information about astrophysical turbulence. Future research should use different methods in order to find statistical characteristics of different phases of the ISM (interstellar medium).

5. ANALYSIS OF OBSERVATIONAL DATA

1. *Study of Turbulence in Thin Layers*

The structure functions of the velocity field were first obtained by Hoerner (1951) and Munch (see Munch, 1955) for the Orion Nebula. Their data is also referred to by Kaplan and Pikelner (1970) (see Figure 2).

These data are usually thought to contradict the hypothesis of turbulent ISM. The usual way of thinking is as follows: if only a very thin layer of the Nebula was observed, relation (12) is valid, but in this case the observable velocity range should be very small, $D(r) \sim (\varepsilon r)^{2/3}$ according to Kolmogorov's theory, where ε is the rate of energy transfer through the cascade of vortices. If the layer is thick,

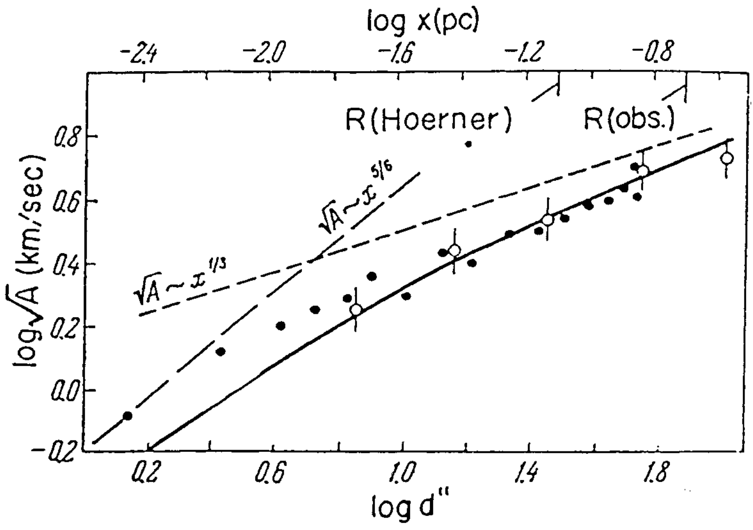

Figure 2 Structure functions for the velocity field in the Orion Nebula according to Hoerner (open circles) and Munch (filled circles). The figure is taken from Kaplan and Pikelner (1963). The line $\sqrt{A} \sim x^{1/3}$ corresponds to a thin emitting layer in the case of the Kolmogorov turbulence, $\sqrt{A} \sim x^{5/6}$ is a result of numerical simulations which corresponds to the spectrum in the thick-layer case.

numerical calculations, based on the Kolmogorov law, show a spectral index different from that observed.

Although they seem to be convincing, these arguments do not take into account some very important points. For one thing, when discussing observational astrophysical data one should be very careful in applying Kolmogorov's law. Another reservation is the fact that in a very thin observable layer the velocity range can be very wide. If the observable layer is a slice of the turbulent volume, its visual thickness does not impose any limits on the observed velocity range if the region is active and expanding.

This example shows that interpretation of observational data concerning turbulence needs much care.

As for the brightness fluctuations, the first set of practical studies was conducted by Pikelner and Shajn (1954). The authors made an estimate of the turbulence spectrum in the case of a thin emitting layer and a thicker one. Unfortunately, no multifrequency observations have been conducted yet to distinguish between these two cases. These two cases discussed could be separated by observations of different emission lines.

2. *The Study of Turbulence in Astrophysical Volumes*

S. B. Kaplan was the first to propose the use of synchrotron fluctuations for the investigation of the magnetic field of the Galaxy. This program has begun to being fulfilled only in the eighties (Ptuskin and Chibisov, 1980; Dagkesamansky and Shutenkov, 1987; Chibisov and Lazarian, 1987; Lazarian and Shutenkov, 1990), when an adequate mathematical formalism of solving the inverse problem

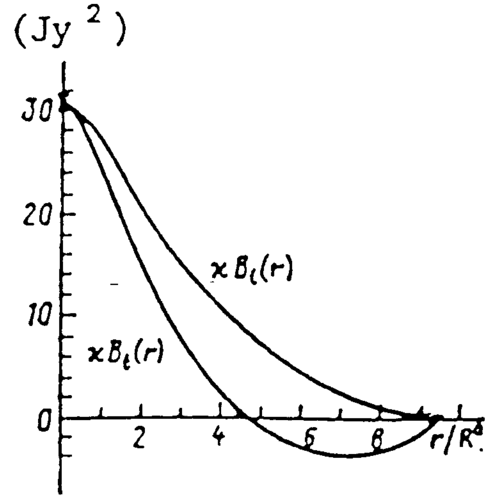

Figure 3 The longitudinal $B_l(r)$ and transverse $B_t(r)$ correlation functions of the random magnetic field (Lazarian and Shutenkov, 1990) plotted against the fraction of the distance between the correlating points to the size of the emitting region. The two functions emerge due to the vector nature of the magnetic field.

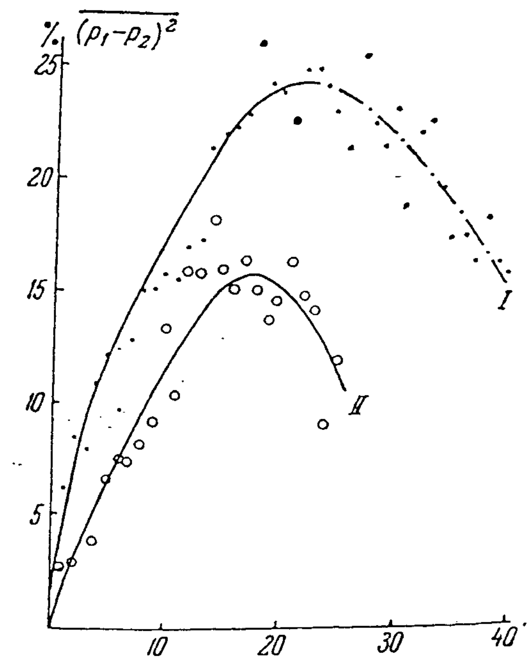

Figure 4 The structure function for the degree of interstellar polarization toward the galactic anticenter and galactic center (Klimishin and Bazilevich, 1963) (from Kaplan and Pikelner, 1963).

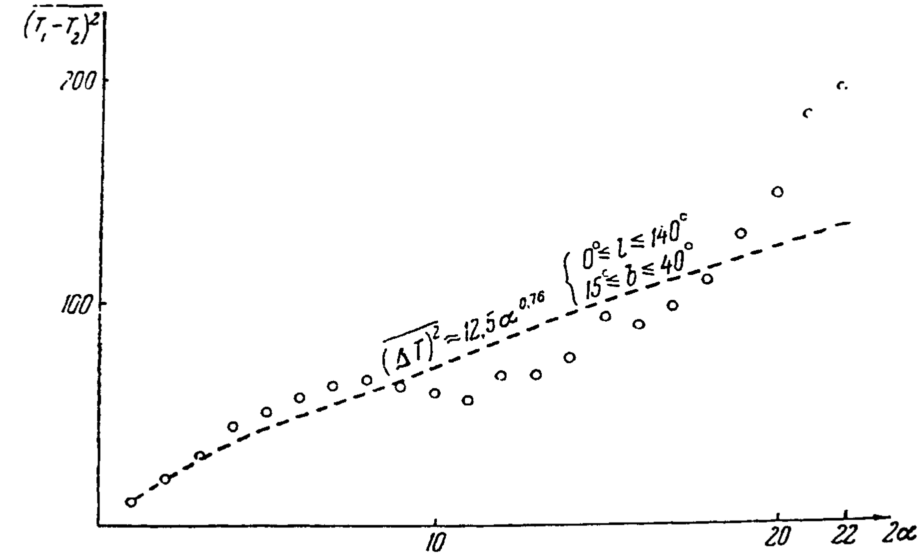

Figure 5 The structure function of the fluctuations of nonthermal radio emission at $\lambda = 75$ cm (Klimishin and Bazilevich, 1963) (from Kaplan and Pikelner, 1963).

for the turbulence was developed. Results for the longitudinal and transversal correlation functions of the random magnetic field are shown in Figure 3. The development of the methods of investigating turbulence in astrophysical volumes was slowed down by the fact that the first observational results on the fluctuations of polarization (Figure 4) and synchrotron intensity (Figure 5) seemed to contradict any reasonable assumptions about their possible behaviour. For example, the decrease in the structure function of polarization at large angular separations was not understood, neither was the increase of the structural function of synchrotron emission.

More careful analysis shows that this behaviour is due to the specific geometry of the lines of sight.

Let us consider first the case of synchrotron intensity. The lines of sight cross turbulent vortices and cross each other at the point of observation. As the angle grows there are still points on the line of sight, the distance between which is less than the characteristic scale of the turbulence. That is why the structure functions of the observed intensity are constantly growing with the angle between the lines of sight θ.

As for the structure functions of the polarization degree one should take into account the fact that the polarization of the radiation from one region can be "neutralised" by the radiation polarized in a perpendicular direction coming from another region.[21] That is why if the volume around the observer is surrounded by regions which emit the perpendicularly polarized light, the decrease of the

[21] The superposition of polarized emission from the regions with different directions of the polarization can be unpolarized.

correlation between the points in these regions results in the increase of the observable correlation of the polarized intensity. Thus the structure functions of the polarization degree can decrease with the angle θ increasing. These arguments should not be considered as any proof of the results obtained, but are to show only that the results do not contradict the existence of turbulence in the ISM. New, more reliable and modern measurements are needed.

Carefully taking into account the geometry of the lines of sight, it is possible to clarify the three-dimensional structure of the ISM. One case of such a tomography is mentioned in a recent paper of Chibisov and Lazarian (1991b). Further discussion will be done elsewhere. Turbulence investigation in the ISM is only at its infancy. Much should be done before the statistical properties of astrophysical turbulence will be well understood even in the case of our Galaxy. For example, turbulence can have a number of characteristic scales, the superbubbles (see Norman and Ikeuchi, 1989) representing the largest scale of a few hundred parcecs, while irregularities with the correlation size ~ 7 pc (Baker, 1973) can correspond to the characteristic size of the typical cloud in the HI disc (~ 5 pc according to Baker and Burton, 1975). The turbulence characteristics are likely to be different in different parts of the Galaxy. One can expect to find peculiar characteristics in star forming regions (McKee *et al.*, 1992).

Only simultaneous analysis of the fluctuations of different parameters can help to obtain an adequate picture of the turbulence in the ISM. There are already some indications that fluctuations of different observable parameters are used simultaneously to recover the structure of magnetic fields (Myers and Goodman, 1991, Heiles, 1992). This trend is likely to continue and be used with a wide variety of different kinds of fluctuations.

6. CONCLUSIONS

The investigation of turbulence through the fluctuations of the ISM emission is still very important even after forty years since S. A. Kaplan and S. B. Pikelner proposed to use them for this purpose.

At the time when S. A. Kaplan and S. B. Pikelner were involved in this research, it was generally believed that astrophysical turbulence should be analogous to the turbulence in the laboratory. The problem was to confirm the assumption of the turbulent nature of the ISM by finding, for example, the Kolmogorov law for the turbulence spectrum. Any discrepancies with this law were considered unacceptable for the turbulent behaviour. However, nowadays the ISM turbulence is found to be a much more complicated phenomenon. The statistical characteristics are needed not to prove or disapprove the turbulent nature of the ISM, but to find the properties of the turbulence, which can differ greatly from those in the laboratory.

The improvement in the resolution power of the telescopes, the VLA interferometry and synthesis techniques makes available the high spatial resolution which one needs for the investigation of the turbulence. This gives confidence that the fundamental ideas of these two men will soon be implemented.

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APPENDIX THE INVESTIGATION OF THE MHD TURBULENCE

In the main sections of the paper the turbulence in the atomic and molecular medium were discussed. The line emission was made use of. But this emission is not the only type of emission which can be used for this purpose. For example, it is possible to discuss the synchrotron emission from thin supernova shells and filaments. The case of three dimentional MHD turbulence is discussed elsewhere (Chibisov and Lazarian, 1991; Lazarian and Shutenkov, 1990).

For the sake of simplicity consider the intensity of the synchrotron emission proportional to the square of the magnetic field intensity. This means that the spectral index of the cosmic rays is considered to be equal to 3, which is close to the real spectral index of the cosmic rays in the Galaxy (Ginzburg and Syrovatskii, 1964).

The magnetic field in the synchrotron emitting layer is considered to consist of the regular, **H**, and random, **h**, components.[22]

The correlation function of intensity,

$$K(\mathbf{e}_1, \mathbf{e}_2) = \langle \delta I(\mathbf{e}_1)\, \delta I(\mathbf{e}_2) \rangle, \tag{26}$$

(δI is a variation of the synchrotron intensity) consists of isotropic and anisotropic parts (Chibisov and Ptuskin, 1980):[23]

$$K(\theta, \varphi) = K_0(\theta) + K_2(\theta) \cos(2\varphi), \tag{27}$$

where φ is the angle between the vectors $\mathbf{e}_1$, $\mathbf{e}_2$ and the direction of the regular magnetic field **H**.

For K_0 and K_2 the following expressions are valid:

$$\bar{K}_0 = 2B_l^2(r) + 2B_t^2(r) + 4H_n^2 B_t(r), \tag{28}$$

$$\bar{K}_2 = 4H_n^2(B_l(r) - B_t(r)), \tag{29}$$

where

$$\bar{K}_i = K_i/\tau_h, \tag{30}$$

τ_h is a constant factor proportional to the density of the relativistic electrons and the thickness of the emitting region, H_n is the component of the magnetic field, perpendicular to the line of sight, $i = 0, 2$.

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[22] Special procedure of averaging should be used in the case of complex topology of the regular magnetic field (Lazarian, 1989).

[23] The layer is considered to be perpendicular to the line of sight. If the layer is not perpendicular to the line of sight, the higher harmonics appear.

A relation similar to (27) is valid for the structure function of the synchrotron intensity as well (Lazarian, 1989):

$$D(\theta, \varphi) = D_0(\theta) + D_2(\theta)\cos(2\varphi). \tag{31}$$

The expression for D_0 is rather complex, while the expression for D_2 is similar to (29):

$$D_2/\tau_h = -8H_n^2[B_l(r) - B_t(r)]. \tag{32}$$

The use of the expressions (30) or (31) gives the opportunity to find the characteristics of the random magnetic field in thin synchrotron emitting shells and filaments in the ISM. Practically one should use the solenoidality condition (8) along with (29) to find the longitudinal and transversal characteristics of the correlation or structure functions from their difference:

$$B_l = -\frac{1}{2H_n^2}\int_\infty^r \bar{K}_2/x\, dx, \tag{33}$$

$$B_t = -\frac{1}{2H_n^2}\int_\infty^r \bar{K}_2/x\, dx - \frac{\bar{K}_2}{4H_n^2}. \tag{34}$$

Similar results can be obtained from equation (32) with the use of the solenoidality condition (8).

The value of H_n^2 can be found using the expression (28):

$$H_n^2 = 1/2\left(\frac{\Phi_1^2 + \Phi_0^2}{\bar{K}_0 + 2\Phi_0}\right)^{1/2}, \tag{35}$$

where

$$\Phi_0 = \int_\infty^r \bar{K}_2/x\, dx + \bar{K}_2/2, \tag{36}$$

$$\Phi_1 = \int_\infty^r \bar{K}_2/x\, dx. \tag{37}$$

These results are of importance for the investigation of the random and regular magnetic field in the thin supernova remnant shells and thin synchrotron emitting filamentary structures in our Galaxy.

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